



Extending Innovative Practices for "Flipping Classrooms" into Recitations: Using a Variety of Representational Modes for Instructions

Dr. Jia-Ling Lin, University of Minnesota, Twin Cities

Jia-Ling Lin is a research scientist in the STEM Education Center at the University of Minnesota. Her research focuses on two distinct but highly correlated areas: innovative instructional model development and its impact on undergraduate engineering and science learning. Jia-Ling was a scientist specializing nanotechnology and surface science at the University of Wisconsin-Madison. She later served as the director of the Undergraduate Learning Center in engineering at the University of Wisconsin-Madison before moving to Minnesota.

Prof. Paul Imbertson, University of Minnesota, Twin Cities

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Abstract

We extend the design of teaching approaches for flipping classrooms into recitation sessions to support students’ development of content knowledge, problem solving skills, and metacognitive knowledge. We focus on co-development of cognitive and metacognitive knowledge and skills and apply a models and modeling approach embedded in instruction to promote such development. Students’ learning outcomes show evidence that multiple representation tools positively influence teaching and learning. It supports problem-centered learning and helps strengthen associations of instructional goals of lectures and recitations.

Introduction

In many institutions for higher education, the dominant educational model for engineering and science instruction continues to be a combination of (i) lecture, (ii) recitation, and (iii) weekly homework problem sets. In recent years, many engineering educators have questioned this traditional model because they have witnessed a lack of correlation between instructional expectations and students’ learning outcomes.^{1,2} To address this issue, engineering educators and educational researchers have been working together seeking innovative approaches that engage students in meaningful learning.³⁻¹⁰ “Flipping classrooms” is one of the emerging instructional approaches that attempt to replace traditional lectures in an effort to engage students in active learning. Educational research shows that when students passively receive content knowledge from instructors in large lecture halls, meaningful learning seldom happens. On the contrary, rote learning is the result of such traditional teachings. Learning in classrooms dominated by instructors’ lectures often leads to knowledge retention in the short term, but fails to prepare students to apply what they learned to real world situations.¹¹ In order to flip classrooms properly and to ensure that true learning takes place, a pedagogical model for quality teaching is required.¹² Supported by grounded learning theories and pedagogical frameworks, we have designed and developed an instruction model to promote quality teaching and learning in flipped classrooms.¹³ We conducted a series of studies that improved the implementation of the model through iterative cycles. The four practices utilized in the model include *Anticipating, Monitoring, Connecting & Contrasting, and Contextualized Lecturing*.

In the current study, we expand on the design of teaching approaches for flipping classrooms into recitation sessions to improve associations of instructional goals for both lectures and recitations. Without a strong association between lectures and recitations, practices in flipped classrooms will stop short of fully supporting students’ development of content knowledge and skills. The four-practice model employed during the lecture periods focuses on problem-centered learning. Pedagogical principles of the new instructional model are expected to facilitate learning beyond “flipping lectures”. We identified challenges while applying the instructional model in lectures and were driven to explore new perspectives and directions for teaching in recitation sections. The necessity of expanding on the practice of *Connecting & Contrasting*, one of the four practices in the model, to support activities both in lecture and recitation periods motivated the current study. The underlying pedagogical principle for the practice of *Connecting & Contrasting* is to make students responsible and accountable for their own learning, for others

(peers and teachers), and for discipline norms. This practice is essential to the instructional goal of moving the whole class forward to improved learning.

Through recent innovations in the practice of delivering recitation sessions students are able to reveal what they do not understand and to practice their skills under the guidance of experts.¹⁴ We expand on this positive first step by developing new instructional techniques. With a focus on improving abilities and fluency with multiple-representation translation, the models and modeling approach supported important practices in learning content knowledge and in problem solving.¹⁵ Such an instructional focus in recitations is expected to better prepare students for engineering practices and research. Through the creation and delivery of models of problem solutions, which employ a variety of representational media, students learn to explain important ideas in several ways and learn to apply these ideas to solve problems in real world situations. We argue that learning activities with multiple representation modes embedded in teaching and learning practices in recitation sessions should improve students' understanding of learning and ensure that students not only acquire knowledge and skills, but also develop skills for knowledge retention and transfer in the long term. As the first part of this study, the current report focuses on three research questions: (1) Is content knowledge co-developed with problem-solving skills and metacognitive (knowledge about cognitions in general and one's own cognition) functions under problem-centered learning? (2) Does the distribution of content knowledge among a variety of representational modes facilitate such development? (3) What are the implications in engineering teaching and learning for such development?

Background

Engineering practices in general include iterative cycles of (i) describing a problematic situation, (ii) modeling/abstracting the problem, (iii) defining the problem and tasks, (iv) exploring, designing, and developing solutions, and (v) implementing, testing, and revising/optimizing solutions. From a models and modeling perspective, the development of content knowledge and problem-solving skills that are transferable for work in the real world follows similar cycles in engineering teaching and learning.¹⁵ Modeling involves cycles of model design, model construction, model testing, model evaluation, and model revision, and is central to professional practices in many disciplines, such as mathematics, science, and engineering. Modeling practices are ubiquitous and diverse, ranging from the construction of physical models in science, engineering, and technology to the development of abstract symbol systems.¹⁶ The abundant presence and the variety of models in these disciplines suggest that modeling can help students develop understanding about a wide range of important ideas. However, modeling is largely missing from school instruction. Figure 1(a) shows engineering practice cycles and Figure 1(b) shows cycles of modeling based learning.

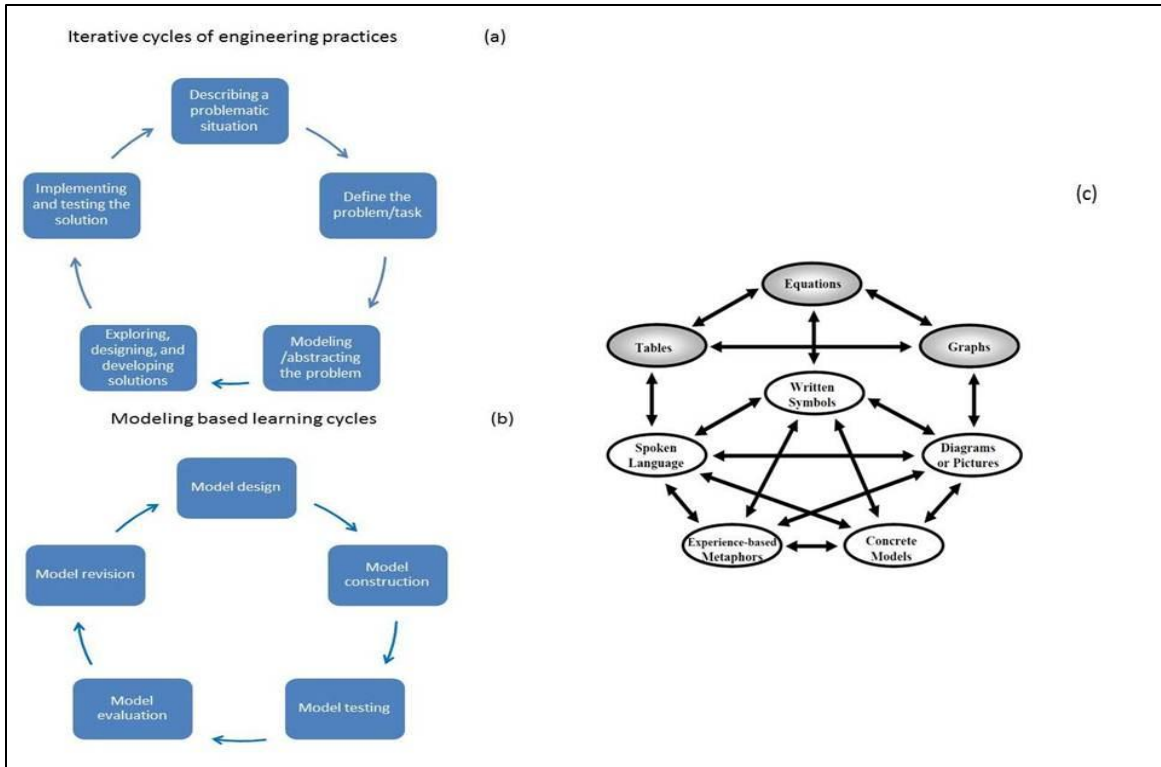


Figure 1. (a) Engineering practice cycles; (b) Model based learning cycles; (c) The Lesh Translational Model¹⁵

