Experiences in an Undergraduate Laboratory Using Uncertainty Analysis to Validate Engineering Models with Experimental Data

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Abstract

Traditionally, the goals of engineering laboratory instruction have been to introduce the students to the use of various measurement devices along with the associated methods to interpret the results in the context of experimental uncertainties. There is usually an emphasis on the demonstration of fundamental engineering principles in applications-oriented projects. Often, theoretical engineering models are used to compare predicted outcomes with the experimental results in order to demonstrate the appropriateness and/or limitations of the theoretical model. When making these comparisons, the uncertainty associated with the experiment measurements is usually included; however, there is usually no consideration of the uncertainty associated with the theoretical model calculations. Students in the Mechanical Engineering (ME) program at Mississippi State University (MSU) are applying the concept of engineering model validation using uncertainty analysis into the undergraduate laboratories in addition to graduate research projects. In this paper, experiences are discussed which illustrate how this approach has been implemented into the undergraduate laboratory classes. The methodology is developed for the model validation, and a case study from our senior mechanical engineering laboratory is presented which illustrates how the uncertainty of the model is combined with the experiment results to provide a comparison.

Keywords: Uncertainty, Experimentation, Modeling, Validation, Laboratories

Background

At MSU, this model validation approach was first implemented into the ME undergraduate laboratory program to provide a bridge between the theoretical aspect of the traditional engineering courses and the practical demonstration of these principles through experimentation. An appreciation of the errors inherent in experimental results is critical, and uncertainty analysis concepts are integrated into the curriculum in an effort to quantify the validity of the test data. This process provides a logical methodology to interpret test results through the application of uncertainty analysis in the planning, design, construction, debugging, execution, data analysis, and reporting phases of experiments [1]. Accuracy of the experiments is investigated along with the appropriateness of a theory or model and its simplifying assumptions. This concept is an extension of the verification and validation research that is currently being done for CFD and other computational design codes [2, 3]. The approach is communicated at the undergraduate level through a three-laboratory sequence consisting of Experimental Orientation (EO), Experimental Techniques I (ET I), and Experimental Techniques II (ET II).

In the undergraduate curriculum, EO gives the students an introduction to the use of instrumentation for basic measurements, to the acquisition and processing of the measurement data, and to the concept of uncertainty associated with the instrumentation selection and the measurement process. The second course in the sequence, ETI, concentrates on identification of the key parameters needed to guide the design of experiments using uncertainty analysis. These concepts merge into ETII, which provides the opportunity for students to combine the knowledge gained in both EO and ETI to compare theoretical predictions with the measured outcome. The complete description of these courses has been presented elsewhere [4].

For the undergraduate ETII class, students select from a range of mechanical systems. Some of the systems are based on potential classroom demonstration units such as a pump test stand, a tensile test machine, or a heat

exchanger apparatus. The mechanical system can also be applied to modeling the physics of games such as predicting variability in the ultimate trajectory of softballs, water rockets, or golf balls. Or the mechanical systems can be chosen to augment ongoing research projects to explore the application of uncertainty analysis in the understanding of the experimental results and associated engineering models.

In the next section, some experiences from the ETII laboratory at MSU are summarized to show the application of model validation using experimental data. Following this section, the methodology of model validation is given along with an example.

Experiences
One of the ETII projects considered the prediction and measurement of the efficiency of a residential gas furnace. The students developed an energy balance model for the system that included the air temperature difference from the inlet to the exit, the air flow rate, and the natural gas flow rate and heat input. The air temperatures were measured with thermocouples, the air flow was measured with a volumetric flow meter, the gas flow rate was measured with an orifice, and the gas heating value was found from standard tabulated values. The uncertainty of each of these inputs was used with the model expressions to find the uncertainty of the model predictions.

The efficiency was measured directly with a flue gas analysis device. The initial comparisons of the measured and predicted efficiencies were not good, but the uncertainty for the model was high. The controlling variable for the model uncertainty was the exit temperature. The students modified the exit temperature measurement from a single thermocouple to a thermocouple grid to better determine the average air exit temperature with less uncertainty. This improvement gave good comparisons of the predicted and measured efficiencies considering the uncertainties of both.

