# First-year Project Experience in Aerospace: Apogee Determination of Model Rockets with Explicit Consideration of Drag Effect 

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# First Year Project Experience in Aerospace Engineering: Apogee Determination of Model Rockets with Explicit Consideration of Drag Effect 


#### Abstract

This paper describes a student team project to learn engineering solution methods for determining the apogee of a model rocket when the drag effect is considered explicitly. Model rocketry is a powerful tool for instructors who wish to incorporate science, engineering, and mathematics into a fun, engaging, and challenging activity for students. In this project, students construct their own rockets and launch them using commercial model rocket engines to determine the apogee of the flight. The apogee can be determined using a number of methods: trigonometry, onboard altimeters, analytical calculations, and simulation. This paper emphasizes numerical and other analytical prediction methods using spreadsheet programming instead of a full analytical solution that requires higher mathematics. Students got a practical introduction to many engineering concepts they will later study. These concepts include thrust, impulse, drag force, payload, ascent and descent (with and without a parachute) times, speed, and acceleration. The importance of the future courses in physics are emphasized. These activities constitute one of two team projects of a 1.5 credit portion of a two-credit course in exploration of engineering and technology at the Old Dominion University in Norfolk, VA. Students learn many skills they need later in their studies and professional practice such as spreadsheet data entry and mathematical operations. Not the least of which is teamwork, a skill that they acquire as they organize into groups with specialized responsibilities for the purpose of launching their rockets, collecting data to be processed, and writing a report. Metric units were used.


## Introduction and Educational Goals

Model rocketry is at once miniature astronautics, technological recreation, and an educational tool. A model rocket is a combined miniature version of a real launch vehicle. A model rocket is a very convenient metaphor to illustrate many important engineering concepts and principles in a fun and exciting way. Once a model rocket leaves the launcher, it is a free body in air. Model rockets have been used as student projects for decades. Other similar publications [1, 2, 10, 11, 17, 20, and 21] report engineering projects in the same general area, but this project is unique in the literature. Mathematics and physics aspects of model rocketry are reported in references $3,5,12$, $14,16,18$, and 23 . Figure 1 shows a cross section of a model rocket. The main educational goal of this project is to study the major methods to estimate the apogee and attempt to confirm the results using extensive field experiments. Student teams performed all analysis, programming, construction, and field work after they were instructed using smaller models. Uncertainty in prediction methods is a lesson well depicted in this project and a life lesson for students.


Figure 1. Single stage model rocket with an engine
Model Rocket Flight Details: A model rocket has three flight phases: powered flight, coasting flight, and recovery. Figure 2 shows the three phases of the flight. The rocket is launched by the ignition of the engine.


Figure 2. Model rocket flight profile [6].

The powered phase of the flight lasts until the engine has consumed all its propellant. During this phase, the model rocket accelerates and moves in response to the forces of thrust, gravity, drag and lift. In order for this phase of flight to be successful, the rocket must be stable. The fins enable the rocket to correct the flight path when it is momentarily perturbed. When perturbed, an angle develops between the rocket's longitudinal axis and the freestream velocity vector called the angle of attack. At angle of attack, the fins will create a lift force, generated by the relative wind, which causes the rocket to align its longitudinal axis with the relative wind again.

This project utilized Estes D12-5 engines. Model rocket engines consist of a propellant powder pressed into a nonmetallic cylindrical casing. The propellant is black powder made of potassium nitrate, sulfur, and charcoal. The rocket gets its power from a chemical reaction which is a nonstoppable controlled explosion that sends gas out the nozzle at a high speed. The appendix shows the specifications for the engine. It was noted that an impulse of 20 N -sec claimed by the manufacturer is disputed by an independent test carried out by the National Association of Rocketry (NAR). This project also confirmed that D12-5 engine's impulse is less than 20 according to field experiments using an altimeter. The impulse value is random with a mean of 16.84 and a standard deviation of 0.53 N -Sec. Similarly, burn times are also random and longer than 1.60 seconds stated by the manufacturer. The average mass of an engine was 47 grams (vs. 43.1 per manufacturer) in this project. The average mass of a used engine was 18 grams. The majority of the difference of 29 grams is due to lift propellant, but it also includes the chemicals for the tracer smoke and parachute ejection. Hence, the manufacturer's propellant mass value (21 grams) was used in calculations. Figure 3 shows a D12-5 engine on a scale.


Figure 3. Mass determination of a D12-5 engine

An altimeter (AltimeterTwo) was used as a payload. Figure 4 shows the payload. The appendix shows the specifications of the altimeter which worked very well in this project.


Figure 4. Mass determination of the payload
A stable rocket always flies into the relative air flow; the presence of wind blowing across the launch field affects the flight path. The relative wind is the sum of two components -the airflow opposite to the direction of the rocket's motion and aligned with the longitudinal axis and flow with a direction orthogonal to the longitudinal axis. The net result is a relative wind that causes the rocket's flight path to curve away from the pure vertical and into the upwind direction. This effect is called "weathercocking" [6, 22]. The rocket’s aerodynamic drag is very important. The drag force is expressed as follows:
$\mathrm{D}=0.5 \rho \mathrm{~V}^{2} \mathrm{C}_{\mathrm{d}} \mathrm{A}(1)$
Where $\rho$ is the density of the air. $\mathrm{C}_{\mathrm{d}}$ is the coefficient of drag that depends on the shape and surface finish of the rocket. V is the velocity and A is the frontal area of the model. A high thrust engine will cause a rocket to experience more drag than a low thrust engine because the rocket will reach higher velocities, but higher thrust engines will still make the rocket reach a higher apogee. The coasting phase begins when the propellant is exhausted. The delay element is a timing device that controls the deployment of the recovery system. Recovery system deployment occurs near the apogee in most cases. During the coasting phase, the rocket slows down since the engine no longer produces thrust. The smoke that is observed comes from the smoke-tracking and delay element of the engine. The recovery phase starts as soon as the smoketracking and delay element is exhausted. Model rockets can be dangerous if not handled safely. Each student turns in a homework early in the semester by handwriting the entire safety code of
the NAR. The students are also asked to find and report a model rocket accident in the same homework.

## Commercially Available Model Rockets

Model rockets are available in two common forms: ready to fly (RTF) and to be constructed from a kit. Figure 5 below shows the RTF models used for demonstration and practice calculations. The RTF model rockets in Figure 5, from left to right, are the Fat Jack, the Rattler-7, the MaxTrax, and the Skytrax which comes with its own payload bay. A payload can be an altimeter as in Figure 5 or anything else that is allowable including an insect. The MaxTrax has a built-in altimeter, but it is very unreliable.