Education researchers have long believed that the structure of any domain knowledge can be characterized in certain modes of representation, and that representational fluency underlies some of the most important abilities associated with what it means to understand a given conceptual system.^{11, 15-19} Bruner defined the three modes of representation as (i) enactive mode (physical representation), (ii) iconic mode (pictorial representation), and (iii) symbolic mode (written symbols).²⁰ Lesh and Doer indicated that meanings of conceptual systems tended to be distributed across a variety of representational media.¹⁵ The Lesh Translation Model, consistent with other research findings emphasizes the importance of various ways of representing content knowledge in mathematics education. See Figure 1(c). Translations of a conceptual system between different representational modes do not take place unless learners understand the concept under consideration in the given mode.¹⁷ Though the Lesh Translational Model shown in Figure 1(c) was designed explicitly to provide teachers with guidelines while developing a hands-on, activity-oriented environment in elementary mathematics classrooms, the implication of a models and modeling perspective goes beyond precollege math classrooms.¹⁵ Because meanings of conceptual systems can be projected into and distributed across a variety of representational media, multiple representational tools are expected to facilitate teaching and learning in significant ways.¹⁹ Research findings support the necessity of representational fluencies across various engineering practices and show evidence that an important element of design and problem solving in engineering involves shifting back and forth among a variety of relevant representations.^{15,21} However, how teaching approaches, structured around multiple representational tools, can be employed to improve learning in engineering and other STEM classrooms have not been studied in detail.

Problem-centered learning in flipped classrooms emphasizes the shared authority and responsibility of teaching and learning in a learning community.^{8, 12, 13} It gives students ownership of their learning through collaborative learning in classrooms, and makes students accountable for their learning as well. We have studied classroom discourse in flipped classrooms and concluded that in a learning environment such as “flipped classrooms” group problem solving with interventions through dialogues among students and between students and the course instructor engaged students in deep learning. It helped students improve their learning on several fronts. (1) It enables co-constructing knowledge and communicative teaching, which changes both students’ learning beliefs and behaviors considerably. (2) It refines roles of the instructor highlighting both the authoritative and responsive natures of instructors in “flipped classrooms”.^{12, 13} Despite many positive influences on engineering student learning in flipped lecture halls, challenges remain. The design and the implementation of the new instructional model are multi-faceted because of the complex nature of learning. Learning of content knowledge and skills is a metacognitive process for competencies transfer.²²⁻²⁵ Our previous study showed limited gains for students in metacognitive knowledge. We encountered challenges in helping students improve understanding of what counts as learning. Even when individual variations in course performance are accepted in flipped classrooms, the onset of changes in learning behaviors and beliefs for each student can be very different. Students who are not able to align their learning goals with teaching objectives of the instructional model have a difficult time adapting to new learning methods. How we help students improve learning gains in metacognitive knowledge and epistemological understanding of the nature of knowledge and learning is important to the success of new instructional approaches for flipping classrooms. We need to find ways to support metacognitive challenges in constructing knowledge and problem solving for all students. Table (I) shows a framework that was created to guide us in designing and implementing instructional approaches that support the co-development of cognitive and metacognitive abilities. Recitations offer us the opportunity.

Table I. Metacognitive Framework for Cognitive Activities in Problem Solving

Cognitive Activities in Problem Solving ²⁶							
Metacognitive Challenges ²⁵		<i>Focus</i>	<i>Define</i>	<i>Design & Devise</i>	<i>Execute</i>	<i>Evaluation</i>	
	<i>Interpreting meanings of the problem and defining task</i>	Important information and features	Underlying concepts	Strategies & Approaches			
		Structure	Target quantities & Goals	Knowledge organization and applicability			
	<i>Monitoring & Self-regulation</i>					Following through	
						Tracking the progress	
Going through cycles of learning activities when necessary							
<i>Reflection</i>				Procedural efficiency	Quality		
					Generality		

Research Method

The design-based-research (DBR) method was applied, which intertwined the three goals of research, design, and pedagogical practice in this study.^{27, 28} A productive partnership between the course instructor and researchers allowed iterative cycles of designing and revising research questions as well as testing new instructional techniques and research plans in real educational settings. The study was conducted in an intermediate level electrical engineering course, “Linear Systems, Circuits, and Electronics” in fall 2014. The class had three 50-minute lecture periods and a recitation section of 50 minutes every week. Forty-four students enrolled in the course, and were divided into 8 groups in the lectures. Students were enrolled in two recitation sessions. The lecture instructor taught one session and another professor from the Electrical and Computer Engineering Department taught the other. Both sessions used the same problems provided by the lecture instructor. Students were instructed to read certain chapters from the textbook before coming to the lecture, and were expected to be prepared to work on problems within their small groups during lectures. The instructor applied the four-practice model for “flipping classrooms”. The lecture instructor and the researcher met regularly every week.

Instructional Approaches in Recitations

The practices of reasoning in engineering education are well-stocked with representational artifacts such as symbols, equations, graphs, tables, and diagrams. Each of these representational modes highlights specific characteristics of subject topics and show different levels of effectiveness in fostering knowledge construction. Students coming into the class bring their perceptions regarding learning and problem solving. Many students believe that learning in the context of problem solving is about identifying a target quantity and finding its numerical value. They thus focus learning on acquiring all needed formulas and equations. Other students favor lecture notes provided by an authoritative figure to build a comfort zone. They limit their thinking within the boundaries of the comfort zone.¹⁶ There are students, although not a majority early in the semester, who believe that learning is about exploring new ideas and constructing knowledge themselves under the guidance of experts. In our previous studies, we focused on the development of classroom discourse while applying the four-practice instructional model in flipped classrooms. We reported how individual talks in a small group as well as in a big class revealed how students learn through dialogues between students and between students and the instructor.^{8, 13} We found a gradual shift of focus for small group discussions, from factual knowledge to conceptual and procedural knowledge when students were engaged in problem-centered collaborative learning throughout the semester. We also found that the way problems were framed influenced the breadth and the depth of students’ talks during small group discussions. Given the fact that it is not uncommon for one instructor to preside over a class of 100 students in engineering classes, we recognized challenges in moving the whole class forward during the practice of *Connecting & Contrasting* in lecture periods. At times, it failed to help those students who need more support for both their cognitive and metacognitive development. Recitations, with a smaller class size, provide us a context to develop instructional approaches to move the whole class toward advanced learning. In the following, we describe our approach to extending innovative instructions for flipping classrooms into recitations.

(I) Keeping Lectures and Recitations on the Same Page

Problem-centered learning and the Lesh Translational Model (shown in Figure 1(c)) engage connections of lectures and recitations. During the lecture period, students were involved in group discussions and dialogues while working on problems. Spoken language was a central tool that supported students' learning activities, particularly making inquiries. Classroom discourse also allows instructors to discover what students know and how they know it. Applications of multiple representational tools were coordinated in lectures. During and after group discussions, students translated their spoken language into written symbols, equations, graphs, diagrams, etc. in which they created a written group report. Each group was required to hand in the work, and the group work was graded based on efforts and attempts, but not the "correctness" of the work. Problem-centered learning was applied in recitation sessions as well. Students were encouraged to ask questions and share their ideas with peers. However, they were asked to complete the work individually. The small size of the session, about 20 students or so, provided abundant time and space for interactions between students and the instructor. Students watched closely how experts (faculty instructors) organized knowledge and approached engineering problems using corresponding multiple representational tools. They got a chance to revisit their work in lectures and reflected on connections of learning between lectures and recitations. While working on their own solution models, they were coached by the instructor, learning how to use different representational tools to analyze the problems from different angles. They learned to create and assess quality solutions by working side-by-side with the instructor. It is important that the **lecture instructor** provide recitation materials. This is to ensure the continuity for learning between lectures and recitations. Because the instructor knows what concepts present learning challenges to students through interactions during the lecture period, the instructor purposefully creates practice problems for recitations. Instructions and learning materials for recitations are to help students reinforce their conceptual understanding and learn to deliver quality solutions.

