AC 2009-7: ENGINEERING ECONOMICS: TIME FOR NEW DIRECTIONS?

John Ristroph, University of Louisiana, Lafayette

Engineering Economics: Time for New Directions?

John H. Ristroph
University of Louisiana at Lafayette

Abstract

This paper questions the appropriateness of devoting a significant portion of an engineering economics course to the use of factors, a computational technique originally necessitated by slide rules. An alternative pedagogy is proposed that reduces this component of the course to allow more coverage within engineering economics courses of important topics such as cash flow estimation, as well as benefiting students who only receive an introductory treatment within other engineering courses.

Introduction

The teaching of engineering has evolved over time, and today's students are expected to have an understanding of the phenomena that they model and the means to evaluate those models. Some calculations can be done quickly on a calculator, whereas others, such as least squares curve fitting, typically are done on a computer. In fact, some topics might not be taught at all if computational aides were not available, such as optimization or finite differences. This paper examines the evolution of engineering economics and technology-driven opportunities for improvement.

Factors

A hundred years ago, tables were used to provide the values of interest formulas that were difficult to evaluate using slide rules. The slide rules have disappeared, but not the tables. At first, tables seem to be a convenience, but they require spending valuable classroom time to teach the mechanics of factors. For example, consider the problem shown in Figure 1. Given deposits of $3,000 at times 2, 3, …, 30, what equal amounts can be withdrawn at times 39, 40, …, 63? This is a three-step problem using factors.

1. Determine the equivalent (i.e., the account balance) at time 30:

$$E_{30} = 3,000(F|A, i, 30\text{-}1) \qquad (1)$$

2. Compute the equivalent at time 38:

$$E_{38} = E_{30}(F|P, i, 38\text{-}30) \qquad (2)$$

3. Calculate the final answer:

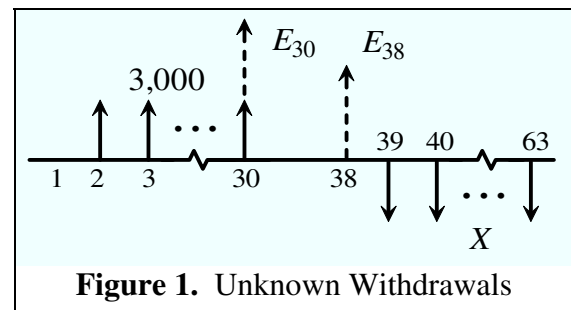$$X = E_{38}(A|F, i, 63\text{-}38) \qquad (3)$$

Figure 1. Unknown Withdrawals

Teaching the students how to solve this problem might involve the following classroom activities:

1. Develop the formulas for F|A, F|P, and A|F.
2. Explain that F|A requires:

a. the equivalent to be placed at the time of the last series flow, and

b. the last parameter to equal the number of series flows, the time of the last flow minus one period before the first flow.

3. Note that F|P has a last parameter equal to the number of periods, the time of the compound amount minus the time of the prior amount.

4. Present why F|A needs:

a. the prior amount placed before the first series flow, and

b. the last parameter to equal the number of series flows, the time of the last flow minus one period before the first flow.

Unknown deposits are shown in Figure 2. The typical three-step solution uses factors different from the unknown withdrawals problem, even though the mathematical relationship between deposits and withdrawals is the same:

$$E_{38} = 3,000(\text{P|A, i, 63-38}) \qquad (4)$$

$$E_{30} = E_{38}(\text{P|F, i, 38-30}) \qquad (5)$$

$$X = E_{30}(\text{A|P, i, 30-1}) \qquad (6)$$

This requires time in the classroom, as do arithmetic and geometric series, but the use of factors saves little, if any, homework time, given the capabilities of modern calculators and computers.
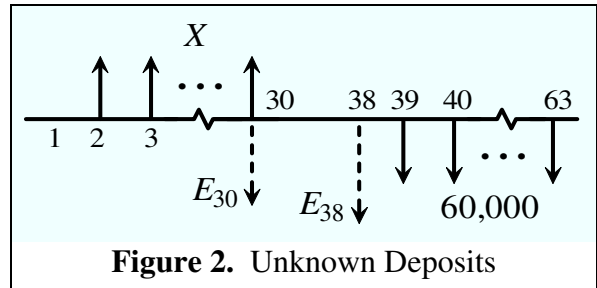