Figure 5. RTF model rockets used for practice

The project used QWEST Corporation's Big Dog (B \& D) model that had to be constructed. The $B \& D$ is a relatively large model, but it does not have a payload bay as in the SkyTrax above. The cone had to be fitted with hooks and wires to hang the altimeter and tiny holes had to be drilled on the upper part of the tube to allow air in for pressure differential detection. The B \& D model has four fins, a length of 82.55 cm and a mass of 198.45 grams according to the manufacturer which also specifies the maximum diameter as 4.80 cm . Measured diameters were larger due to painting and 5.10 cm was used in calculations. In addition, a launch lug with a diameter of 0.635 cm was attached.

## Dynamics of Model Rockets

If we assume a vertical flight with zero degree of angle of attack and ignore the lift as a force to simplify calculations, there are three force factors on a model rocket as shown on the right in Figure 6 below.

1) Thrust ( T ) from the engine acts on the back of the model and makes it accelerate,
2) Weight (W) is a force that slows the model in its vertical flight. This force decreases slightly over time due to propellant consumption and is the product of gravitation acceleration and mass
3) Aerodynamic drag (D) force is a force due to air friction and separated flow which acts to slow the model.


Figure 6. Actual and simplified (right) free body diagrams of a model rocket in Flight [2]
During the powered flight, all three forces act upon the model, but the thrust is zero during the coasting flight. Freshmen are introduced to Newton's three laws of motion or reacquainted as some were exposed to them in high school physics. These laws are stated below:

1. Objects at rest will stay at rest, and objects in motion will stay in motion in a straight line at constant velocity unless acted upon by an unbalanced force,
2. Force is equal to mass times acceleration. F = ma(2). This equation applies to launching the rocket off the launch pad. Thrust is a forward propulsive force that moves an object and is produced by the engine. As the engine ignites and thrust develops, the forces become unbalanced. The rocket then accelerates. Rocket propellant is burned and converted into gas that expands and then
escapes from the rocket. Acceleration is the rate at which the gas escapes. The gas inside the engine accelerates as it leaves the engine. The greater the amount of propellant burned, and the faster the gas produced can escape the engine, the higher the resulting thrust.
3. For every action there is always an opposite and equal reaction. A model rocket will lift off if it expels gas out of its engine. The rocket pushes on the gas and the gas pushes on the rocket. The action, then, is the expulsion of gas out of the engine. In return, the reaction is the departure of the rocket skyward.

## Model Construction and Pre-Launch Activities

In fall 2015, 104 students were randomly assigned to groups of 3 to 5 and each group was given a kit to construct the model. Figures 7 through 12 show various stages of model construction and launch preparation.


Figure 7A. Construction of the B\&D model mockets by student teams


Figure 7B. Painting


Figure 8A. Engine Mount Insertion and Painting of the Models.


Figure 8B. Holes drilled to allow air for altimeter to work


Figure 9. Drill work and wiring for payload (altimeter) insertion


Figure 10. Nose cone with an unattached payload


Figure 11A. Determination launch masses excluding the payload and wiring (engine included)


Figure 11B. Determination launch masses excluding the payload and wiring (engine included)


Figure 12. Determination of the launch lass of another model rocket
Figures 13 and 14 show the moments just ahead of marching to a remote parking lot on campus. The university parking service was able to make only one-half of a remote lot available on a Sunday. This lot is by no means an ideal place, but the urban location of the campus leaves few alternatives. Figure 15 shows the instructor and two teams.


Figure 13. B\& D model rockets ready for launch


Figure 14. The main launch event on 10/25/2015


Figure 15. Make up launch event in November 2015
The altimeter was set before each launch as shown below in Figure 16.


Figure 16. Altimeter is set to launch

Figure 17 shows an in-house built launcher as hobby store launchers were not always reliable.


Figure 17. Home built launcher to reduce misfires and provide consistent current
Figures 18 through 21 show various launch and flight scenes. On-board altimeter collects the data shown in Table 1.


Figure 18. The first B\&D model rocket descends


Figure 19. A B\&D model rocket accelerates and clears the "tower"


Figure 20. Another model rocket takes off


Figure 21. It ascends

## Flight Trial Launch Results

Table 1 below shows the data for 27 launches of B\&D model rockets in fall 2015. Five rockets with altimeters were lost due to various reasons. Teams with lost rockets were allowed to use another team's data. While this project concentrated on the determination of apogee, a wealth of additional flight data (speed, acceleration, and flight times) were also obtained as shown in Table 1. The appendix includes two sample raw data collection sheets for fall and spring semesters.

## Apogee Prediction Methods

This section provides the variables and the formulas used by the students. Five non-physical apogee determination methods are also explained. Students learn that physical data, as in Table 1 above, should be collected after other methods, analytical and simulation based, are first applied in order to reduce costs and improve physical experiment.

## Definition of Variables

```
mon Lift-off mass (kg)
mp}=\mathrm{ Propellant mass (kg)
mb
W
W
T = Thrust (N)
D = Drag force (N)
Vmax }\mp@subsup{\mp@code{N}}{= Maximum speed at burnout (m/sec)}{\mathrm{ )}
Vmax }=\mathrm{ Maximum speed (MPH)
Vav}= Average speed (m/sec
Sb
St}=\mathrm{ Total apogee without drag (m)
S
S = Expected apogee with drag (m)
g = Acceleration of gravity (m/sec}\mp@subsup{}{}{2}
d = Drag effect factor
Cd= Drag coefficient
I = Total impulse (N-Sec)
B
\Delta \mathrm { t } = \text { Time increment for numerical analysis (sec)}
a = Acceleration (m/sec}\mp@subsup{}{}{2}
V = Speed (m/sec)
\rho = Air density (kg/m}\mp@subsup{}{}{3}
A = Area of rocket cross section (m}\mp@subsup{}{}{2}
r = Radius of the rocket (m)
```