Another ETII experiment was the use of an alternating fatigue device to determine the fatigue life of Aluminum 6061 T-6 for various stress levels. The engineering model for this test was the standard stress versus number of cycles to failure fatigue chart for this material. The initial comparison of the test and model results for this experiment was poor. The values of stress for the test were estimated based on the displacement of the specimen and on the specimen dimensions. This approach led to reasonably large uncertainty values for the stress. Also, the number of cycles to failure was determined with a mechanical counter, which had an uncertainty of 5%. An improvement to the test was made by gluing a strain gage to the specimen and by using a computer data acquisition system to monitor the strain (stress) during the test and to measure the number of cycles to failure. This improved test procedure yielded results with less uncertainty that agreed well with the published data.

In the next section, we present the methodology for using the uncertainty of test results and engineering model predictions to determine the model validity. Following the methodology, we give a detailed example of another ETII experiment.

Methodology
Some of the previous work related to the application of uncertainty analysis in undergraduate engineering laboratory courses is documented in References 1, 4, 6-9. These efforts have been directed primarily toward quantifying the uncertainty of the result of the experiment. The methodology for applying uncertainty analysis to the experimental result is summarized below, where the nomenclature has been updated to reflect the latest accepted version.

In nearly all experiments, the measured values of different variables are combined using a data reduction equation (DRE) to form some desired result. A general representation of a data reduction equation is

$$r = r(X_1, X_2, ..., X_J) \tag{1}$$

where r is the experimental result determined from J measured variables X_i. Each of the measured variables contains systematic (fixed) errors and random (varying) errors. These errors in the measured values then propagate through the DRE, thereby generating the systematic and random errors in the experimental result, r. Uncertainty analysis is used to estimate the random and systematic standard uncertainties of the result, s_r and b_r, respectively, and the corresponding expanded uncertainty of the result, U_r.

If it is assumed that the degrees of freedom for the result is large (>10), which is very appropriate for most engineering applications, then the "large sample assumption" [6] applies and the 95% confidence expression for U_r is

$$U_r = 2\sqrt{b_r^2 + s_r^2} \qquad (2)$$

The systematic standard uncertainty of the result is defined as

$$b_r^2 = \sum_{i=1}^{J}\theta_i^2 b_i^2 + 2\sum_{i=1}^{J-1}\sum_{k=i+1}^{J}\theta_i\theta_k b_{ik} \qquad (3)$$

where

$$\theta_i = \frac{\partial r}{\partial X_i} \qquad (4)$$

The systematic standard uncertainty estimate for each X_i variable is the root-sum-square combination of its elemental systematic standard uncertainties

$$b_i = \left[\sum_{j=1}^{M}b_{i_j}^2\right]^{1/2} \qquad (5)$$

where M is the number of elemental systematic standard uncertainties for X_i and where each b_{i_j} is the standard deviation level estimate of the systematic uncertainty in variable X_i resulting from error source j. The standard deviation level systematic uncertainty estimate for an error source is usually made by making a 95% confidence estimate of the limits of the error for that source and dividing that estimate by 2 [6]. The second term in Eq. (3) accounts for systematic errors that have the same source and are correlated. The factor b_{ik} is the covariance term appropriate for the systematic errors that are common between variables X_i and X_k and is determined from [10] as

$$b_{ik} = \sum_{\alpha=1}^{L}b_{i_\alpha}b_{k_\alpha} \qquad (6)$$

where variables X_i and X_k share L identical systematic error sources. The random standard uncertainty of the result is defined as

$$s_r^2 = \sum_{i=1}^{J}\theta_i^2 s_i^2 \qquad (7)$$

where s_i is the sample standard deviation for variable X_i (sample standard deviation of the mean if X_i is a mean value or sample standard deviation if X_i is a single reading).

This same basic methodology can be applied to the engineering model in order to estimate the uncertainty associated with the calculated result from the model. The engineering model has input values that have uncertainties. These uncertainties cause an uncertainty in the calculated result. The model may also have an uncertainty based on how well it matches the physics of the experiment. This additional uncertainty cannot be estimated prior to the validation process and is therefore the primary reason for doing a validation study on the engineering model.