(II) Integrating and Interacting with Multiple Representational Modes

One important strategy in recitations was to stress the application of multiple representations and engage interactions with multiple representational modes. A given idea in science, engineering, and mathematics can be represented in a variety of modes. Different representations of the same phenomenon can highlight different features or properties of the phenomenon. Applying different representations provides students distinct conceptual resources and problem-solving affordances in the context of a specific learning task.¹⁹ Learning to represent the same concept in different modes is one way to construct understanding and is an effective way to learn transferable knowledge and skills. We asked students to build "models" while solving problems instead of "seeking solutions". Under the terms of "models" and "modeling", problem solving processes are perceived as cycles of designing, creating, testing, evaluating, and learning. By integrating teaching and learning through interactions with multiple representation tools, students focus problem solving on creation, quality, generality, and usability of their work.

Data collection

(1) Students were asked to participate in two online surveys, one at the beginning and one at the end of the semester. Eighteen students responded to the first survey and 17 to the second survey.

(2) Students' team worksheets and copies of exam papers were collected and analyzed. Some individual worksheets were collected in recitations.

(3) Students' verbal discourse, while working on problems posed by the instructor within their small groups in lectures was observed by researchers and was audio recorded. Lectures were video recorded, and recitations were video and audio recorded.

Data analyses

Coding schemes to analyze students' exam papers were developed using several key concepts from the Lesh Translational Model.¹⁵ Usage of various representational modes was counted and analyzed along with characterizations of the usage. Following the framework shown in Table I, the focus of the data analysis was placed on how different modes were applied in interpreting the problem, and how and if representation modes were applied to facilitate problem solving. Four representational modes were selected: (i) LA: Labeling the provided diagram with written symbols; (ii) EQ: Equation; (iii) TX: Written text; and (iv) DI: Students' own diagrams, pictures, and graphs. If none of the four mentioned above was used, NA was applied. For usage counting, two scores, "0" and "1" were used. "0" means no usage at all. For analyzing fluency of translation between modes and/or shifting back and forth between different modes, a scoring system of "0", "0.5", and "1" was used. "0" means not at all; "0.5" means somewhat, that is, a mode was used but some key concepts were omitted; "1" means fluent. Table II shows the definition of representational mode applied in this study. Table III shows the scoring rubrics.

Table II. Definition of Representational Mode

Representational Mode	Definition
Label (LA)	Labeling the provided diagram with written symbol and text
Equation (EQ)	Equation and formula
Text (TX)	Written words
Diagram (DI)	Drawing and sketch
NA	No representational mode applied

Table III. Scoring System for Multiple Representational Mode Usage Analysis

Score	Usage counting	Usage fluency	Usage accumulation
0	No usage	Not at all	0
0.5		Using one mode but omitting some concepts	0.5
1	Showing usage	Fluent	1
>1			1.5-4; Using more than one mode

Results

Figures 2-11 compare students' usage of the identified four modes in the first midterm and the final exam. Frequencies and varieties of representational mode usage were analyzed. The data analyses were intended to help us understand if the usage of representational modes facilitated learning and/or vice versa. Figures 3 and 4 show how students applied multiple representational

tools while working on one problem in the first exam. The problem asked students to use phasor to analyze an AC RCL circuit and is shown in Figure 2. Students took the test at the end of the 4th week of the semester.