|  | M4 | W8 | W10 | W6 | M1 | W5 | M9 | M13 | M2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Launch Sequence | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Launch Mass (kg) | ? | 0.236 | 0.246 | 0.236 | 0.241 | 0.241 | 0.241 | 0.241 | 0.231 |
| Apogee (m) | 103 | 99 | 94 | 98 | 101 | 98 | 82 C | 93 | 98 |
| Parachute Ejection (m) | 102 | 94 | 85 | 79 | 98 | 56 | 62 | 89 | 75 |
| Maximum Speed (m/sec) | 39 | 37 | 38 | 38 | 38 | 38 | 38 | 37 | 39 |
| Descent Speed (m/sec) | 5 | 5 | 5 | 8 | 5 | 3 | 4 | 5 | 7 |
| Thrust Time (sec) | 2.70 | 2.25 | 2.16 | 1.94 | 2.10 | 2.20 | 2.50 | 2.50 | 1.90 |
| Coast Time (m/sec) | 3.30 | 3.20 | 3.10 | 3.20 | 3.30 | 3.10 | 2.70 | 2.80 | 3.10 |
| Apogee to Eject Time(sec) | 0.60 | 1.10 | 1.50 | 2.20 | 1.00 | n/a | 2.50 | 1.20 | 2.50 |
| Total Flight Time (sec) | 24.30 | 23.50 | 22.20 | 16.70 | 25.40 | 23.20 | 20.40 | 24.20 | 18.30 |
| Peak Acceleration (G) | 6.0 | 6.0 | 6.3 | 6.3 | 6.1 | 8.4 | 6.5 | 6.3 | 6.8 |
| Average Acceleration (G) | 2.0 | 1.7 | 1.8 | 2.0 | 1.8 | 1.7 | 1.9 | 1.7 | 2.0 |
| Model Rocket ID | M10 | W13 | M12 | M15 | M3 | M5 | W3 | M14 | W9 |
| Launch Sequence | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| Launch Mass (kg) | 0.236 | 0.236 | 0.246 | 0.236 | 0.251 | 0.246 | 0.236 | 0.236 | 0.241 |
| Apogee (m) | 111 | 110 | 87 | 87 | 80 | 91 S | 113 | 93 | 91 |
| Parachute Ejection (m) | 109 | 110 | 78 | 77 | 70 | 81 | 109 | 68 | 76 |
| Maximum Speed (m/sec) | 41 | 40 | 25 | 36 | 35 | 35 | 40 | 37 | 38 |
| Descent Speed ( $\mathrm{m} / \mathrm{sec}$ ) | 5 | 4 | 13 | 6 | 5 | 6 | 5 | 4 | 6 |
| Thrust Time (sec) | 2.18 | 2.21 | 1.79 | 1.92 | 2.16 | 2.13 | 2.17 | 1.92 | 1.88 |
| Coast Time (m/sec) | 3.40 | 3.90 | 2.80 | 2.90 | 2.70 | 3.00 | 3.30 | 3.10 | 3.00 |
| Apogee to Eject Time(sec) | 0.70 | n/a | 1.70 | 1.70 | 1.60 | 1.60 | 1.10 | 2.60 | 2.00 |
| Total Flight Time (sec) | 27.50 | 29.00 | 11.9 P | 19.10 | 19.20 | 20.00 | 27.40 | 22.40 | 19.80 |
| Peak Acceleration (G) | 7.0 | 6.8 | 3.6 | 6.5 | 5.7 | 5.6 | 6.8 | 6.7 | 5.4 |
| Average Acceleration (G) | 1.9 | 1.8 | 1.5 | 1.9 | 1.6 | 1.7 | 1.9 | 2.0 | 2.1 |
| Model Rocket ID | M7 | W12 | M8 | W10 | W10-R | W4 | W1 | W2 | W? |
| Launch Sequence | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| Launch Mass (kg) | 0.226 | 0.241 | 0.251 | 0.246 | 0.245 | 0.237 | 0.239 | 0.236 | 0.235 |
| Apogee (m) | 104 | 94 | 100 | 94 | 99 | 108 | 96 | 112 | 101 |
| Parachute Ejection (m) | 103 | 87 | 93 | 85 | 96 | 106 | 89 | 108 | 77 |
| Maximum Speed ( $\mathrm{m} / \mathrm{sec}$ ) | 39 | 35 | 36 | 36 | 38 | 39 | 37 | 40 | 40 |
| Descent Speed (m/sec) | 4 | 5 | 6 | 5 | 4 | 5 | 5 | 9 | 8 |
| Thrust Time (sec) | 2.21 | 2.15 | 2.20 | 2.16 | 2.15 | 2.33 | 2.18 | 2.19 | 1.96 |
| Coast Time ( $\mathrm{m} / \mathrm{sec}$ ) | 3.30 | 3.10 | 3.10 | 3.10 | 3.00 | 3.10 | 3.10 | 3.40 | ? |
| Apogee to Eject Time(sec) | 0.40 | 1.30 | 1.40 | 1.50 | 1.10 | 0.90 | 1.30 | 1.00 | 2.40 |
| Total Flight Time (sec) | 27.60 | 24.00 | 21.50 | 22.20 | 26.60 | 25.70 | 24.70 | 18.60 | 17.00 |
| Peak Acceleration (G) | 6.6 | 5 | 5.6 | 6.3 | 6.5 | 6.1 | 6 | 6.8 | 7 |
| Average Acceleration (G) | 1.8 | 1.7 | 1.7 | 1.8 | 1.8 | 1.8 | 1.7 | 1.9 | 2.1 |

Table 1. Fall 2015 Launch Data

## 1) Simplified Analytical Spreadsheet Model

F in equation 2 is the sum of all applied forces in Figure 6; $m$ is the mass of the model and a is the resulting acceleration. $I=m_{b} V_{m a x}^{1}-m_{0} V_{0}(3), v_{0}=$ velocity at lift-off which is zero. Impulse is also equal to force $x$ time. Thrust $=$ total impulse/burn time or $T=\left(I / B_{t}\right)$ (4). Using equation 2, the net force ignoring the drag is $\mathrm{T}-\mathrm{W}=\mathrm{m}$ a. Then, $\mathrm{T}-\mathrm{W}=(\mathrm{W} / \mathrm{g})\left(\mathrm{Vmax}_{1} / \mathrm{B}_{\mathrm{t}}\right)(5)$ where $V_{m a x}^{1}=a B_{t}$ and $m=(W / g)$. This yields: $\operatorname{Vmax}_{1}=[(T / W)-1] g B_{t}$ (6). The term $\mathrm{Vmax}_{1}$ is the velocity at the end of the impulse or thrust period and called the burnout velocity. It is also the maximum velocity the model can attain. At lift-off, the propellant to total rocket mass ratio is about $5 \%$ for most model rocket and engine combinations. This is contrary to full-scale rockets. This issue is discussed in class by looking up some larger rocket mass ratios on the web. The ratio of the propellant's mass to the model rocket's is very small and during the flight the mass stays fairly constant. As the force is fixed and the mass is almost constant, acceleration can be assumed constant. The velocity is acceleration times the thrust time $\left(V=a B_{t}\right)$ where, $B_{t}$ is the end of the thrust duration. The average velocity during thrusting is the average of initial and maximum velocities or just half of the maximum velocity:
$\mathrm{V}_{\mathrm{av}}=\mathrm{Vmax}_{1} / 2$ (7).
While thrusting for $B_{t}$ seconds, the model rocket will climb a distance of $S_{b}=V_{a v} B_{t}$ When the engine stops thrusting, the model rocket starts its coasting flight and climbs up an additional distance of $\mathrm{S}_{\mathrm{c}}=\left[\left(\mathrm{Vmax}_{1}\right)^{2} /(2 \mathrm{~g})\right]$ (9). The apogee is the sum of both distances, $S_{t}=S_{b}+S_{c} \quad(10)$.