Considering the model result, m, to be a function of K input values, Y_i, as indicated by Eq. (8)

$$m = f(Y_1, Y_2, \ldots, Y_K)$$ (8)

the uncertainty of the model result would then be determined from the uncertainty propagation equation [6] as

$$U_m = 2\left[\sum_{k=1}^{K}\theta_k^2 s_k^2 + \sum_{k=1}^{K}\theta_k^2 b_k^2 + 2\sum_{k=1}^{K-1}\sum_{j=k+1}^{K}\theta_k\theta_j b_{kj}\right]^{1/2}$$ (9)

where θ_k is the derivative of the model with respect to each input quantity Y_i and the s_k and b_k factors are the random and systematic standard uncertainties for the model input variables. Note that property values and empirical coefficients will have uncertainties as well as the input variables.

In order to determine the validation of the model with respect to the result of the experiment, r, a comparison error, E, is defined as

$$E = r - m$$ (10)

The uncertainty associated with this comparison error is

$$U_E^2 = U_r^2 + U_m^2$$ (11)

The basic concept in the validation process is a comparison of E and U_E. If $|E|$ is less than U_E, then the comparison is within the noise level of the uncertainty, and the level of validation of the model is U_E. If $|E|$ is much larger than U_E, then there is probably justification for improving either the selection of the governing equations for the model or the initial simplifying assumptions [3]. In this case, the sign of E gives some indication of the needed correction to the model.

Another benefit of the validation process is the determination of how each of the sources of uncertainty affects the uncertainty of the comparison error. The uncertainty percentage contribution, UPC, for each error source is determined as

$$\frac{\theta_i^2 b_i^2 \times 100}{(U_E/2)^2} \; ; \; \frac{2\theta_i\theta_k b_{ik} \times 100}{(U_E/2)^2} \; ; \; \frac{\theta_i^2 s_i^2 \times 100}{(U_E/2)^2}$$ (12)

where the sum of all of the UPC's is 100%. The factors in Eq. (12) come from Eqs. (3), (7), (9), and (11). These parameters show the percentage contribution that each error source has on the square of the total uncertainty of the comparison error. A review of the UPC's will identify which uncertainties are controlling the total uncertainty and which uncertainties are having a negligible effect on the total uncertainty. This information can be used to identify where improvements need to be made in the uncertainties of the experiment variables or in the model input variables in order to reduce the magnitude of the uncertainty of the comparison error.

In the next section, the validation process is illustrated with an example experiment from the ME laboratory course at MSU.

Example
One assignment for an ETII team was to experimentally and theoretically determine the head loss, h, for a straight section of a plastic pipe. The flow apparatus is shown in Figure 1. Pressure differentials across both the orifice plate flow meter and the straight pipe section were measured using a manometer and a differential pressure transducer, respectively.

For the experiment, the result was the measured head loss in the pipe, Δhp_r, over a range of flow rates. For the engineering model, the following expression was used to predict the pipe head loss, Δhp_m,

$$\Delta hp_m = f \frac{8L\left[C(\Delta ho)^{0.5}\right]^2}{g\pi^2 d^5} \tag{13}$$

where L is the length of the pipe, C is the orifice flow coefficient, Δho is the orifice head loss, g is the acceleration of gravity, d is the pipe diameter, and f is the friction factor. The Haaland relationship [11] was used for the friction factor

$$f = \frac{0.3086}{\left\{log\left[\left(\frac{6.9}{Re_d}\right)+\left(\frac{\varepsilon}{3.7d}\right)^{1.11}\right]\right\}^2} \tag{14}$$

where ε is the roughness of the pipe and Re_d is the Reynolds number based on the pipe diameter

$$Re_d = \frac{4\rho C(\Delta ho)^{0.5}}{\pi d\mu} \tag{15}$$

where ρ is the water density and μ is the water viscosity.

The values for all of the variables associated with the experiment and the engineering model are given in Table 1. All head values are in terms of inches of water. Also given in the table are the uncertainty estimates for each variable.

