## Enhancing Math and Stat Courses with Surveying Engineering Problems

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#### Abstract

This paper aims to enhance mathematics and statistics courses taken by engineering students with real life numerical problems and examples. Mathematics and statistics courses instructors often report that students may lack the motivation to study, participate, and engage in their courses. Students often fail to connect Mathematics with real life problems, and they have limited comprehension of how what they learn in class can be applied in real life. To enhance mathematics and statistics courses we aim to enrich instruction and assignments with real life problems from surveying engineering. Surveying engineering is a major with frequent use of trigonometry and statistics problems. Surveyors routinely collect distance and angle measurements to determine coordinates and shapes, and make area and volume calculations. Many of those surveying tasks are related to mapping and boundary determination, concepts that many students are already familiar with. In addition, surveyors collect measurements and use statistics to check accuracy requirements and ensure that their instruments are performing according to manufacturer standards. The introduction of real-life surveying problems into mathematics courses answers the question so many mathematics instructors hear every day, "where am I going to use this?". The WIP paper will discuss the importance of students connecting math with real life problems, provide examples of surveying problems in mathematics and statistics, present the courses that we have identified for integrating surveying content, and planned implementation and dissemination. Furthermore, surveying is a profession with a low public profile and only a few accredited programs exist in the US. Thus, as a secondary objective, engineering students of any major will learn about surveying and be exposed to real surveying problems. This can help students identify themselves as future surveyors and aid in recruitment and enrollment.


## Introduction

This is a work in progress (WIP) paper that aims to enhance mathematics courses such as college algebra, trigonometry, and calculus with real life (RL) problems from surveying which, for instance, incorporate area, volume, and slope calculations. Mathematics and statistics instructors often complain that students are less motivated in their classes as it is difficult for them to relate their math problems to RL problems [1], [2], [3]. It is not a surprise that students often believe that math is boring, difficult, and useless, which indicates why students lose interest in these courses, leading to underachievement [4]. An important reason why students lose interest is because they do not see a connection between what they learn with real science and engineering problems [5]. Traditional word problems without real-world context are lacking in assigning a significant role to the problem and in developing meaning for mathematics [6], [7]. In addition, students often approach math courses with fear and anxiety [2], [3]. For instance, students who reported higher levels of math anxiety correlated to lower levels of performance than peers who
stated lower levels of math anxiety [8]. Their anxiety has even reached a popular cartoon strip. The very famous cartoonist Gary Larson's "Hell's Library "has an entire library of mathematical word problems with the devil as the head librarian [1]. Another example of word problem dismay is [9] who describes word problems as a deep space "Black Hole" in which students enter and never come out when attempting a RL problem. It is interesting to note that student anxiety or aversion to mathematics can begin as early as middle school, believing that certain students are good at math and therefore their efforts will not make any difference [10], [11]. This perception can follow students through their college education limiting their motivation and success in math and statistics. Mathematics and Statistics serve as foundation for all programs in Science, Technology, Engineering, and Mathematics (STEM); therefore, lower success in mathematics can also affect their decisions to follow a STEM career [12], but also undermine their success in engineering courses [13], [14]. The issue of anxiety and lack of confidence in their abilities can be worse for female students who are often raised under the stereotypic view that females are not good at math [15].