The discussion so far neglects lift and drag forces. The aerodynamic drag has a big effect on the actual altitude. Stine's text [22] states that aerodynamic drag lowers the computed drag-free maximum altitude of a model rocket by 50 percent (for low-powered models) to as much as 80 percent (for high-powered models). Then, the actual apogee is: $S=S_{t} * d$ (11). Then, $d$ is the fraction of $S_{t}$ that is the actual apogee. The value of $d$ ranges from 0.20 to 0.80 , but the value of coefficient "d" is never found in the engine specifications of any manufacturer. Steps outlined in equations 3 through 11 constitute the simplified analytical method implemented by each project team before the drag effect is considered explicitly in other methods. A d value of 0.50 seemed to yield the correct apogee value determined later by more advanced methods. The "d" variable is often confused with $\mathrm{C}_{\mathrm{d}}$ variable used in other methods.

Figure 22 shows the EXCEL code for the method 1.

## 2) Numerical Analytical Spreadsheet Model

Thrust: $\mathrm{T}=\mathrm{I} / \mathrm{B}_{\mathrm{t}}$
Weight Force:
$\mathrm{W}_{0}=-\mathrm{m}_{\mathrm{o}}{ }^{*} \mathrm{~g}$ before burn out
$\mathrm{W}_{\mathrm{b}}=-\mathrm{m}_{\mathrm{b}}{ }^{*} \mathrm{~g}$ after burn out
Drag force:

$$
\begin{equation*}
\mathrm{D}=-0.5^{*} \mathrm{Cd}_{\mathrm{d}} * \rho * \mathrm{~A} * \mathrm{~V}^{2} \tag{15}
\end{equation*}
$$

Acceleration:
$\mathrm{a}=\left(\mathrm{T}+\mathrm{W}_{0}+\mathrm{D}\right) / \mathrm{m}_{0}$ before burn out
$\mathrm{a}=\left(\mathrm{W}_{\mathrm{b}}+\mathrm{D}\right) / \mathrm{m}_{\mathrm{b}}$ after burn out
Speed: $V=V$ (before) $+a^{*} \Delta t$
Apogee: $\mathrm{H}=\mathrm{H}$ (before) $+\mathrm{V}^{*} \Delta \mathrm{t}$

Apogee is reached at an iteration just before the speed becomes negative indicating the model is no longer climbing. Figures 23 through 26 show various sections of the EXCEL code for this method. Many students with no programming or other EXCEL experience had some difficulty in understanding both methods and the idea of programming in general, but they were very pleased to eventually understand and apply EXCEL to this project. Reference [19] provides a comprehensive treatment numerical solution approach to model rocket calculations.

Each method was run for two burnout or thrust times: manufacturer's (1.65 seconds) and typical field value ( 2.16 seconds) as shown in Table 1. A drag coefficient of 0.95 was used. This value is justified in the appendix.
3) Web Based Calculator: This tool [8] applies Fehskens-Malewicki_[15] single stage model rocket equations. This method is an example of another simplified analytical approach. The equations of this method are listed in the appendix, but not used by the students in this project. Figure 33 shows the input and output screen using two burn times.
4) Web Based Numerical Analytical Model : A web based "simulator" [7] was also used. Figures 31 and 32 show the application and the output curves. This tool appears to be a numerical method instead of a simulation as its title implies.
5) RockSim Simulation: Each team constructed the model rocket on a computer using the RockSim model rocket simulation software [19] as shown in Figure 29. Figure 30 shows the output of this simulation software.
6) Trigonometric Method: This method was only applied to the last launch (No. 27). Using hand-held tracking devices, two experienced students measured the apogee angles from two opposite locations each located 200 feet away from the launch pad.

Physical Experiment can be considered as the Method 7.

|  | A | B | c | $\square$ | E | F | G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Altitude Prediction Program |  |  |  | ENGN 110 |  |  |
| 2 | PROGRAMMER NAME: Z. CLARKE |  |  |  |  |  |  |
| 3 | Inputs are in blue |  |  |  |  |  |  |
| 4 | ENGINE DATA |  |  | Units |  |  |  |
| 5 | Engine Name | D12-5 |  |  |  |  |  |
| 6 | Total Impulse | 17.00000 | $\mathrm{N}-\mathrm{sec}$ | metric |  |  |  |
| 7 | Thrust Duration | 1.65000 | Seconds |  |  |  |  |
| 8 | Delay Time | 5.00000 | Seconds |  |  |  |  |
| 9 | Engine Mass | 0.04710 | kg | metric |  |  |  |
| 10 | Propellant Mass | 0.02500 | kg | metric |  |  |  |
| 11 | g value in metric | 9.81000 | $\mathrm{m} / \mathrm{sec}^{\wedge} 2$ |  |  |  |  |
| 12 | Drag effect factor "d" | 0.50000 |  |  |  |  |  |
| 13 | MODEL DATA |  |  |  |  |  |  |
| 14 | Model Name | Skytrax |  |  |  |  |  |
| 15 | Model + Engine + Plugs + Igniter + Paper | 0.23500 | kg |  |  |  |  |
| 16 | Wires + Payload | 0.01100 | kg |  |  |  |  |
| 17 | Lift-off mass of model (mo) | 0.24600 | kg |  |  |  |  |
| 18 | (including the engine \& altimeter) |  |  |  |  |  |  |
| 19 | Burnout mass of model ( mb ) $=\mathrm{mo}-(\mathrm{mp} / 2)$ | Eq 1 | 0.234 | kg |  |  |  |
| 20 | Burnout mass as weight $=$ mass $\times \mathrm{g}$ | Eq 2 | 2.291 | N |  |  |  |
| 21 | Thrust (T) | Eq 3 | 10.303 | N |  |  |  |
| 22 | Vmax - Max Velocity at Burn out | Eq 4 | 56.62 | $\mathrm{m} / \mathrm{sec}$ |  |  |  |
| 23 | Vmax in more familiar MPH units | Eq 5 | 127.39 | MPH | Model rockets ca |  |  |
| 24 | V -average - on the way from ground to the burnout point | Eq 6 | 28.31 | $\mathrm{m} / \mathrm{sec}$ | go very fast. Be | eful ! |  |
| 25 |  |  |  |  |  |  |  |
| 26 | $\mathrm{Sb}=$ where the burnout occurs | Eq 7 | 46.71 | m | above ground |  |  |
| 27 | (ascend during thrusting in feet) |  |  |  |  |  |  |
| 28 |  |  |  |  |  |  |  |
| 29 | $\mathrm{Sc}=$ Coasting distance above the burn -out point | Eq 8 | 163.39 | m |  |  |  |
| 30 | (ascend during coasting in feet) |  |  |  |  |  |  |
| 31 |  |  |  |  |  |  |  |
| 32 | $\mathrm{St}=$ | Eq 9 | 210.10 | m | if taking off from | place |  |
| 33 | (total ascend or apogee) |  |  |  | like the moon wit | no air |  |
| 34 | Estimated "d" in equation 10 for DRAG |  |  |  |  |  |  |
| 35 | $\mathrm{S}=$ | Eq 10 | 105.05 | m | $=$ |  |  |
| 36 | (expected apogee) |  |  |  |  |  |  |