The systematic standard uncertainty for the pipe head loss came from the combination (using Eq. (5)) of the elemental systematic standard uncertainties for the pressure transducer of 0.1 in. for zero shift and 0.1 in. for the calibration curve fit. The random standard uncertainty for the pipe head loss was based on the standard deviation of

the mean for the 36 readings made with the pressure transducer for each run. This value was nominally constant for each run, so the same value was used for all runs.

The orifice head loss systematic standard uncertainty was based on the accuracy of the manometer used for these measurements. The random standard uncertainty was estimated from the variation of the water column during readings. The typical variation was about ±4 mm yielding an estimate of the standard deviation of ±2 mm or 0.08 in.

The systematic standard uncertainty estimates for the length and diameter were based on the devices used to make the measurements, a scale and a micrometer, respectively. The orifice was calibrated prior to running the experiment using a catch basin of known volume. The systematic standard uncertainty for the flow coefficient is the standard deviation of the mean for the calibration process.

The pipe wall was relatively smooth; however, considering the variation in relative roughness estimates near the smooth wall limit, this value could be uncertain by as much as 50% with 95% confidence. The water density and viscosity were based on tabulated values at the water temperature. The water temperature was about 65°F, but possible variation in the water temperature of ±5°F led to uncertainty estimates of 7% for the water viscosity and 0.3% for the density, both at 95% confidence. The systematic standard uncertainty estimates for these three variables were taken as one-half of the 95% confidence estimates.

The results for each run for both the experiment and the engineering model are given in Table 2. The Reynolds number range for this test was 22,000 to 48,000, and the pipe head loss varied from 5 to 20 inches of water over this range. The comparison of the measured and predicted pipe head loss values is given in Figure 2. The comparison looks very good, but uncertainties should be taken into account in order to validate the model. This validation comparison is shown in Figure 3, where the comparison error, E, (from Eq. (10)) is plotted along with the uncertainty in the error, U_E, (from Eq. (11)). For this test, $|E|$ was less than U_E for all runs; therefore, the model is validated at the level of U_E for this pipe diameter and smoothness over this range of Reynolds numbers. The validation uncertainty varies from 0.43 in. at the low Reynolds number end to 0.75 in. at the upper end.

This level of validation is based on the uncertainties of the experiment result and the model input variables. An investigation of the UPC's for each of the error sources will show which uncertainties are dominating the determination of the error uncertainty. Table 3 gives the UPCs for the two limits of the Reynolds number range for the test. At the lower Reynolds number, the systematic standard uncertainty of the pipe head loss measurement dominates U_E. To reduce this uncertainty, a better transducer would be needed that had less zero shift and a more linear calibration curve. The random standard uncertainty of the orifice head loss measurement also is significant at the low Reynolds number limit. To improve this uncertainty, a stable and well calibrated pressure transducer could be used to make the measurement. At the higher Reynolds number, the orifice flow coefficient uncertainty dominates U_E. To reduce this value, a better calibration of the orifice would have to be performed. If these three uncertainties were reduced, then there would be less uncertainty in both the experiment and the model, and the uncertainty in the error would be less. Then the comparison range for E would be smaller, but it is very likely that improved pressure transducers and a better orifice calibration would reduce the comparison error.

Conclusion
Understanding the limitations of physical models is key to the successful practice of engineering. In the ETII laboratory in ME at MSU, the students are given an opportunity to investigate the validity of models by comparing the predictions with experimental results. The uncertainty of both the model and experiment results are used to assess the model validity and to identify ranges where different or improved models are needed or to show that improved variable uncertainties are needed to reduce the validation uncertainty.