Figure 2. Unknown Deposits

Alternative for Single Payments and Uniform Series

An alternative solution procedure for problems which currently use single payment and uniform series factors builds on early step-by-step examples of compounding that frequently are included in an initial presentation of compound interest. Such numerical examples quickly can be extended to show that an account's compound amount or balance B_n at time n is given as a function of its cash flows c_t at time t as:

$$B_n = \sum_{t=0}^{n} c_u (1+i)^{n-t} \qquad (7)$$

In the unknown withdrawal problem shown in Figure 1, the balance at time 63 after the last cash flow is 0, so:

$$0 = 3,000 \sum_{t=2}^{30} (1+i)^{63-t} - X \sum_{t=39}^{63} (1+i)^{63-t} \qquad (8)$$

The sum of a geometric series is a known from basic calculus courses to be:

$$\sum_{j=a}^{z} r^j = \frac{r^{z+1} - r^a}{r-1}. \qquad (9)$$

Using equation (9) to simplify the sum on the left side of equation (8) begins with letting r equal $1+i$ and observing that the exponent $63-t$ goes from 61 to 33 as t goes from 2 to 30. Since $r^{61} + r^{60} + \cdots + r^{34} + r^{33}$ equals $r^{33} + r^{34} + \cdots + r^{60} + r^{61}$, a and z in equation (9) are 33 and 61, respectively. Similarly, the sum on the right side of equation (8) can be simplified to obtain:

$$0 = 3{,}000 \frac{(1+i)^{62} - (1+i)^{33}}{i} - X \frac{(1+i)^{25} - 1}{i} \tag{10}$$

This results in the easily solved equation:

$$X = 3{,}000 \frac{(1+i)^{62} - (1+i)^{33}}{(1+i)^{25} - 1} \tag{11}$$

Equation (11) can be evaluated using a calculator, probably more quickly than looking up and entering factors. The unknown deposit problem shown in Figure 2 has the same mathematical structure, so it also can be solved by using the balance equation and the formula for a geometric series. This simple approach replaces most of the classroom time spent developing six factors and training students in their use.

Extending the Alternative Approach

The simplicity of this pedagogy extends to computational aspects of problems involving present worth, equivalent annual worth, multiple period series, and gradients. Each of these is examined below and then summarized in Table 1.

Present worth problems require the evaluation of :

$$PW = \sum_{t=0}^{n} c_t (1+i)^{-t} \tag{12}$$

Irregular flows can be evaluated using equation (12) directly, and uniform series can be evaluated by using the formula for a geometric series with r equal to $(1+i)^{-1}$.

Equivalent annual worth expresses present worth as annual flows at times 1, 2, …, n (the end of the planning horizon) that produce the same compound amount as present worth, so

$$PW(1+i)^n = \sum_{t=1}^{n} EAW(1+i)^{n-t} . \tag{13}$$

Applying the formula for a geometric series with r equal to $1+i$ results in the well established relationship between *PW* and *EAW*:

$$EAW = PW \frac{(1+i)^n i}{1 - (1+i)^n} \tag{14}$$

Multiple period series have equal flows that occur regularly, but not every compounding period, such as annual flows with monthly compounding. Let i_p be the rate per compounding period with P compounding periods per cash flow. Problems involving compound amounts contain the sum $\sum_{t=a}^{z} (1+i_p)^{P(n-t)}$, so a geometric series with r equal to $(1+i)^P$ can be evaluated. Similarly, discounting problems contain $\sum_{t=a}^{z} [(1+i_p)^P]^{-t}$ which can be written as $\sum_{t=a}^{z} [(1+i_p)^{-P}]^t$, a geometric series with r equal to $(1+i_p)^{-P}$.