Figure 22. Simplified analytical solution

|  | B\&D | w/ D12-5 | Engine |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5.1 cm DIA |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | Amount | UNITS |  |  |  |  |  |  |  |  |  |  |  |  |
| Mass at rest (m) | 0.235 | kg | Incl. engine |  |  | Thrust Force = Impulse/B | /Burn Time (UP | (UP or positive) |  |  |  |  |  |  |
| Altimeter + wires | 0.011 | kg | Payload |  |  |  |  |  |  |  |  |  |  |  |
| Launch Mass | 0.246 |  | Incl. Payload |  |  |  |  |  |  |  |  |  |  |  |
| Propellant | 0.021 | kg |  |  |  | Weight Force $=$ Mass * ${ }^{\text {G }}$ | Gravity Acce | leration (DOWN or | r Negative) |  |  |  |  |  |
| Mass after Burnout | 0.225 |  |  |  |  | Drag Force $=-0.5$ Drag | ag Coeff * Air | Density* Area * Sp | peed^2 (DOWN | or Negative) |  |  |  |  |
| Area | 0.0020428 | $\mathrm{m}^{\wedge} 2$ |  |  |  |  |  |  |  |  |  |  |  |  |
| rho (air density) | 1.22 | $\mathrm{kg} / \mathrm{m}^{\wedge} 3$ |  |  | Acceleration $=$ | (Thrust - Weight - Drag)/M | /Mass or N | NET FORCE/MASS |  |  | Max acceleration | Max Speed |  |  |
| Gravity Acceleration | 9.81 | $\mathrm{m} / \mathrm{sec}^{\wedge} 2$ |  |  |  |  |  |  |  |  | 32.072 | 45.684 |  |  |
| C (drag coefficient) | 0.95 | no unit | . $77+25 \%$ | (lug effect) |  | Speed ( $t$ ) $=$ Speed ( $(\mathrm{t}-1)+$ | +a * Delta t |  |  |  | or | $\mathrm{m} / \mathrm{sec}$ |  |  |
| weight-(m*g) | -2.41326 | Newtons |  |  |  | Height(t) $=$ Height (t-1) + | + Speed (t) * | Deltat |  |  | 3.269 |  |  |  |
| Weight after burnout | -2.20725 | Newtons |  |  |  |  |  |  |  |  | G |  |  |  |
| Deltat | 0.05 | 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| Engine impulse | 17 | N-S |  |  |  | It is NOT a free fall after | a apogee. |  |  | Apogee: | 110.100 | meters |  |  |
| Burn time | 1.65 | 5 |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| ) |  |  |  |  |  | Time (sec) |  | Engine Thrust ( N ) | Drag Force ( N ) | Net Force ( N ) | Acceleration (m/sec^2) | Speed ( $\mathrm{m} / \mathrm{s}$ ) | Apogee (meters) | Apogee(Feet) |
|  |  |  |  |  |  | 0.00 |  | 10.303 | 0.000 | 7.890 | 32.072 | 0.000 | 0.000 | 0.000 |
|  |  |  |  |  |  | 0.05 |  | 10.303 | 0.000 | 7.890 | 32.072 | 1.604 | 0.080 | 0.263 |
|  |  |  |  |  |  | 0.10 |  | 10.303 | -0.003 | 7.887 | 32.060 | 3.207 | 0.241 | 0.789 |
|  |  |  |  |  |  | 0.15 |  | 10.303 | -0.012 | 7.878 | 32.023 | 4.808 | 0.481 | 1.578 |
|  |  |  |  |  |  | 0.20 |  | 10.303 | -0.027 | 7.862 | 31.961 | 6.406 | 0.801 | 2.629 |
|  |  |  |  |  |  | 0.25 |  | 10.303 | -0.049 | 7.841 | 31.875 | 8.000 | 1.201 | 3.941 |
|  |  |  |  |  |  | 0.30 |  | 10.303 | -0.076 | 7.814 | 31.764 | 9.588 | 1.681 | 5.514 |
|  |  |  |  |  |  | 0.35 |  | 10.303 | -0.109 | 7.781 | 31.630 | 11.169 | 2.239 | 7.346 |
|  |  |  |  |  |  | 0.40 |  | 10.303 | -0.148 | 7.742 | 31.472 | 12.743 | 2.876 | 9.436 |
|  |  |  |  |  |  | 0.45 |  | 10.303 | -0.192 | 7.698 | 31.291 | 14.307 | 3.592 | 11.783 |