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J. A. Schneider

J. A. Schneider is an Assistant Professor in the Mechanical Engineering Department at Mississippi State University. Her primary area of research is the engineering of the microstructure by control of the processing parameters to obtain the desired mechanical performance of structural materials. Much of her research centers on characterization of the microstructual evolution during either the processing or service life of the material

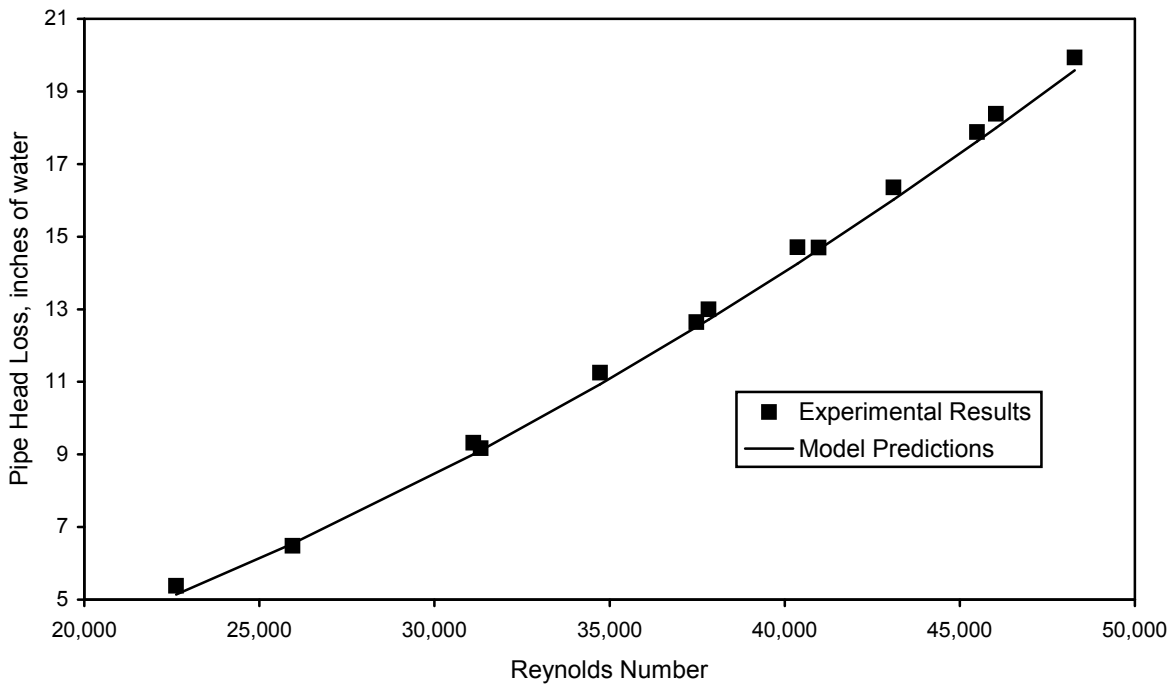Figure 1. Fluid Friction Apparatus

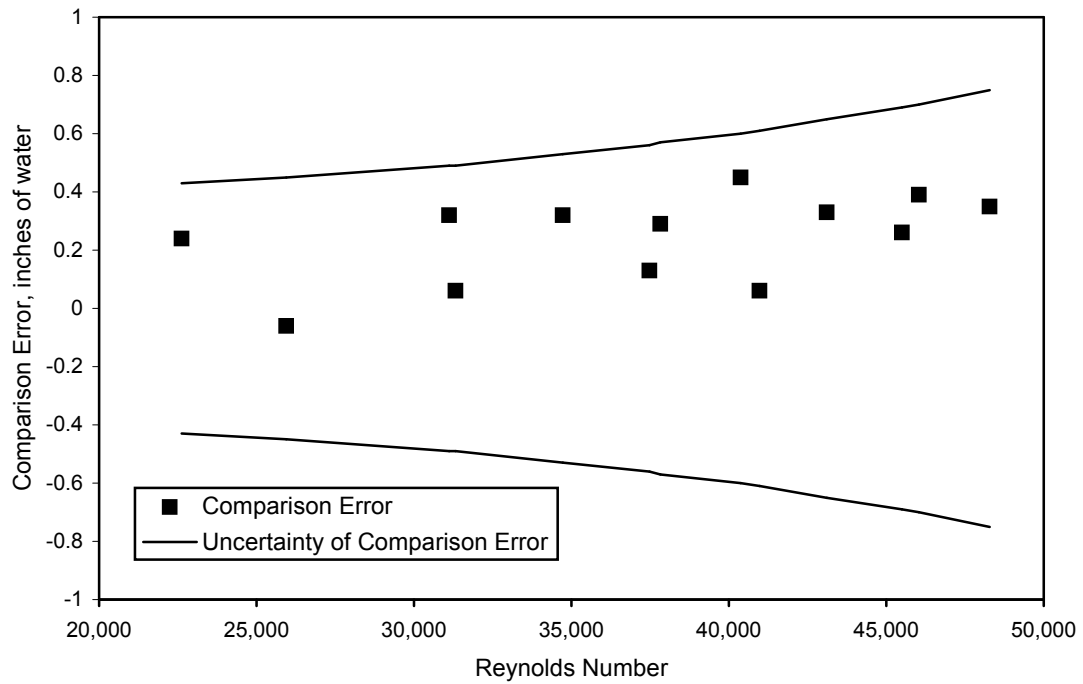Figure 2. Results for Pipe Head Loss Experiment and Model