Enhancing math problems with RL examples can provide deeper understanding of math concepts and skills [5], [3]. It can also motivate students to learn more about how math can be used in real life, experiencing a positive attitude towards math. For instance, [5] developed learning modules for college math courses based on engineering problems, and used virtual technologies to visualize them, engaging students and enhancing their learning. Redmond [11] enhanced middle school math courses with engineering problems and found that this intervention made a significant impact on students' confidence in science and mathematics, awareness and interest in engineering, and an interest as a potential career. Gleason [14] developed a summer bridge program for incoming students with hands-on activities to help them relate math concepts with real world engineering problems, leading to increased success in calculus courses and retention in the STEM fields. Premadasa and Bhatia [1] show that students prefer problems that intrigue them or problems that they can relate to, increasing the chances of knowledge retention. The importance of taking a generic math problem and relating it to a real-life problem while in college cannot be over emphasized enough. When some students enter into an engineering major, they lack the mathematics background needed to enter directly into calculus one. This means they must usually take one to two semesters of college algebra and/or trigonometry. It is in these prerequisite courses that students would benefit the most from practical surveying problems. Normally in these courses, students practice routine generic problems. The problem consists of a formula such as the Law of Sines or the general equation for a parabola and a set of numbers are given which are then plugged into a formula and some desired quantity is calculated. The numbers lack units of measurement, and the problem lacks any real-life practicality, and it reinforces the concept that math is plugging plain numbers into a formula. While this type of problem solving is essential initially to help students become familiarized with a formula, the transition to real-life problems must always follow. The introduction of these RL problems also helps students overcome two basic problems. One, the basic problem of reading comprehension and just as important, converting English into a mathematical equation [16], [17], [7]

In our first efforts to enhance math and statistics with RL problems, we have selected to draw RL problems from surveying engineering. Two reasons contributed in selecting surveying: (1) Surveying offers a great wealth of simple RL trigonometric and statistical problems that students can find interesting and easy to understand. As part of their daily activities, surveyors collect angle and distance measurements using modern instruments. Using such measurements they can calculate coordinates, height of buildings, shapes, and make volume calculations [18]. Surveying tasks include mapping of man-made features (buildings, roads, etc.), topography, and boundary determination. Many of those concepts are already familiar to students, assisting in making the connection between math formulas and real-life application. In addition, surveyors collect angle and distance measurements and use statistics to check accuracy requirements and ensure that their instruments are performing according to manufacturer standards. Therefore, making surveying a great resource for RL problems. (2) Penn State Wilkes-Barre is one of the few institutions that offer surveying. There are about 20 programs (2-year or 4 -year) in the US that are ABET accredited by the Engineering Accreditation Commission (EAC), Engineering Technology Accreditation Commission (ETAC), or the Applied and Natural Science Accreditation Commission (ANSAC) [19]. Penn State Wilkes-Barre has both a four-year Surveying Engineering degree and a two-year Surveying Engineering Technology degree. Surveying is a profession with low public profile [19]; therefore, engineering and nonengineering students working on surveying problems would be able to learn about this major and increase the awareness of surveying among engineering and non-engineering students.

The paper has the main objective to discuss the concept of integrating Math and Statistics courses with Surveying Engineering, and to outline the main steps of the implementation plan. We discuss our developed a framework for classroom implementation that is characterized by the acronym DSR, which stands for Discuss, Solve, and Reflect. The paper also provides numerical examples of surveying related RL problems relevant for different Math and Statistics courses.

## Future Implementation Plan

As educators we see those students entering college wanting a science degree, arrive in two general groups. Group one is students proficient in math, and place directly into calculus and the second group are those that are not. These students enter a series of intermediate to college algebra and trigonometry courses to eventually place into calculus. In addition, all science majors require statistics so this course will also be given RL problems that relate to statistical procedures. Table 1 shows the list of courses that have been identified as candidates for implementation. These courses have been selected due to the connection between math concepts and surveying RL problems. In our first implementation, we plan to use RL problems in Math 22 and 26 (Spring of 2022). These are the courses with the most connections with surveying engineering. In future semesters, we will be gradually adding more courses that are integrated with surveying RL problems (Table 2), increasing the impact of this project. We expect to add RL problems to one course each semester. In addition, in the future we plan to draw RL
problems from other engineering disciplines such as Civil Engineering, increasing the variety of RL problems and providing a diverse experience to students. For instance, a very common civil engineering problem is to have a site surveyed and then the total impervious area is calculated for a storm-water management plan. When setting a concrete foundation on a rock subsurface the slope of the subsurface is needed to determine if the concrete foundation should be pinned with steel rods to the subsurface. This requires elevations to be surveyed. The total area of a site is also needed to determine the size of a storm-water detention basin which requires a volume calculation. All of these civil engineering related problems require a surveyed job site at the start of any new project. In addition, we plan to add examples from other engineering disciplines, as the group explores the synergies between the Math and Statistics courses in Table 1 and engineering disciplines.