Figure 23. The numerical method

|  | 1.35 |  | 9.882 | -1./U8 | 3.869 | 24.9/6 | 39.98/ | 29.168 | 95.695 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1.40 |  | 9.882 | -1.819 | 5.758 | 24.500 | 41.212 | 31.228 | 102.455 |
|  | 1.45 |  | 9.882 | -1.933 | 5.644 | 24.019 | 42.413 | 33.349 | 109.413 |
|  | 1.50 |  | 9.882 | -2.047 | 5.530 | 23.533 | 43.590 | 35.529 | 116.563 |
|  | 1.55 |  | 9.882 | -2.162 | 5.415 | 23.043 | 44.742 | 37.766 | 123.903 |
|  | 1.60 |  | 9.882 | -2.278 | 5.299 | 23.426 | 45.913 | 40.061 | 131.435 |
|  | 1.65 |  | 9.882 | -2.399 | 5.178 | 22.912 | 47.059 | 42.414 | 139.154 |
| m, w change | 1.70 | engine stop | 0.000 | -2.520 | -4.619 | -21.585 | 45.980 | 44.713 | 146.697 |
|  | 1.75 |  | 0.000 | -2.406 | -4.505 | -21.051 | 44.927 | 46.960 | 154.067 |
|  | 1.80 |  | 0.000 | -2.297 | -4.396 | -20.542 | 43.900 | 49.155 | 161.269 |
|  | 1.85 |  | 0.000 | -2.193 | -4.292 | -20.057 | 42.897 | 51.300 | 168.305 |
|  | 1.90 |  | 0.000 | -2.094 | -4.193 | -19.594 | 41.918 | 53.395 | 175.182 |
|  | 1.95 |  | 0.000 | -1.999 | -4.099 | -19.152 | 40.960 | 55.443 | 181.901 |
|  | 2.00 |  | 0.000 | -1.909 | -4.008 | -18.730 | 40.023 | 57.445 | 188.466 |
|  | 2.05 |  | 0.000 | -1.823 | -3.922 | -18.327 | 39.107 | 59.400 | 194.882 |
|  | 2.10 |  | 0.000 | -1.740 | -3.840 | -17.942 | 38.210 | 61.310 | 201.150 |
|  | 2.15 |  | 0.000 | -1.661 | -3.761 | -17.573 | 37.331 | 63.177 | 207.274 |
|  | 220 |  | ก กnก | -1 586 | -2 685 | -1720n | 26. 470 | 65 กn1 | 212756 |

Figure 24. Mass is reduced by the amount of propellant at burn out


Figure 25. The Apogee is 110 meters as the speed becomes negative ( 1.65 sec burnout)

| --. | -...- | --...- | ----- |  | 0.664 | 106197 | 348.416 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5.35 | 0.000 | -0.002 | -2.209 | -9.817 | 0.664 | 106.197 | 348.416 |
| 5.40 | 0.000 | -0.001 | -2.208 | -9.812 | 0.173 | 106.206 | 348.444 |
| 5.45 | 0.000 | 0.000 | -2.207 | -9.810 | -0.317 | 106.190 | 348.392 |
| 5.50 | 0.000 | 0.000 | -2.207 | -9.811 | -0.808 | 106.149 | 348.259 |
| 5.55 | 0.000 | -0.001 | -2.208 | -9.813 | -1.299 | 106.085 | 348.046 |
| 5.60 | 0.000 | -0.002 | -2.209 | -9.819 | -1.790 | 105.995 | 347.753 |

Figure 26. The Apogee is 106 meters as the speed becomes negative ( 2.16 sec burnout)

Figures 27 and 28 show the altitude and speed as a function of time for the numerical method using a burn time of 1.65 seconds. The maximum speed is reached around 5 seconds which is the sum of the most thrust and the coast times for the field or the physical experiment data in Table 1.


Figure 27. Altitude vs. time plot for the numerical method


Figure 28. Speed vs. time plot for the numerical method

## Rocksim- H:/Documents/ENGN 110/Rocket project/Quest_Big_Dog.rkt

File Edit View Rocket Simulation Help


| Rocket design attributes | Rocket design components | Mass override | Cd override | Flight simulations |
| :--- | :--- | :--- | :--- | :--- |



Figure 29. Model rocket ronfiguration and input for the RockSim simulation software of reference 19.

## Quest Big Dog - Simulation results

## Engine selection

[D12-5]

## Simulation control parameters

Flight resolution: 800.000000 samples/second
Descent resolution: 1.000000 samples/second
Method: Explicit Euler
End the simulation when the rocket reaches the ground.

## Launch conditions

Altitude: 0.00000 Ft .
Relative humidity: 50.000 \%
Temperature: $59.000 \mathrm{Deg} . \mathrm{F}$
Pressure: 29.9139 In.
Wind speed model: Calm (0-2 MPH)
Low wind speed: 0.0000 MPH
High wind speed: 2.0000 MPH

## Wind turbulence: Fairly constant speed (0.01)

Frequency: $0.010000 \mathrm{rad} /$ second
Wind starts at altitude: 0.00000 Ft .
Launch guide angle: 0.000 Deg.
Latitude: 0.000 Degrees

## Launch guide data:

Launch guide length: 60.0000 In .
Velocity at launch guide departure: $53.9730 \mathrm{ft} / \mathrm{s}$
The launch guide was deared at: 0.317 Seconds
User specified minimum velocity for stable flight: $43.9993 \mathrm{ft} / \mathrm{s}$
Minimum velocity for stable fight reached at: 41.3987 In .

## Max data values:

Maximum acceleration:Vertical (y): $378.069 \mathrm{Ft} . / \mathrm{s} /$ sHorizontal (x): $1.643 \mathrm{Ft} . / \mathrm{s} / \mathrm{sMagnitude}: 378.069 \mathrm{Ft} . / \mathrm{s} / \mathrm{s}$
Maximum velocity:Vertical (y): $159.4399 \mathrm{ft} / \mathrm{s}$, Horizontal ( x ): $2.1433 \mathrm{ft} / \mathrm{s}$, Magnitude: $159.5749 \mathrm{ft} / \mathrm{s}$
Maximum range from launch site: 26.29281 Ft .
Maximum altitude: $414.71016^{\circ} \mathrm{Ft}$. ..

## Engine ejection charge data:

Using a delay time of : 5.000 Seconds
Velocity: $39.8422 \mathrm{ft} / \mathrm{s}$
Altitude: 388.80045 Ft .

## Recovery system data

P: Parachute Deployed at: 6.651 Seconds
Velocity at deployment: $39.8422 \mathrm{ft} / \mathrm{s}$
Altitude at deployment: 388.80045 Ft .
Range at deployment: -26.29281 Ft .

Figure 30. The output of the RockSim simulation software of reference 19.