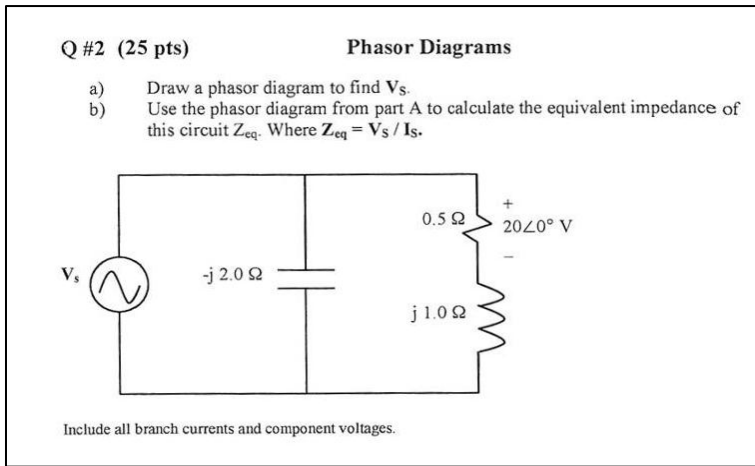


Figure 2. Question 2 in the 1st midterm exam

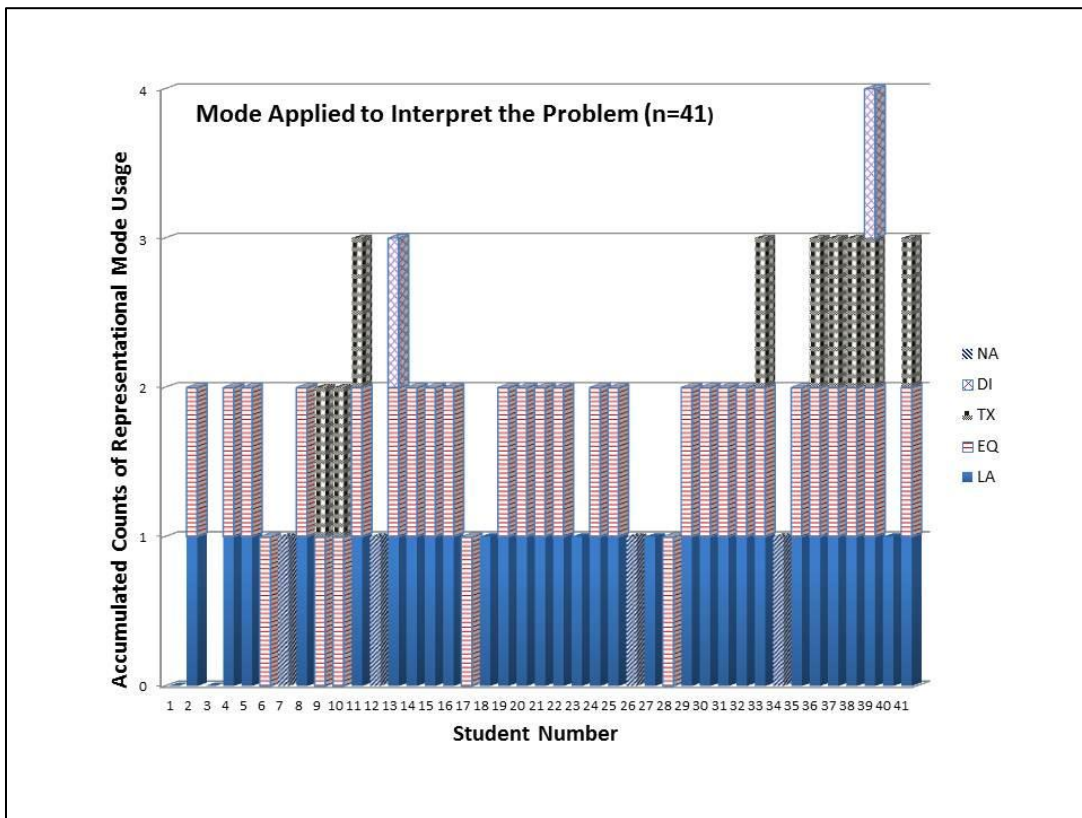


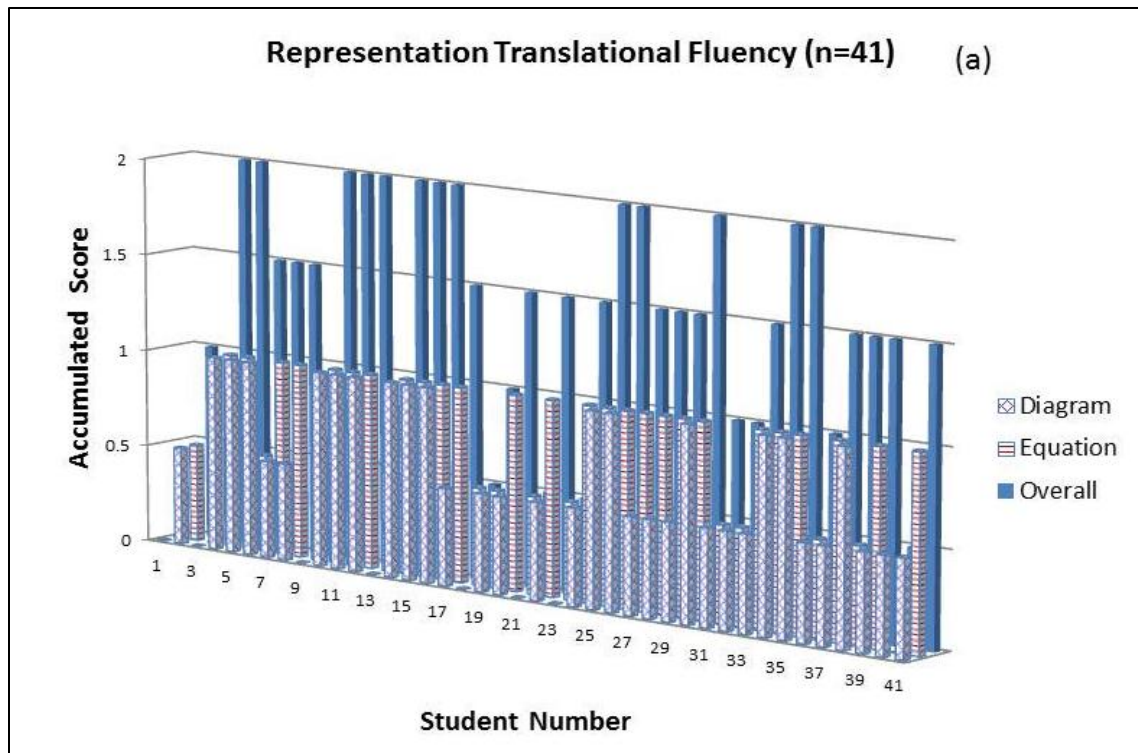
Figure 3. Accumulated counts of representational mode usage for the problem shown in Figure 2. (NA: none; DI: diagram; TX: text; EQ: equation; LA: written symbols on provided diagram)

Figure 3 displays how different representational tools were used in interpreting the problem shown in Figure 2. We used 41 out of 44 exam copies because of the scanned copy quality. As explained in the section of “data analyses” and Table III, a binary system of “0” and “1” was applied for usage counting. When one mode was used, a score of “1” was assigned. The counting is accumulated if more than one mode was applied. The vertical axis shows the accumulated counts of the mode usage. We noticed that seven students did not use any of these modes, (NA), to show how they interpreted the given information. Two of the seven NA students displayed the complete solution earning a perfect score.

Figure 4(a) shows how different representational modes were applied during problem solving. Students were specifically asked to use the phasor diagram to find V_s , and to use the phasor diagram to calculate the equivalent impedance, Z_{eq} . A score of “1” for “DI” means that the correct answer was displayed in the phasor diagram. The correct diagram included other quantities, such as V_L , V_R , I_R , I_C , etc. as well. A score of “0.5” means some of the key items were absent in the phasor diagram when the mode was applied. A score of “0” means no idea at all in using phasors to solve the problem. A score of “2” was used to indicate fluent translations and connections from the mode of “DI” (diagram) to the mode of “EQ” (equation). The vertical axis shows the accumulated score. Figures 4(b) and (c) categorize and review how these modes were applied. Fifteen out of 41 students (37%) displayed the required phasor diagram and earned a score of “1”, and 13 out of 15 who correctly applied the phasor linked the diagram to the equation to calculate the impedance earning an accumulated score of “2”. A difference of 0.16, which is the difference between the DI average score of 0.72 and the EQ average score of 0.88, in a scale of 0-1, is observed. The relative score of 0.16 compares the fluency level of using DI with EQ. Both (b) and (c) also show that 63% of the class (26 students) included equations in their work, almost twice as much as compared to 37% (15 students) who used the diagram. Yet, only half of these 26 students were able to apply the diagram. Figure 4(c) summarizes students’ usage of modes. Out of 28 students who used multiple representations, 13 students were fluent in translating from one mode to the other. The data show that students are comfortable in using equations, but not the diagram, a consistent finding as we mentioned earlier.²⁹

Figures 5(a), (b), and (c) illustrate how understanding of the three concepts, I_c , V_s , and I_s , played a role in problem solving and representational fluency. While 71% students included I_c in the phasor diagram, shown in (a), only 42% included V_s and I_s in the diagram. As shown in Figure 5(c), we notice that 17 students, about 2/3 of 26 students who identified equations for calculating Z_{eq} correctly included V_s and I_s in the phasor diagram. The remaining 1/3 of the group of 26 students remembered the formula, but failed to show that they understood what the meaning of each element of the equation is.

Data in Figures 7-11 show usages of multiple representations in the final exam. The problem from the exam is shown in Figure 6. As displayed in Figures 7(a) and (b), 40% of the class used and labeled the provided diagram (LA), 12% used equations (EQ), 9% used text (TX), and 37% drew diagrams (DI). Among them, 5 students used more than one mode to interpret the problem. Six students did not show how to interpret the problem (NA), and only one earned the grade above the average.



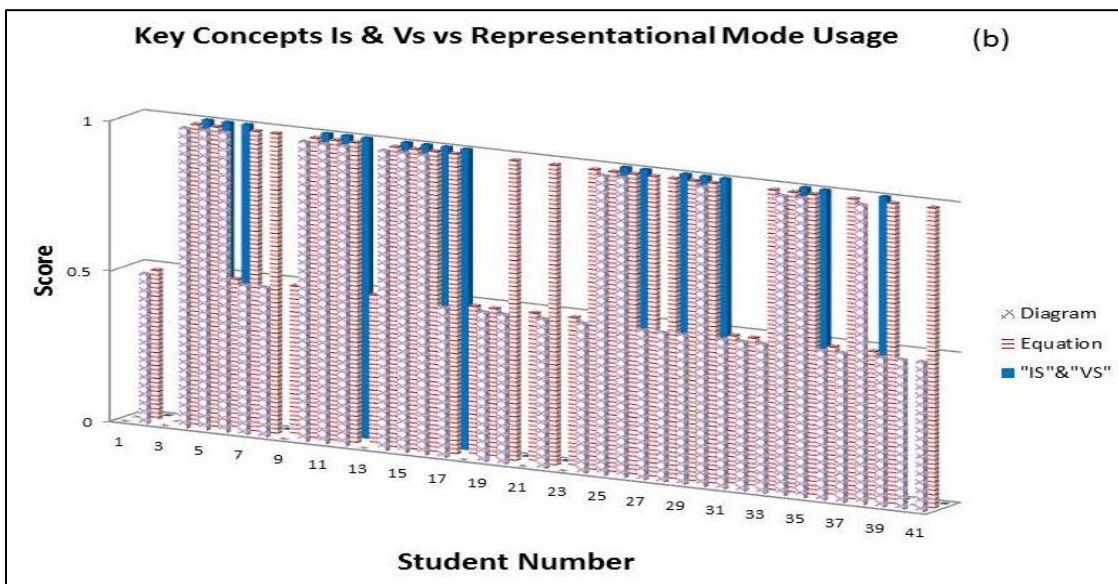
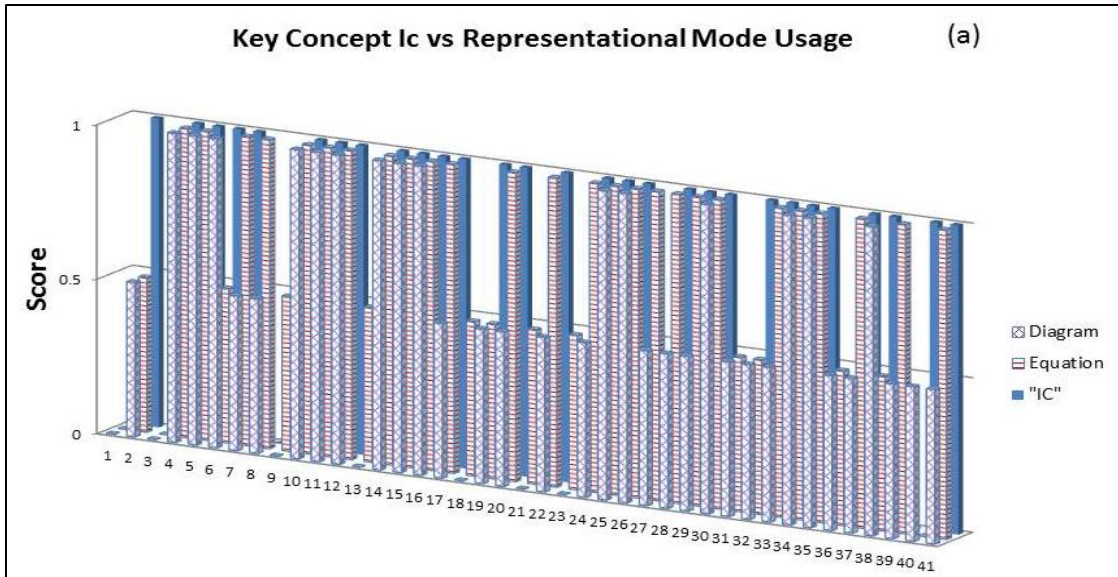
(b)