Table 1. Course listings and planned implementation. Mathematics course descriptions can be found here: https://bulletins.psu.edu/university-course-descriptions/undergraduate/math/ . Statistics course descriptions can be found here: https://bulletins.psu.edu/university-coursedescriptions/undergraduate/stat/

| Courses | Content | Approximate number of students <br> (number of engineering students in the <br> course) <br> / Number of sections |
| :--- | :--- | :--- |
| Math 22 College <br> Algebra | Linear and quadratic <br> equations. The exponential <br> and logarithmic functions | $15(5)$ / one section |
| Math 26 Plane <br> Trigonometry | Right triangles, angles, <br> Law of Sines and Cosines | $15(5)$ / one section |
| Math 41 College <br> Algebra and <br> Trigonometry | Combination of Math 22 <br> and Math 26 | $10(5-10)$ / one section |
| Math 140 Calculus I | Derivatives, optimization, <br> and Numerical Integration | $20-30(10-15)$ / two sections of 10-15 |
| Math 141 Calculus II | Numerical Integration | $20-30(20-25)$ / one section |
| Math 220 Matrices | Simultaneous equations | $30(25) /$ one section |
| Math 250/251 <br> Differential Equations | Differential equations | $30(25) /$ one section |
| Statistics 200 | Method of least squares, <br> confidence intervals, <br> means and standard error | $30-50(5-10) /$ two sections of 15-25 |
| Statistics 401 | Method of least squares, <br> confidence intervals, <br> means and standard error | $10(10)$ / one section |

From previous years of 8 different classes of Math 22 College Algebra an overall approximate test average was a $78 \%$, and for 8 different classes of Stat 200, the approximate test average was a $75 \%$. Therefore, about half of the years had grades less than $75 \%-78 \%$. The average grade scores reflect that there is room for improvement. From instructor's observations the reason for the lower average grades, is due to the non-engineering students who feel less motivated to engage with the course content. On the other hand, the engineering students understand that they will be using the Math concepts in future courses, and they tend to make stronger efforts.

Integration of RL problems in math will start with a few problems for each course. Each semester more examples will be added, with the hope of having RL problems throughout the entire course. These RL problems will be graded and included in their overall course average. All the RL problems will involve numerical measurements with appropriate units such as degrees, meters, and area or volume units. Solution of these problems will require a logical set of steps with correct "mathematical language" such as using variables with an equal sign indicating the correct answer with correct units.

Table 2. Course integration and implementation plan.

| Semester | Courses |
| :--- | :--- |
| Fall 2021 | Implementation Plan and Framework is Constructed |
| Spring 2022 | Math 22 and 26 |
| Summer 2022 | Math 26, 22 and, 140 |
| Fall 2022 | Stat 200, Math 22, and 26 |
| Spring 2023 | All of the listed courses if possible |
| Summer 2023 | Math 26, 22 and, 140 |
| Fall 2023 | All of the listed courses if possible |
| Spring 2024 | All of the listed courses if possible |
| Summer 2024 | Math 26, 22 and, 140 |
| Fall 2024 | All of the listed courses if possible |
| Spring 2025 | All of the listed courses if possible |

For the implementation and integration of RL problems in Math and statistics courses we have developed a framework, characterized by the acronym DSR, which stands for Discuss, Solve, and Reflect. Each of these elements are presented below:
(1) D : Discuss. The instructor discusses the RL problem, how the engineer would use math and statistics to address the problem, and the impact of the RL problem on the society and community. If the instructor can also demonstrate the instruments used for data collection, then use of such instruments in the RL problem will be covered.
(2) S : Solve. The students work on the problem, they device a plan to solve the problem, and implement their plan. This would follow Polya's four-step method of solving mathematical problems, thus reinforcing that concept.
(3) R: Reflect. Students reflect on the problem, and they use information related to the RL problem to check if the values calculated are reasonable. Discussion about the RL problem and its impact on society and community is re-enforced.

