

## **Finite Element Analysis for the Engineering Technology Student**

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Finite element analysis (FEA) is a tool widely used by engineering professionals. It can also be a valuable educational tool for illustrating the distribution of stress, strain, and temperature in a component. However, it is also a tool that can be misused by those that have not received proper training. It has been offered as an elective in the Mechanical Engineering Technology program at the University of Pittsburgh at Johnstown for a number of years.

Fundamentally, FEA is a numerical method for solving complex engineering problems. The most common problems are related to structural analysis, heat transfer and fluid flow. Technically, the solution involves dividing a structure into discrete elements which are joined at common points called nodes, writing equations that describe the behavior of each element, applying loads and boundary conditions, assembling an overall stiffness matrix and solving the set of resulting simultaneous equations. However, through the use of FEA computer codes, the engineer is no longer burdened with the tedious process of writing equations for each element nor with the matrix algebra required to obtain the solution.

Using computer-based codes, the analyst must simply be able to generate solid models of a component, discretize the model into elements, apply loads and boundary conditions, wait for the computer to perform the analysis and interpret the results. Most of the modern commercial FEA computer codes in use today make this a relatively simple process. However, there is a good deal more to FEA than mechanically stepping through this process. A well-trained engineer has an appreciation, if not a complete understanding, of the mathematics behind the computer code, the importance of an adequate mesh, the critical nature of the correct application of loads and boundary conditions, and an ability to check analytical results using basic engineering hand calculations.

### **Why Teach FEA?**

There are many reasons to introduce FEA to undergraduate engineering students. At a minimum, it is an opportunity to give students a marketable job skill. However, more importantly, it is an opportunity to give students valuable fundamentals for the correct use of FEA. Most commercial FEA codes have excellent tutorials that enable an engineer to obtain a solution, but they usually stop short of teaching the supporting

theory. This theory provides not only an appreciation of the underlying mathematics, but also reinforces the importance of assigning proper boundary conditions. Without these boundary conditions, the students quickly see that they are asking the computer to solve an indeterminate matrix, in other words, to divide by zero. When it doesn't work in their hand calculations, they no longer expect it to work in the computer. Furthermore, through classroom instruction, emphasis can be placed on proper modeling techniques and the importance of verification. It is paramount that students learn to be critical of their results and not get in the habit of accepting computer-generated answers on faith.

At the University of Pittsburgh at Johnstown (UPJ), FEA is offered as a senior-level elective in the Mechanical Engineering Technology department during the spring semester. It is an outstanding opportunity to reinforce basic knowledge that the students have acquired from Statics through our senior-level course, Machine Design. Starting at the most basic level, Statics, students are reminded that unless they satisfy the equilibrium equations their model will not work. Beam-type models can be used to vividly illustrate stress distributions in beams whether under torsion or bending. The students can model stress concentrations, Figure 1, and calculate their own stress concentration factor.

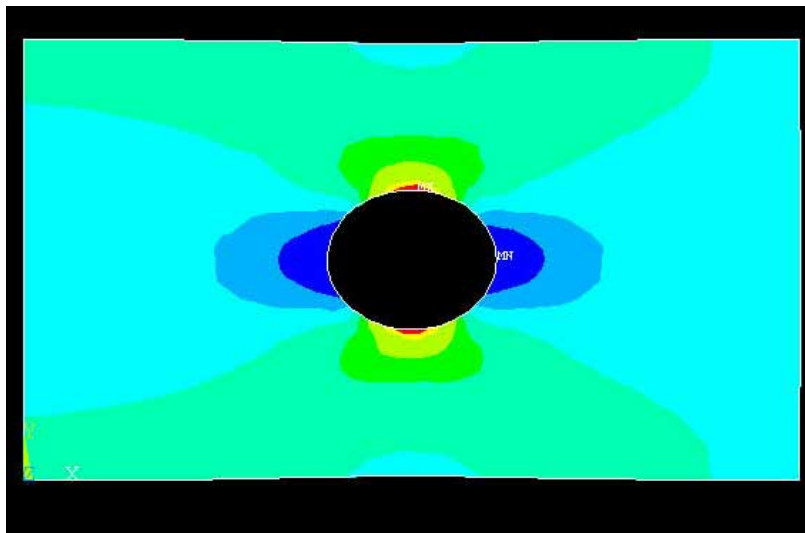


Figure 1 – Stress Distribution in a Plate with an Axial Load

Furthermore, they see that if the mesh density is inadequate in the region of interest, they will not be able to obtain satisfactory results. By starting with simple models that are readily verified with hand calculations, the students gain confidence in their FEA skills and their engineering knowledge and by the end of the term are ready to tackle fairly complex problems.