Model Rocketry Simulator

| Rocket Details and Simulation Constants: |  |
| :--- | :--- |
| Rocket Name: | B\&D |
| Date: | $02-01-2016$ |
| Rocket Mass (dry): | $0.245: 44$ |
| Engine Type: | 31 |
| Mass of Engine Casing: | 0.0241 |
| Propellant Mass: | 0.0211 |
| Thrust: | 10.21 |
| Impulse: | 17 |
| Rocket Diameter: | 0.051 |
| Air Density: | 1.22 |
| Drag Coefficient: | 0.95 |
| Parachute/sheet Diameter: | 0.30 |
| Time Increment: | 0.05 |
| Mass Decrement: | 0.012672411764706 |
| Gravitational Constant: | 9.8 |
| Area of Rocket: | 0.0020428206229968 |
| Parachute/sheet Area: | 0.07068583470577 |
| Burn Time: | 1.6650342801175 |
| Eject Time: | 5 |


| Time | Drag Force | Thrust | Net Force | Mass | Acceleration | Velocity | Altitude | Area | Velocity (mph) | Air Density |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | ---: | ---: |
| $t$ | Fd | Ft | F | $M$ | Acc | $V$ | $Y$ | Area | mph | rho |
| 0 | 0.00000 | 10.21 | 7.36604 | 0.29020 | 25.38263 | 1.26913 | 0.09518 | 0.00204 | 2.83905 | 1.22000 |
| 0.05 | 0.00191 | 10.21 | 7.37034 | 0.28957 | 25.45303 | 2.54178 | 0.25409 | 0.00204 | 5.68597 | 1.21999 |
| 0.10 | 0.00765 | 10.21 | 7.37081 | 0.28893 | 25.51047 | 3.81731 | 0.47684 | 0.00204 | 8.53932 | 1.21997 |
| 0.15 | 0.01725 | 10.21 | 7.36742 | 0.28830 | 25.55477 | 5.09505 | 0.76354 | 0.00204 | 11.39762 | 1.21994 |
| 0.20 | 0.03073 | 10.21 | 7.36015 | 0.28767 | 25.58579 | 6.37433 | 1.11424 | 0.00204 | 14.25939 | 1.21990 |
| 0.25 | 0.04810 | 10.21 | 7.34899 | 0.28703 | 25.60340 | 7.65450 | 1.52897 | 0.00204 | 17.12313 | 1.21986 |
| 0.30 | 0.06935 | 10.21 | 7.33395 | 0.28640 | 25.60751 | 8.93488 | 2.00772 | 0.00204 | 19.98733 | 1.21980 |
| 0.35 | 0.09449 | 10.21 | 7.31502 | 0.28576 | 25.59806 | 10.21478 | 2.55046 | 0.00204 | 22.85047 | 1.21974 |


|  |  |  |  |  |  | 1 | 1 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4.86 | 0.01564 | 0.001 | -2.65082 | 0.26916 | -9.856/61 | 2.92022 | 100.046/11 | 0.002041 | 6.532531 | 1.20122 |
| 4.85 | 0.00999 | 0.00 | -2.64717 | 0.26910 | -9.83712 | 2.42836 | 100.15583 | 0.00204 | 5.43225 | 1.20721 |
| 4.90 | 0.00691 | 0.00 | -2.64409 | 0.26910 | -9.82567 | 1.93708 | 100.24040 | 0.00204 | 4.33325 | 1.20719 |
| 4.95 | 0.00440 | 0.00 | -2.64158 | 0.26910 | -9.81633 | 1.44626 | 100.30045 | 0.00204 | 3.23529 | 1.20718 |
| 5.00 | 0.00245 | 0.00 | -2.63963 | 0.26910 | -9.80910 | 0.95581 | 100.33598 | 0.00204 | 2.13814 | 1.20718 |
| 5.05 | 0.00107 | 0.00 | -2.63825 | 0.26910 | -9.80398 | 0.46501 | 100.34700 | 0, 00204 | 1.04157 | 1.20717 |
| 5.10 | 0.00025 | 0.00 | -2.63743 | 0.26910 | -9.80094 | -0.02444 | 100.33353 | 0.00204 | -0.05467 | 1.20717 |
| 5.15 | -0.00000 | 0.00 | -2.63718 | 0.26910 | -9.80000 | -a 51444 | 190.29556 | 9.00204 | -1.15080 | 1.20717 |
| 5.20 | -0.00031 | 0.00 | -2.63687 | 0.26910 | -9.79885 | -1.00438 | 100.23309 | 0.00204 | -2.24680 | 1.20718 |
| 5.25 | -0.00118 | 0.00 | -2.63600 | 0.26910 | -9.79561 | -1.49416 | 100.14614 | 0.00204 | -3.34244 | 1.20718 |
| 5.30 | -0.00262 | 0.00 | -2.63456 | 0.26910 | -9.79028 | -1.98368 | 100.03471 | 0.00204 | -4.43748 | 1.20719 |
| 5.35 | -0.00461 | 0.00 | -2.63257 | 0.26910 | -9.78287 | -2.47282 | 99.89885 | 0.00204 | -5.53170 | 1.20721 |

Figure 31. Input and output screens for the web based numerical tool [7].


Figure 32. Altitude and speed as functions of time using the web based numerical tool [7]

Rocket Altitude Calculator (Flash ver. .0.9)


Help


Rocket Altitude Calculator (Flash ver. 0.9 )


Figure 33. Input and output screens of the web based calculator [8]

## Physical Experimental vs. Other Results

Table 1 shows that the actual burn times were usually more than the manufacturer's specified value of 1.65 seconds for engine D12-5. Table 2 shows the apogee values for two burn times: 1.65 and 2.16 seconds for each of the five non-physical methods using a launch mass of 0.246 kg that includes the engine, payload, and attachments. The apogee values in field data have a mean of 98 meters with a standard deviation of 9 meters. The burn times have a mean of 2.0 seconds and a standard deviation of 0.2 seconds. Physical experiments generally agree with the results shown in Table 2.

| Method No. | Method | Burn Time: 2.16 Sec. | Burn Time: 1.65 Sec. |
| :---: | :--- | :---: | :---: |
| 1 | Simple Analytical | 96 | 105 |
| 2 | Numerical Analytical | 106 | 110 |
| 3 | Web Based Calculator | 118 | 122 |
| 4 | Web Based Numerical | N/A | 100 |
| 5 | RockSim Simulator | N/A | 126 |

Table 2. Apogee Comparison of the Five Methods Used by Student Teams
The methods 1 and 2 (programmed by each student team) and the method 4 are in good agreement with the field or the physical experiment results while the methods 3 and 5 are within statistical agreement. It should be noted the actual launch masses varied as shown in Table 1, but there was no clear correlation between the launch mass and the apogee as measured by the altimeter. The method 5 assumes ideal conditions and results in the highest apogee value. The top three altimeter values were 113 (Table 1) and 125 and 119 meters observed in spring 2016.

## Conclusions

The outcomes include an understanding of how each of the apogee prediction methods works, a realization that engineering results are sometimes inconsistent, and appreciation for teamwork. Other outcomes include gaining a firm belief that engineering data is often resource intensive, facility dependent, and it must be carefully recorded and saved for future use. All launch results are entered into a spreadsheet and posted on a web-based educational management program, for all teams to share. Documentation of lessons learned is a major outcome also. Just as in the real practice, model rocket launches are subject to many unexpected and surprising problems including loss of a vehicle with costly altimeter or other sensors onboard. Each team carefully notes and reports the problem encountered and remedy, if any, to the instructor who enters the information to the master flight log spreadsheet. Practice activities use RTF rockets with progressively bigger engines and higher average thrusts. Students get a practical introduction to many engineering concepts they will encounter later on in their studies and can decide if engineering excites them based on experience, rather than just textbook learning.