	Diagram (DI)		Equation (EQ)	
	DI score	Student number	EQ score	Student number
	0	(7)	0	(7)
	0.5	19	0.5	8
	1.0	15	1.0	26
Average score	0.72	34	0.88	34

(c)

	Multi-mode	DI Score "1"	EQ Score "1"	Overall Score "2"
Student number	28	15	26	13
Percentage	68.3%	36.6%	63.4%	31.7%

Figure 4. (a) Representational fluency (the problem shown in Fig. 2); (b) Average score for Diagram (DI) and Equation (EQ) usage; (c) Summary of representational fluency shown in (a).



(c)

Student Number	Diagram (DI) Score "1"	Equations (EQ) Score "1"	Total DI & EQ Score "2"	I_c shown in Phasor	V_s & I_s Shown in Phasor
Number	15	26	13	29	17
Percentage	37%	63%	31.7%	70.7%	41.5%

Figure 5. (a) Key concept of I_c vs multiple representational usage; (b) Key concepts of I_s and V_s vs multiple representational usage. (c) Summary of mastery of key concepts in terms of representational mode usage shown in (a) and (b). (The problem is shown in Figure 2).

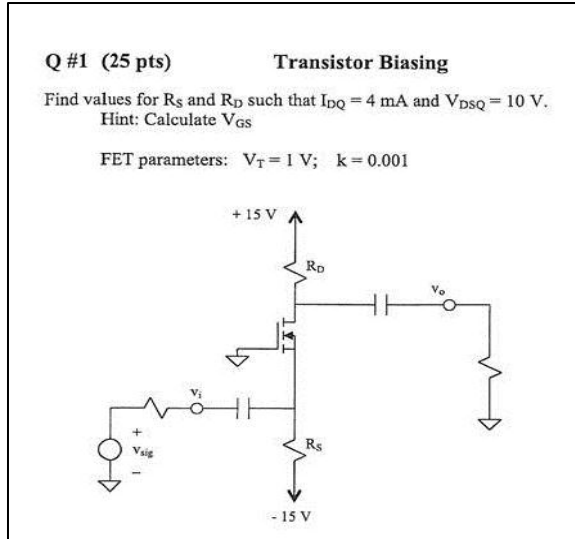


Figure 6. Question 1 of the final exam.

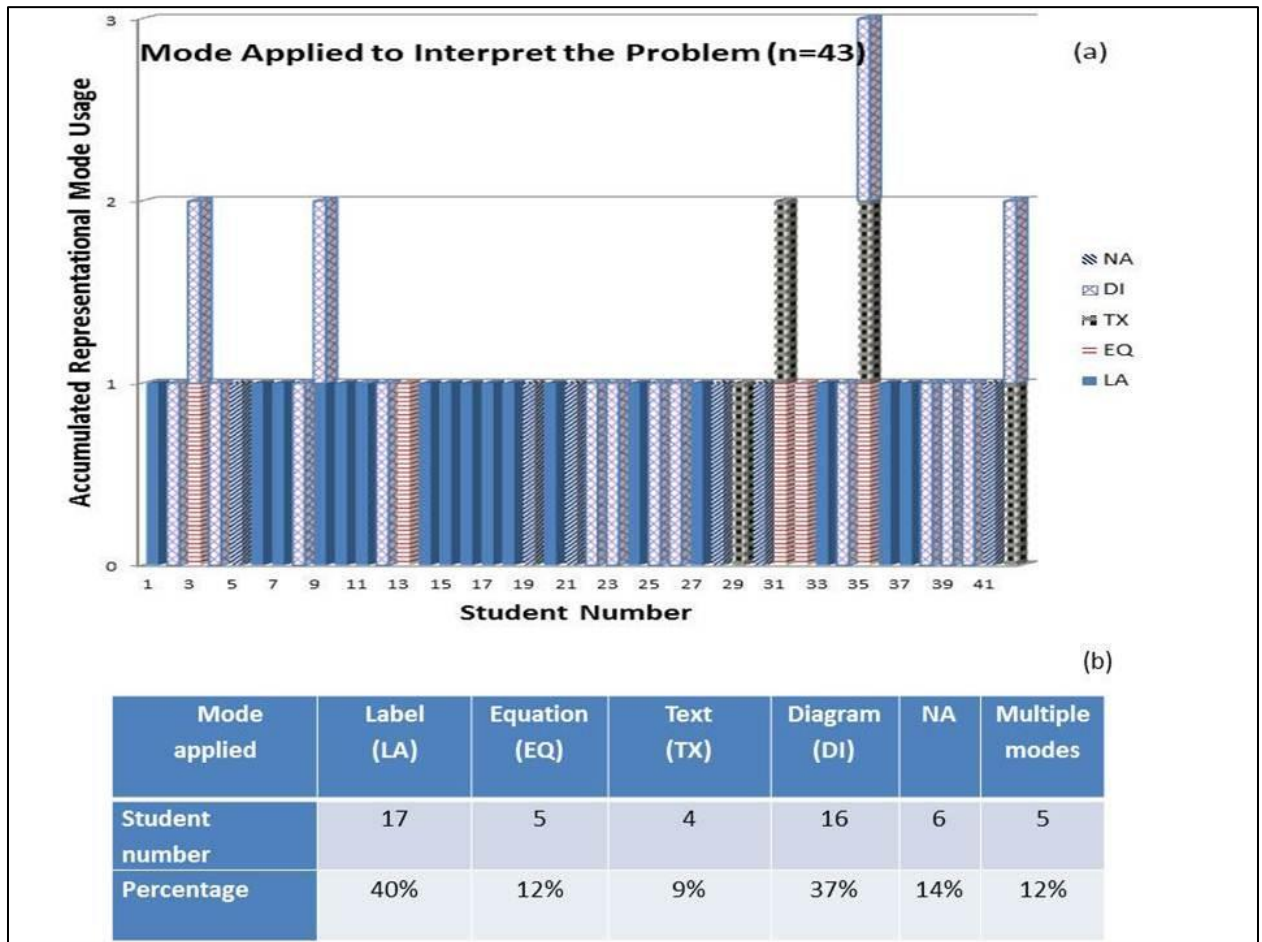


Figure 7. (a) Accumulated usages of representational mode for problem shown in Fig. 6; (b) Summary of the data shown in (a).