Both formative and summative assessment will be sought for this project. At the beginning of the semester, we will administer an online survey for students. The survey will focus on self-efficacy with questions such as (1) How confident do you feel about your ability with Math? (2) How motivated are you to follow the content in Math courses? (3) How stressed are you when trying to solve Math problems? (4) How would you rate your ability with Math? In addition, we will ask students about their major, so we can identify and track engineering and non-engineering students, as well as their gender and race for identifying different perceptions and competences based on these factors. At the end of the academic year a similar survey will be sent to students in order to identify trend changes after the application of RL problems. We will be asking students the following questions in the post-survey: (1) Did the RL problems help you understand how math is utilized in the world outside of the classroom? (2) Did the RL problems help motivate you to study more? (3) Do you feel that the RL problems helped you achieve a better overall grade in this course?

Course grade and exam averages will be compared with previous years for courses with past year test courses and class grades. In courses where there is more than one section, two groups will be formed. A test group that will follow the enhanced math and statistics courses and a control group that will follow traditional instruction. In this case, a pre-test and post-test will be given to both test and control groups that will allow us for quantitative assessment.

## Examples of numerical problems

In this section we provide three problem examples that are very common in surveying that are relevant to Math and Statistics courses. Below the description and solutions of those problems are provided as an example within the developed DSR framework.

Problem 1: Finding the height H of a building. A surveyor makes two measurements, an angle measurement $(\mathrm{Z})$ and a distance measurement ( D ) to determine the height of the building. The instrument height during the measurements is h.i. The ground between the pole and the instrument is horizontal. Compute the height of the building.


Figure 1. Problem 1: Finding the height H of a building.

- Discus: Instructor information to students: The instructor would pause and discuss why surveyors need to measure the height of buildings, and how is this accomplished with modern instrumentations.
- Solve: $\mathrm{H}=\mathrm{h} . \mathrm{i} .+\mathrm{D} / \tan (\mathrm{Z})=5.10+150.00 / \tan \left(74^{\circ} 44^{\prime} 40^{\prime \prime}\right)=46.01 \mathrm{ft}$
- Reflect: Students and instructor further discuss about the surveying, and students are trying to make sense of their result. For instance, is the height of the building reasonable based on the measurements given?

Problem 2: Area estimation of parcels. A surveyor needs to measure the total area of six parcels that are located next to an irregular shaped river. The surveyor measured the right-angle offsets ( $h_{0}, h_{1}, h_{2}$, et.) to a nearby road and the offset distances ( $\mathrm{a}, \mathrm{b}, \mathrm{c}$, etc.) along the road. Compute the total area.


Figure 2. Problem 2: Area estimation of parcels. Problem based on Ghilani and Wolf [18]

- Discuss: Instructor information to students: The instructor would pause and discuss how surveyors are being hired to locate and establish property boundaries, and how using measurements taken from modern instruments they can locate their coordinates and estimate the area of land parcels.

Solve: Area $=\frac{1}{2}\left[a\left(h_{0}+h_{1}\right)+b\left(h_{1}+h_{2}\right)+c\left(h_{2}+h_{3}\right)+\cdots\right]=\frac{1}{2}[60(7.2+11.9)+$ $80(11.9+14.4)+100(14.4+6.0)+30(6.0+6.1)+105(6.1+11.8)+$ $60(11.8+12.4)]=4,490 f t^{2}$

- Reflect: The instructor and students discuss the importance of accurately locating boundaries as mistakes can mean loss of property and land value for the owners.