Table 1. Summary of Alternative Pedagogy			
Flow Type	*Time*	*Calculation*	*Notes*
Single Flow	t	Exponent	For PW: $(1+i)^{-t}$ For FW: $(1+i)^{n-t}$
Uniform Series	$a, a+1, ..., z$	$\sum_{j=a}^{z} r^j = \dfrac{r^{z+1} - r^a}{r-1}$	For PW: $r = (1+i)^{-1}$ For FW: $r = 1+i$.
Multiple Period Series	$a, a+P, ..., z$	$\sum_{j=a}^{z} r^j = \dfrac{r^{z+1} - r^a}{r-1}$	For PW: $r = (1+i_p)^{-P}$ For FW: $r = (1+i_p)^{P}$ i_p is the rate per compounding period with equal flows every P periods.
Geometric Gradient	$a, a+1, ..., z$	$\sum_{t=a}^{z} b(1+g)^{t-a}(1+i)^{-t} = b(1+g)^{-a}\sum_{t=a}^{z}\left(\dfrac{1+g}{1+i}\right)^{t}$	Base flows of b at a increases by g. Use preceding formulas with $r = \left(\dfrac{1+g}{1+i}\right)$.
Arithmetic Gradient	$a, a+1, ..., z$	$\sum_{t=1}^{z-a+1} t(1+i)^{-t} = \dfrac{(1+i)^{z-a+2} - i(z-a+2)-1}{i^2(1+i)^z}$	Flows are $d, 2d, ...$ Factor power of $1+i$ to form sum.
EAW or *PW*	$1, 2, ..., n$	$EAW = PW \dfrac{(1+i)^n i}{1-(1+i)^n}$	Convert between economic measures.

Geometric gradients at times $a, a+1, ..., z$ lead to sums such as $\sum_{t=a}^{z} b(1+g)^{t-a}(1+i)^{n-t}$ or

$\sum_{t=a}^{z} b(1+g)^{t-a}(1+i)^{-t}$, where b is the base of the series. If the first sum should be encountered, factor $(1+i)^n$ to express it in terms of the second one:

$$(1+i)^n \sum_{t=a}^{z} b(1+g)^{t-a}(1+i)^{-t} = \sum_{t=a}^{z} b(1+g)^{t-a}(1+i)^{n-t} \tag{15}$$

Then factor $(1+g)^{-a}$ from the second sum to obtain:

$$\sum_{t=a}^{z} b(1+g)^{t-a}(1+i)^{-t} = b(1+g)^{-a}\sum_{t=a}^{z}\left(\frac{1+g}{1+i}\right)^{t} \tag{16}$$

If $i = g$, then $\sum_{t=a}^{z}\left(\dfrac{1+g}{1+i}\right)^{t}$ equals z-a+1; otherwise equation (9) can be used with r equal to $\dfrac{1+g}{1+i}$.

Arithmetic gradients at times a, $a+1$, …, z lead to sums involving the forms

$$s(1+i)^{-a+1} \sum_{t=1}^{z-a+2} t(1+i)^{-t} \text{ for present worth or } s(1+i)^{n-a+1} \sum_{t=1}^{z-a+2} t(1+i)^{-t} \text{ for compound amounts,}$$

where s is the arithmetic rate of change of the series. From calculus it is known that

$$\sum_{t=1}^{z-a+1} t(1+i)^{-t} = \frac{(1+i)^{z-a+2} - i(z-a+2) - 1}{i^2(1+i)^z}, \tag{17}$$

so this type of problem merely requires factoring sums into the form of equation (17).