We did find that students applied various modes in making sense of the problem in meaningful ways and observed a significant improvement, as compared to their usage in the first midterm. Early in the semester, most students just put down some symbols when they used the provided circuitry diagram. No connections between the diagram and the phasor under study were shown. See data shown in Figures 2-5. To understand if this improvement helps their learning, we further examined the usage during problem solving processes. Figure 8 shows that students went back and forth between different representations to seek solutions. On average, half of the class applied more than one mode during the problem solving process. More than half of the group who earned a grade above the class average for this problem displayed efficient applications of multiple representations that facilitated problem solving. Figure 9 displays a few samples of students' work that depicts how translations between different representations influence the quality of the solution.

Figure 10 shows how students applied different representational modes to explain their solution for another problem in the final exam. The problem is shown in Figure 11. The first part of the question asked students to draw a circuit for a Cascode Amplifier, and the second part required explanations on how the Cascode Amplifier was able to overcome the frequency limitation imposed by Miller capacitance. We reviewed students' work and displayed the result in three groups based on their score of the problem. We noticed that only two out of 42 students failed to draw a correct circuit diagram as required. Thirty-three students displayed sound understanding of underlying concepts, such as frequency responses, gains, etc. Even though students were not asked to use other than text to explain their answer, 17 out of 42 students (40%) used additional modes in their explanations. By plotting the usage pattern vs the grade, we observed a similar trend shown in Figure 8: students who scored higher than the class average utilized more representational tools to express and support their ideas. Samples of students' work of applying several representational modes are included in Figure 11.

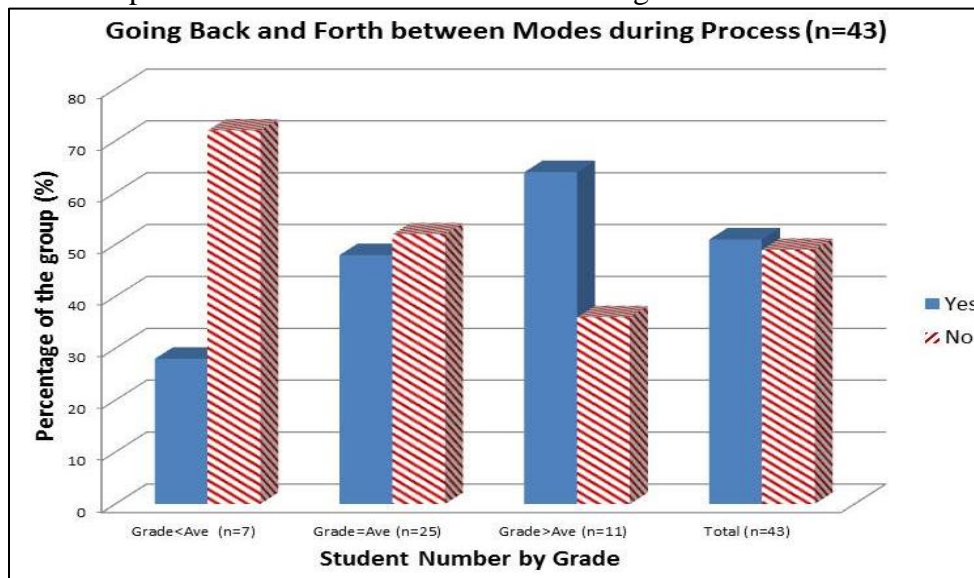


Figure 8. Percentage of the student groups that shifted usages of representational modes

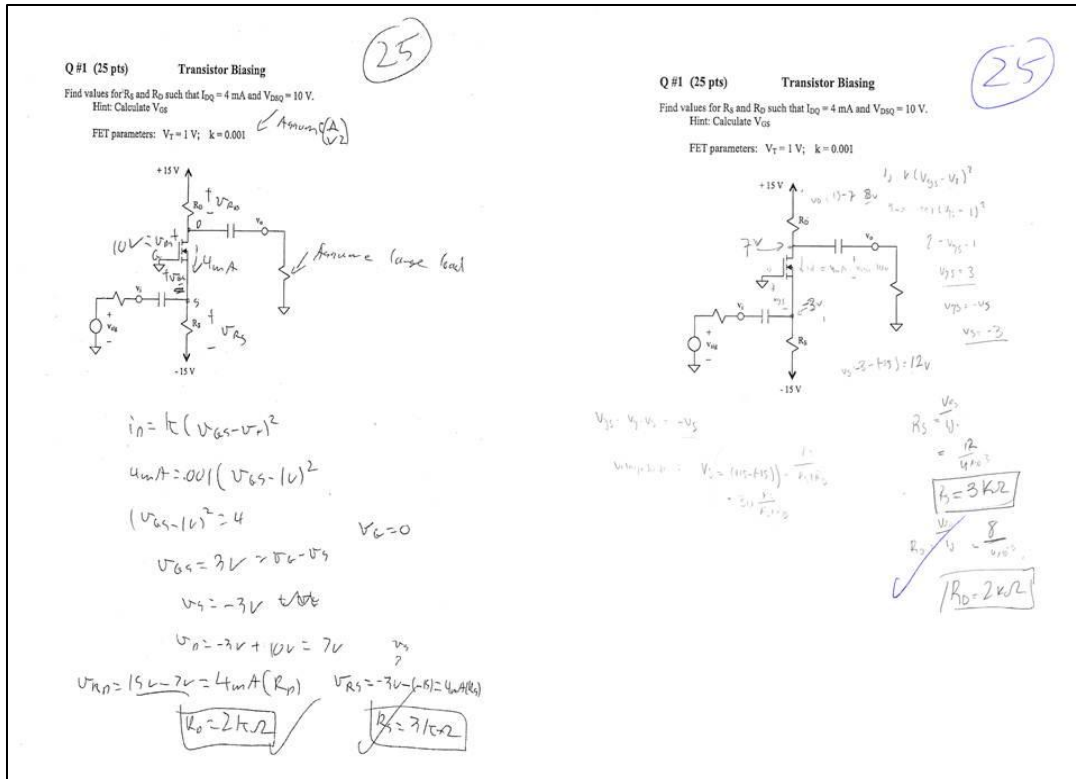


Figure 9. Samples of students' work show applications of multiple modes during problem solving processes.