Figure 3. Validation of Pipe Head Loss Model

Table 1. Variable Values and Uncertainty Estimates

Variable	Value	Systematic Standard Uncertainty	Random Standard Uncertainty
pipe head loss (Δhp_r)	Variable	0.141 in	0.04 in
orifice head loss (Δho)	Variable	0.05 in	0.08 in
pipe length (L)	39.125 in	0.031 in	-
pipe diameter (d)	0.697 in	0.00025 in	-
flow coefficient (C)	11.45 in $^{2.5}$/sec	0.089 in $^{2.5}$/sec	-
roughness (ε)	3.6×10^{-6} in	25%	-
water density (ρ)	999 kg/m^3	0.15%	-
water viscosity (μ)	1.056×10^{-3} Nsec/m^2	3.5%	-

Table 2. Test Results

Run	Orifice Head Loss (in)	Re_d	Pipe Head Loss Measured Δhp_r (in)	Data Uncertainty U_r (in)	Pipe Head Loss from Model, Δhp_m (in)	Model Uncertainty U_m (in)	Comparison Error E (in)	Comparison Error Uncertainty U_E (in)
1	3.14	22,623	5.38	0.29	5.14	0.32	0.24	0.43
2	4.13	25,946	6.48	0.29	6.54	0.34	-0.06	0.45
3	5.94	31,116	9.32	0.29	9.00	0.39	0.32	0.49
4	6.02	31,325	9.17	0.29	9.11	0.40	0.06	0.49
5	7.40	34,730	11.25	0.29	10.93	0.44	0.32	0.53
6	8.62	37,484	12.64	0.29	12.51	0.48	0.13	0.56
7	8.78	37,830	13.00	0.29	12.71	0.49	0.29	0.57
8	10.0	40,373	14.71	0.29	14.26	0.53	0.45	0.60
9	10.3	40,974	14.70	0.29	14.64	0.54	0.06	0.61
10	11.4	43,106	16.35	0.29	16.02	0.58	0.33	0.65
11	12.7	45,498	17.88	0.29	17.62	0.63	0.26	0.69
12	13.0	46,032	18.38	0.29	17.99	0.64	0.39	0.70
13	14.3	48,279	19.93	0.29	19.58	0.69	0.35	0.75

Table 3. Uncertainty Percentage Contributions

Variable	Reynolds Number			
	22,623		48,279	
	Systematic Standard Uncertainty	Random Standard Uncertainty	Systematic Standard Uncertainty	Random Standard Uncertainty
pipe head loss (Δhp_r)	42.6	3.4	15.1	1.2
orifice head loss (Δho)	11.0	28.1	2.8	7.1
pipe length (L)	0.0	-	0.2	-
pipe diameter (d)	0.2	-	0.8	-
flow coefficient (C)	10.5	-	54.9	-
roughness (ε)	0.0	-	0.0	-
water density (ρ)	0.0	-	0.0	-
water viscosity (μ)	4.2	-	17.9	-
	Sum = 100.0		Sum = 100.0	