A pervasive characteristic of the FEA course is its inherent requirement for critical thinking. Whether the models are restricted by computer code limitations, computer processing limitations, or execution time, FEA models, loads, and boundary conditions are rarely an exact duplicate of the true problem. Instead, the students must learn to look at the problem and use techniques that they learn in the class coupled with their judgement to ascertain where they can simplify the model without impacting the validity of the results.

## **FEA Curriculum**

The FEA course at UPJ is divided into two main thrust areas, FEA Theory and FEA Application using a commercially available general-purpose FEA computer code. The specific code selected for instruction is not particularly important. However, the use of a commercially available code provides the students with a marketable job skill. The theory and application are taught in parallel to reinforce key aspects.

The course begins with a general outline of the finite element method. The outline is provided first for the theoretical approach and repeated for the computer-based approach. The similarity between the two methods is emphasized. The displacement or generalized stiffness method is the only theoretical approach that is demonstrated. Although other methods, such as the force or flexibility approach are occasionally used, they are not as popular. Next a simple example problem is developed using a series of springs. First, the direct stiffness method is used to solve the problem by hand. Since matrix algebra techniques are required for the analysis, a brief review is provided. Then the students are asked to repeat the problem, as an in-class exercise, using the FEA program. This simple example requires them to go through all required steps for computer analysis including generation of nodes and elements, assignment of loads and boundary conditions, and post-processing the results. During the discussion, they are shown that the simple spring model can be used to represent any number of uniaxial load conditions, including torsional loads on a shaft, and 1-D heat transfer. Their first out-of-class assignment requires them to repeat the analysis for a slightly more complex set of springs using a computer model and checking the results, by hand, using the direct stiffness method.

Next the students are introduced to the concept of local and global coordinate systems. This skill allows them to analyze simple 2-dimensional structures. For the out-of-class assignment, the students are asked to analyze a simple truss. The required computer analysis introduces them to a new type of element and a more complex set of boundary conditions. The supporting hand calculations are solved using the direct stiffness method and local coordinate systems. Students are also encouraged to check the results using techniques for truss analysis that they learned in Statics. Although deflections are sometimes difficult to check, they can, at a minimum, easily verify the loads in each element and the reaction loads.

At this point, the course departs from an emphasis on theoretical analysis. Up to this point, students have only generated computer models using nodes and elements that they have input by hand. This works well for small models, but is not practical for large 2-

and 3-D models. Most FEA codes have a built-in tool for generating solid models. These models are automatically meshed by the computer code based on instructions by the analyst. Students are now shown how to develop simple 2-D solid models and how to automatically mesh them with plane strain, plane stress, or shell elements to produce the FEA model. Use of the solid modeling is intentionally delayed until this point to make sure that the students have a firm understanding that the FEA model is comprised of nodes and elements versus the solid model that is comprised of geometric shapes. When generation of the two models is taught simultaneously, the students are often confused by the difference between the two.

The solid modeling portion of the course is extensive. Students are taught how to generate complex shapes using fillets and holes. They learn how to control the automatic meshing to decrease the element size in high stress areas. They learn how to check that they have an adequate mesh density. The use of symmetric boundary conditions to reduce model size is introduced, Figure 2.



Figure 2 – Model of Axial-Loaded Plate using Symmetric Boundary Conditions

The advantages of using symmetry are further reinforced through hand calculations using simple truss elements. Students quickly appreciate the value of symmetry when they are creating and solving a generalized stiffness matrix by hand. Emphasis is placed on how to assign boundary conditions to simulate the physical situation. Students have already seen that the set of equations they develop by hand for a given problem cannot be solved unless adequate boundary conditions are included.

Along with the modeling or pre-processing, students are shown how to post-process their results to obtain desired information. This includes stresses in the element coordinate system, principal stresses, and Von Mises stresses. Advanced post-processing techniques include how to obtain linearized and peak stresses and how to prepare plots suitable for a report.

The course concludes with additional theoretical work including how to rotate and transform the stiffness matrices and derivation of the 2-dimensional element. Finally, the students are shown one other approach for generating the generalized stiffness matrix, the minimum potential energy method.

Throughout the course, the students are assigned problems of increasing complexity to analyze. Some of the problems have included a plate with slotted holes, a bracket with a fillet radius, a cylindrical pressure vessel and flange. The final assignment in the course is a special project of the student's choice. They are encouraged to pick a project that will support their senior capstone design project. Some of the projects have included analysis of a composite beam with non-isotropic properties, a Formula One car frame under roll-over loads, a complex alloy car wheel, a spoked bicycle wheel and a motorcycle engine piston. These special projects are a valuable learning experience for the students and a meaningful practical application of their studies. Through these projects, they have to work through one of the most important issues with FEA, how to simplify the model, loads and boundary conditions to something that is practical to analyze yet representative of the actual design.

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