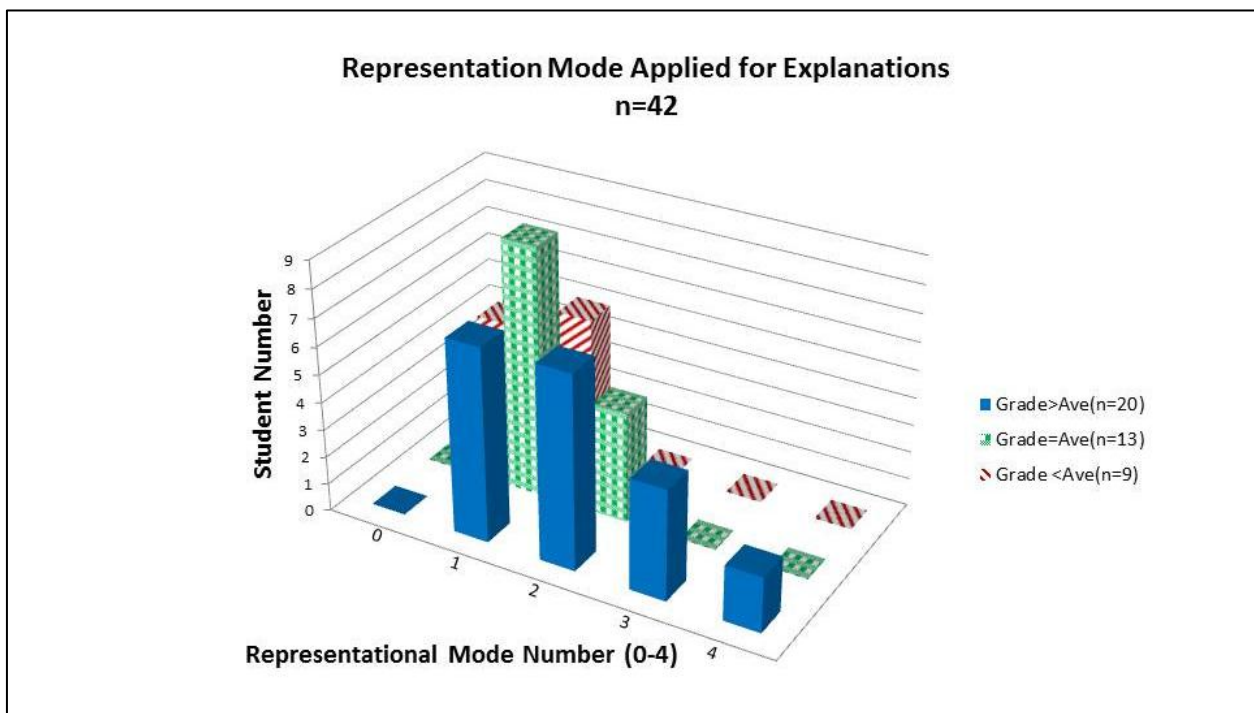


Figure 10. Representational mode applied for Question 4 in the final exam.

Q #4 (25 pts) Cascode Amplifier

To get around the frequency limitation of the common-source amplifier, which is primarily caused by the large Miller capacitance, we can implement a two-stage amplification technique yielding the Cascode Amplifier. The FET version of the Cascode Amplifier consists of a common-source amplifier whose output feeds the input of a common-gate amplifier.

a) Draw a valid circuit for a Cascode Amplifier.
b) How is the Cascode Amplifier able to overcome the frequency limitations of Miller capacitance?

a)

b)

The common source amplifier has low gain at high frequencies. I know the common gate has a high voltage gain, so I think that it must have a higher gain at high frequencies to offset the loss of gain in the common source amplifier. Or else it may have something to do with C_{gd} from the common source cancelling with some of C_{gs} from the common gate.

Q #4 (25 pts) Cascode Amplifier

To get around the frequency limitation of the common-source amplifier, which is primarily caused by the large Miller capacitance, we can implement a two-stage amplification technique yielding the Cascode Amplifier. The FET version of the Cascode Amplifier consists of a common-source amplifier whose output feeds the input of a common-gate amplifier.

a) Draw a valid circuit for a Cascode Amplifier.
b) How is the Cascode Amplifier able to overcome the frequency limitations of Miller capacitance?

a)

b) $A_{v,dc} = \frac{R_D}{R_S}$
If $R_S \rightarrow 0$, A_v is very large

$\omega_c = \frac{1}{R_{eq} C_{eq}}$
 C_{eq} non-inverting

* Total effective resistance seen by caps in CG is decreased due to current R_D and R_S , thus increasing ω_c

Figure 11. Samples of students' work show usage of multiple representational modes for explanation.

Discussions

We have focused on students' work during learning stages of interpreting the problem and processing problem solving, as shown in Table I, because these two phases of learning highlight metacognitive skills of "making sense" as well as "monitoring and self-regulating". Several findings from the current study support the claim that multiple representations hold great promises in designing and supporting instructional techniques in recitation sessions.

(1) Engineering students display varying comfort levels in applying different representational modes to interpret problems and define tasks. Earlier in the semester, it was evident that equations were the preferred mode for many students even though diagrams might be required for the problem under work, as displayed in Figure 3. This observed tendency of selecting equations as a means to approach problems is consistent with previous findings.²⁹ Though the heavier usage of certain modes, for example equations in this case, can be simply considered as learning habits, data in Figure 5 suggest that lack of conceptual understanding may play a role. This shortcoming is likely the main obstacle preventing students from learning to apply different representational modes for problem solving and other learning tasks. Stressing application and fluency in multiple representations helps instructors to identify instructional focus and understand what concepts are missing and what learning challenges are present. In order to help students recognize the relevance of utilizing multiple representation tools in learning content knowledge and problem solving, instructional focus should be placed on helping students understand the meaning of various representations, i.e. what content knowledge and concepts these modes stand for.

(2) Students' performance in the final exam illustrates the co-development of content knowledge, problem solving skills as well as metacognitive knowledge. Figure 7 shows how students applied several modes while attempting to understand what the problem meant. Many students no longer relied on equations because equations were not suitable in this case. This behavior change is indicative of improved understanding of cognitive and metacognitive knowledge because students recognized the relevance of multiple representational tools in their learning. Such co-development is further demonstrated by the data shown in Figure 8. We noticed that students shifted by going back and forth between several representational modes in problem solving. The majority of the group who earned a grade higher than the class average for the problem applied multiple representational tools resourcefully. Again, we observed that students with a better understanding of conceptual knowledge tended to effectively apply multiple representation tools in the process, as shown in Figure 10. Multiple representational tools applied in recitations engage students in integrating learning of content knowledge, problem solving skills, and metacognitive knowledge. As demonstrated by samples of their work, it is shown in Figure 9 how students used multiple representational tools to monitor progresses in problem solving.

(3) Multiple representational tools can be purposefully applied not only in formative assessment, but also in summative assessment of student learning. Before each exam, the instructor and the researcher would spend some time to talk about how student learning could be fairly assessed to help us gain insightful information of what students learned. Testing on students' usage of representational tools was incorporated while designing exam problems. For each exam, students were informed on subject topics to be included. Students were also aware of the significance of reasoning in problem-centered learning, which was stressed in both lectures and recitations. Students understood that they were expected to apply multiple representations because they were exposed to problems framed in various modes including text, equation, diagram, graph, etc. We were pleased to see that students who displayed translational fluency in multiple representation modes displayed deep understanding of content knowledge as well as metacognitive knowledge. The similar trend displayed in Figures 8 and 10 suggests that deeper conceptual understanding should help students understand the relevance of multiple representations in learning. Multiple representational modes embodied in almost all practices of problem solving in recitations offered students learning opportunities to gain competencies in interpreting, applying, and making connections across those representations as problem solving tools and resources. The problem included in Figure 11 was intended to ask students to use text to describe the circuit and reflect on engineering concepts. Students' reasoning and explanations using a variety of modes in a coordinated and fluent way went beyond our expectations.

Summary

A models and modeling perspective in problem solving and engineering learning promotes teaching and learning that facilitates the co-development of content knowledge, problem solving skills, and metacognitive knowledge. Using a variety of representational modes in teaching and learning practices has several implications: (i) it strengthens the association of instructional objectives for lectures and recitations by stressing conceptual systems that are distributed through a variety of representational media; (ii) it requires that problems are framed in ways that engage students in deep discussions during lecture periods and that provide students with resources and insights in problem solving during recitations; (iii) it offers opportunities for both

formative and summative assessments, allowing instructors to evaluate and understand what and how students learn.

Students' comments from two online surveys indicate that they supported new approaches that problematized content and emphasized applications of multiple representational tools. They were critical about problems presented to them, but were not as critical regarding instructors' teaching styles. Some problems helped them learn better in terms of connecting problems with underlying concepts. Others failed to engage them in constructing conceptual knowledge, specifically a few framed by daily life experiences and intended to help them apply analogies.¹³ They told us that they learned as well regardless of teaching styles of instructors. Students' comments are consistent with what we found in the previous studies: the way that problems are framed influences classroom discourse development and problem solving.^{8,13} Their comments are consistent with research findings that multiple representation tools can be used to promote equitable instructions to embrace different learning styles.³⁰

References

¹ National Academy of Engineers (NAE), "Educating the Engineer of 2020: Adapting Engineering Education to the New Century". Washington, DC, USA: National Academies Press, 2005.