One interesting aspect of all foregoing solution procedures is that none require explaining how to create equivalents and position them correctly. Equivalence still has meaning in that flows which accumulate to the same compound amount are equivalent, but the use of equivalents as intermediate steps is no longer necessary if calculations are not restricted to tables.

Advantages of the Alternative Pedagogy

Course syllabi on the Internet[1] indicate that it is not uncommon to devote 15% or more of a 3 hour engineering economics course to learning how to use factors. Factors might provide a computational convenience, but calculators reduce their benefit to at most few minutes of homework time. The proposed pedagogy uses standard mathematical notation and series formulas already known from calculus to reduce classroom time by not developing factors, explaining their notation, and teaching their mechanics. Its treatment of series is useful in other engineering disciplines, and its familiar notation reduces the uniqueness of a course in which money is modeled instead of physical properties, thereby decreasing the difficulty that some students have in adjusting to new concepts.

A faster coverage of financial mathematics allows time for including other topics in engineering economics courses. A panel discussion in session 2239 of the 2007 ASEE Annual Convention explored ways to improving engineering economics classes, and the most common recommendation was to include more material on cash flow estimation. One possibility for doing this is to use the extra time to introduce statistical procedures such as regression so that students minimally will know what data to collect and the type of expertise that is necessary.

Another possible inclusion in engineering economics courses is suggested by a review of several popular textbooks[2,3,4,5] that indicates opportunities to provide students with a better understanding of differences between modeling single banking accounts and industrial growth in which reinvestment occurs in a wide range of projects. This is crucial to setting the discount rate, particularly in smaller companies that still use payback or similar measures. A concise coverage can use Thuesen's approach[5] of:

- explaining the investment opportunities curve of capital budgeting, and
- noting that investments with higher internal rates of return tend to be selected first,
- so differences among mutually exclusive alternatives generally increase or decrease funds available to marginal projects having an internal rate around MARR,
- and then showing that selecting the alternative with the largest PW evaluated at MARR results in the choosing the best project.

This takes little time and greatly enhances understanding of industrial economics. If desired, more detailed expositions are available on the Internet[6].

A more subtle, but significant impact of the proposed pedagogy is on the organization of text-books. Books can be designed so that chapters after those on financial mathematics do not depend on a knowledge of factors. For example, a geometric trend can be used in a chapter on inflation without covering $(P|A,g,i,n)$, since the solution logic only needs to use standard mathematical notation to show sums of discounted amounts. A student can simply put the problem on a spreadsheet instead of using the procedures shown in Table 1. This allows professors much greater latitude in choosing topics to include or exclude.

The proposed method also affects students whose only exposure to engineering economics is an introductory coverage in a non-engineering-economics course. Fundamentals can be taught quickly via the use of standard notation, and spreadsheets can be used to bypass the need for presenting computational tools for series. This will allow graduates to recognize the importance of economics in engineering design and to communicate effectively with practitioners using standard mathematical notation. A little knowledge can be dangerous, however, so it is recommended that such instruction inform students of gaps in their knowledge, such as an understanding of industrial criteria, taxes, inflation, or estimating cash flows.

Conclusion

Factors are a relic of a by-gone era. Modern calculators allow the use of standard mathematical notation and series manipulation techniques common to other engineering disciplines. Further, industrial analyses should incorporate taxes and inflation, and this typically results in single cash flows rather than uniform series or gradients, so it is not desirable to spend too much time on those topics. They are still useful, particularly for financial problems involving borrowing and saving, but consideration should be given to using more efficient teaching methods that allow including additional topics in engineering economics courses. Using standard mathematical notation also facilitates their instruction of students in non-engineering-economics course and their communications with practitioners.

What can the Engineering Economics Division do to enable, but not mandate, change? The primary institutional obstacle is the Fundamentals of Engineering Examination. A recommendation by the EED to include the information in Table 1 in the FEE, along with the currently provided tables, is an important first step in letting professors choose which pedagogy they prefer.

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