Problem 3: Statistical testing. A calibrated baseline ( $\mu$ ) has a length of 400.008. After 20 distance measurements ( $n=20$ ) with an Electromagnetic Distance Measurement (EDM) unit we find that the observed distance $(\bar{y})$ is 400.012 with a standard deviation $(S)$ of 0.002 m [20]. Is the observed distance significantly different from the calibrated distance at 0.05 level of significance?

| D known (m) | D observed (m) | Standard deviation (m) |
| :---: | :---: | :---: |
| 400.008 | 400.012 | 0.002 |



Figure 3. Problem 3: Statistical testing.

- Discuss: Instructor information to students: Surveyors need to calibrate their instruments and ensure that they are working according to manufacturer specifications. This is part of their due diligence and ensuring that properly calibrated equipment is used in engineering projects.
- Solve: $\mathrm{H}_{0}$ (null hypothesis) $\mu=\bar{y} ; \mathrm{H}_{\mathrm{a}}$ (alternative hypothesis) $\mu \neq \bar{y}$

$$
t=\frac{|\bar{y}-\mu|}{S / \sqrt{n}}=\frac{400.012-400.008}{0.002 / \sqrt{20}}=8.944, \text { for } \alpha / 2=0.025 t_{0.025,19}=2.093 .
$$

The rejection region is true (8.944>2.093) and we can reject the $\mathrm{H}_{0}$.
The surveyor will have to repair his EDM or needs to review the field and office procedures.

- Reflect: Students try to make sense of the statistical outcome based on the input data. In addition, the instructor and students discuss how using non-calibrated instruments can lead to incorrect measurements, which in turn can have a significant impact on engineering projects, such as monitoring of engineering structures with implications to the community and safety.


## Discussion and Conclusions

Increasing student motivation and confidence to participate in class activities is important for mathematics and statistics classes that are aimed to build strong knowledge foundations in engineering students. Average grades from previous years indicates an average $75 \%$ to $78 \%$, which reflects that there is room for improvement. For instance, in 2019 the number of numerical math problems in math 140 (Calculus I) was increased by approximately 200 problems for that semester to enhance preparedness for quizzes and exams. Student feedback indicated that $67 \%$ of the total 15 students was favorable of this increase in problems. The increase was with standard math problems with no real life applications. Therefore, the conversion of a portion of these problems to RL will further enhance their engagement and motivation in their course material. It is the authors' observation that such lack of motivation exists mostly in non-engineering students who will not be using Math in their future courses; therefore, these are the students that will benefit the most from our planned implementation.

It is possible that students taking math and statistics classes can feel disconnected, making it challenging to understand the value of the course material, how it can be used in real life, and eventually lower the participation and engagement. To address this issue this paper outlined a plan to integrate RL problems originating from Surveying Engineering. Three numerical problems were presented that cover three different topics in Math and Statistics, namely trigonometry, area calculations, and statistical testing. Such problems will be used with enhanced discussion on "why" and "how" surveyors use math and statistics to complete such activities.

Surveying Engineering offers a wealth of simple RL problems that can cover a wide range of Math and Statistics courses. The paper has identified a total of nine courses that can benefit from surveying RL problems. Implementation of this plan will start in Spring of 2022 with two courses (college algebra and plane trigonometry). The number of participating courses will gradually increase by attempting to add one new course every semester. Surveying is a major
with low public profile; therefore, introducing Surveying Engineering in engineering and nonengineering students would also increase the awareness of this major to students.

In the future we plan to expand the list of RL problems, and draw RL problems from a variety of engineering and non-engineering disciplines. Examples include civil engineering and finance. After examining the synergies between the selected Math and Statistics courses and other engineering disciplines, we would also like to expand our implementations by adding examples from engineering disciplines other than surveying and civil engineering. This would create a diverse bank of RL problems that Math instructors can draw from each year. In addition, this would allow a larger part of the student body to relate to RL problems, as students find different things interesting and intriguing. In the future, we also hope to create a complete bank of RL problems that will be made available to any interested educators. With respect to Surveying Engineering, having multiple instructors from various institutions use surveying related RL problems, would increase the profile of surveying to engineering and non-engineering students and increase the awareness of this interesting major.

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