² L.S. Shulman, "If Not Now, When? The Timeliness of Scholarship of the Education of Engineers", *Journal of Engineering Education*, January 2005, pp. 11-12; Jia-Ling Lin and Donald Woolston, "Important Lessons in Engineering Education Learned from Seven Years of Experience in Undergraduate Academic Support Programs", *Proceeding of the 38th ASEE/IEEE Frontiers in Education Conference*, Oct. 22-25, 2008.

³ B. Olds, M. Borrego, M. Besterfield-Sacre, and M. Cox, "Continuing the dialog: Possibilities for community action research in engineering education". *Journal of Engineering Education*, Vol. 101 (3):407-411, 2012; See other references therein.

⁴ Julie Foertsch, Gregory Moses, John Strikwerda, and Mike Litzkow, "Reversing the Lecture/Homework Paradigm Using eTEACH Web-based Streaming Video Software", *Journal of Engineering Education*, July (2002).

⁵ C. Demetry, Work in Progress – "An Innovation Merging Classroom Flip and Team-Based Learning", *Proceedings, the 40th ASEE/IEEE Frontiers in Education Conference*, October 27 - 30, Washington, DC. (2010).

⁶ S. Zappe, R. Leicht, J. Messner, T. Litzinger, and H.W. Lee, "Flipping the Classroom to Explore Active Learning in a Large Undergraduate Course", *Proceedings, the 116th American Society for Engineering Education Annual Conference & Exhibition* (2009).

⁷ Jeremy Strayer, *The effects of the classroom flip on the learning environment: a comparison of learning activity in a traditional classroom and flip classroom that used an intelligent tutoring system* (Doctoral Dissertation), (2007).

⁸ Jia-Ling Lin, Tamara Moore, and Paul Imbertson, “Introducing an Instructional Model in Undergraduate Electric Power Energy Systems Curriculum-Part (I): “Monological (Authoritative)” vs. Dialogic Discourse in a Problem-Centered Learning Classroom”, the 120th ASEE Annual Conference and Exposition, June 23-26, 2013.

⁹ M. Loftus. “Keep the lecture, lose the lectern: Blended Classes — Mixing Traditional and Digital Teaching — are Gaining Converts”, Connections Newsletter, October, (2013). <http://www.asee.org/papers-and-publications/blogs-and-newsletters/connections/October2013.html#sponsored>

¹⁰ Gregory S. Mason, Teodora Rutar Shuman, and Kathleen E. Cook, “Comparing the Effectiveness of an Inverted Classroom to a Traditional Classroom in an Upper-Division Engineering Course”, IEEE Transactions on Education, Vol. 56(4), November, (2013).

¹¹ Richard E. Mayer, “Applying the Science of Learning to Multimedia Instruction”, Chapter 3, in *Cambridge Handbook of Multimedia Learning*, (R.E. Mayer, Ed.) New York: Cambridge. (2005).

¹² Jia-Ling Lin, Paul Imbertson, and Tamara Moore, “Introducing an Instructional Model for ‘Flipped Classrooms’ -Part (II): How Do Group Discussions Foster Meaningful Learning?” The 121st ASEE Annual Conference and Exposition, 2014.

¹³ Jia-Ling Lin, Paul Imbertson, and Tamara Moore, “Theoretical Concepts, Practices, and Joint Efforts From Engineering Students and Instructors”, the 44th ASEE/IEEE Frontiers in Engineering Education (FIE) Conference, 2014.

¹⁴ J. Kim Vandiver, “Getting More out of Lecture and Recitation Time”, MIT Faculty News, Vol. XIX No. 5 March / April 2007.

¹⁵ Richard Lesh & Helen M. Doerr, “Foundations of a Models and Modeling Perspective on Mathematics”, (Chapter 1), in *Beyond Constructivism: Models and modeling perspectives on mathematics*, (R. Lesh and H. Doerr, editors). Mahwah, NJ: Lawrence Erlbaum Associates; (2003), pp 3-34.

¹⁶ Committee on Developments in the Science of Learning with additional material from the Committee on Learning Research and Educational Practice, National Research Council, “How people learn, Brain, Mind, Experience, and School”; Expanded Edition. (John Bransford, Ann Brown, and Rodney Cocking, Eds). National Academy Press, Washington, DC. (2000).

¹⁷ Kathleen Cramer, “Using a translation model for curriculum development and classroom instruction”, (Chapter 24), in *Beyond Constructivism: Models and modeling perspectives on mathematics*, (R. Lesh & H. Doerr , Eds). Mahwah, NJ: Lawrence Erlbaum Associates; (2003), pp 449-464.

¹⁸ P. Cobb, E. Yackel, & K. McClain, K. (Eds.), *Communicating and symbolizing in mathematics: Perspectives on discourse, tools, and instructional design*. Mahwah, NJ: Lawrence, Erlbaum Associates. (2000).

¹⁹ Tobin White a & Roy Pea , “Distributed by Design: On the Promises and Pitfalls of Collaborative Learning with Multiple Representations”, Vol. 20 (3), J. of the Learning Sciences, pp. 489-547, (2011) DOI: [10.1080/10508406.2010.542700](https://doi.org/10.1080/10508406.2010.542700)

²⁰ J. Bruner, “On cognitive growth”. In J. S. Bruner, R. R. Oliver, & P. M. Greenfield (Eds.), *Studies in cognitive growth: A collaboration at the Center for Cognitive Studies*, (1996), pp. 1–67, New York: John Wiley and Sons, Inc.; J. Bruner, Bruner on the learning of mathematics: A “process” orientation. In D. Aichele & R. Reys (Eds.); *Readings in secondary school mathematics*, (1971), pp. 166–192, Boston, MA: Prindle, Weber & Schmidt.

- ²¹ Tamara et al, "Modeling in Engineering: The Role of Representational Fluency in Students' Conceptual understanding", Vol. 102(1), J. of Engineering Education. (January 2013), pp. 141–178.
- ²² David R. Krathwohl, "Revising Bloom's Taxonomy: An Overview", Theory into Practice, autumn (2002).
- ²³ D.K. Detterman & R.J. Sternberg, (Eds.) *Intelligence, Cognition, and Instruction*. Norwood, NJ: Ablex. (1993).
- ²⁴ R.E. Haskell, *Transfer of Learning*, San Diego: Academic Press. (2001).
- ²⁵ Chris Quintana, Meiland Zhang, and Joseph Krajcik, "A Framework for Supporting Metacognitive Aspects of Online Inquiry Through Software-Based Scaffolding", Published online: 08 Jun 2010. Educational Psychologists, <http://www.tandfonline.com/loi/hedp20>
- ²⁶ <http://groups.physics.umn.edu/physed/Research/CRP/psintro.html>
- ²⁷ P. Cobb, J. Confrey, A. diSessa, R. Lehrer & L. Schauble, "Design experiments in educational research", *Educational Researcher*, 32(1), (2003). 9–13; Diana Joseph, "The Practice of Design-Based Research: Uncovering the Interplay Between Design, Research, and the Real-World Context", *Educational Psychologist*, 39(4), 235-242 (2004); See other references in the same issue; S. Barab (Ed.), Design-based research [Special issue]: *Journal of the Learning Science*, 13(1) (2004).
- ²⁸ Terry Anderson and Julie Shattuck, "Design-Based Research: A Decade of Progress in Education Research?", *Educational Researcher* (<http://er/aera.net>), Jan/Feb, (2012).
- ²⁹ Jia-Ling Lin, Jennifer M. Binzley, Eman Zaki, and Manuela Romero, "What is Important in Physics Learning? Understanding Perspectives and Providing Assistance for Engineering Students", the 119th ASEE Annual Conference and Exposition, June 10-13, 2012.
- ³⁰ L. Lesser, "Reunion of broken parts: Experiencing diversity in algebra", *Mathematics Teacher*, 93(1), (2000), 62–67; W. Roth, & M. McGinn, "Inscriptions: Toward a theory of representing as social practice", *Review of Educational Research*, 68(1), 35–59, (1998).