This paper has discussed a major group project using model rockets in a two-hour per week laboratory that is a part of a two-credit course in exploration of engineering and technology. The paper presented experiences with the rocket project with particular focus on comparing and
contrasting several approaches to apogee prediction. As an aside, students also learned that the metric units were easier to use in aerospace work. The other main lab team project (metal cutting and manufacturing), however, uses the U.S. standard units to provide a balanced engineering unit instruction.

There were several educational goals expected of this project: 1) develop team work skills, 2) gain appreciation for future coursework in physics, statics, dynamics, aerodynamics, flight dynamics, and thermodynamics, 3) get an early understanding on the role of experimental (physical and simulated) and analytical approaches to solve engineering problems, 4) gain practice in writing technical team reports, 5) experience a "real life," hands-on engineering project from start to finish, 6) learn about rockets in general, 7) excite students about engineering and space exploration and improve the overall retention rate. These educational goals were either accomplished or it is too soon to tell, as in the case of goal 7 that seeks to improve retention. Team reports show that goals $1,3,4$, and 5 were accomplished at various levels that ranged from fair to excellent. Teams were required to meet with the instructor and/or the graduate assistant to review draft versions of reports. An anonymous exit survey taken on the last day of classes indicate that a majority ( $70 \%$ ) of the students felt this project was a very good learning experience for all the stated goals. In addition, substantial anecdotal evidence suggests that this project had a positive impact on student learning and retention. Positive student comments about the project were not just limited to student exit survey and course evaluations.

## Recommendations

The urban location of the campus made it difficult to find as much open space as desired to conduct better experiments. The maximum distance from the launch pad was 65 to 100 meters depending on the direction from the launch area. This distance was not long enough for good triangulation. This project would work much better and be more exciting if more open space were available. Limited area made it impossible to use E engines that could send the models to 250 meters instead of around 110 meters feet as is the case now. E engines with twice the impulse were used in spring 2015, but too many rockets were lost due to the inability to track them in the urban environment. An altimeter should be used as a gold standard if there is a high probability that it can be recovered. This probability can be increased by simply adding additional weight as payload to cause more powerful engines to provide a lower apogee. More paint and/or an additional payload can be added to increase weight. This project was repeated in spring 2016. A key chain was used to hang the altimeter to the cone instead of wire used in fall 2015. This change allowed much faster removal and re-installation of the altimeters between successive launches on different rockets. Metal wires used in the fall semester to attach the payload took too long to tie and then cut open them. Table 1 shows that launch masses varied by up to 20 grams among the 27 rockets. All launch masses should be equal. This will make the results more consistent and easier to compare with the methods summarized in Table 2. It also became clear that engine loading and other prep work should be done before the launch day as the combined fall semester activity took over 8 hours. This was too long for many students who had other commitments. Finally, more angular data should be taken to further confirm altimeter data using trigonometry.

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## Appendix

Fehskens-Malewicki Equations of the Method 3

For those of you who enjoy a good equation (and who doesn't?) here are the Fehskens-Malewicki equations for a single-stage rocket:

## Single Stage Fehskens-Malewicki Equations:

burnout velocity:

$$
v_{b}=\sqrt{\frac{F-m g}{k}} \tanh \left[\frac{t_{b}}{m} \sqrt{k(F-m g)}\right]
$$

$$
y_{\mathrm{b}}=\frac{\mathrm{m}}{\mathrm{k}} \ln \left\{\cosh \left[\frac{t_{\mathrm{b}}}{\mathrm{~m}} \sqrt{\mathrm{k}(\mathrm{~F}-\mathrm{mg})}\right]\right\}
$$

$$
\begin{aligned}
& \text { coast altitude: } \\
& \qquad y_{\mathrm{c}}=\frac{\mathrm{m}_{\mathrm{b}}}{2 \mathrm{k}} \ln \left[\frac{\mathrm{k} \mathrm{v}_{\mathrm{b}}^{2}}{\mathrm{~m}_{\mathrm{b}} \mathrm{~g}}+1\right] \\
& \text { coast time: } \\
& \mathrm{t}_{\mathrm{c}}=\sqrt{\frac{\mathrm{m}_{\mathrm{b}}}{\mathrm{gk}}} \tan ^{-1}\left[\mathrm{v}_{\mathrm{b}} \sqrt{\frac{\mathrm{k}}{\mathrm{gm}}}\right]
\end{aligned}
$$

$$
\begin{aligned}
& \text { Where: } \\
& \mathrm{k}=\frac{1}{2} \rho_{\mathrm{D}} \mathrm{~A} \\
& \mathrm{p}=\text { atmospheric density } \\
& \mathrm{C}_{\mathrm{D}}=\text { drag coefficient } \\
& \mathrm{A}=\text { frontal area } \\
& \mathrm{t}_{\mathrm{b}}=\text { burn time } \\
& \mathrm{F}=\text { aperage thrust } \\
& \mathrm{m}=\text { aperage thrusting mass } \\
& \mathrm{m}_{\mathrm{b}}=\text { burnout mass } \\
& \mathrm{g}=\text { accelerationdue to gravity }
\end{aligned}
$$

## Solution Methods Studied in this Project as Depicted by Students



## Engine Data from the Manufacturer and the NAR.

## ESTES D12

| CERTIFIED |
| :--- |
| VALUES |
|  |
|  |
|  |
| STATIC |
| TEST DATA |
|  |


| Total Impulse: Delays: | 17.00 newton-seconds 0, 3, 5, 7 seconds |  |  |
| :---: | :---: | :---: | :---: |
| Propellant Type: Propellant Mass: | Black Powder 21.1 grams |  |  |
| Casing Dimensions: | $24 \mathrm{~mm} \times 70 \mathrm{~mm}$ |  |  |
| Certification Date: Contest Use Date: | Continuing Continuing |  |  |
| Certification Type: | Model Rocket |  |  |
| Date Tested: | 94-September-17 |  |  |
| Total Impulse: | 16.84 newton-seconds 29.73 newtons 1.65 seconds <br> 10.21 newtons |  |  |
| Peak Thrust: |  |  | (\% 4.59) |
| Burn Time: Average Thrust: |  |  | $\left(\begin{array}{ll}\sigma & 0.30)\end{array}\right.$ |
| Average Thrust: |  |  |  |
| Mass After Firing: | 16.0 grams |  |  |
|  | $\begin{aligned} & \text { Average } \\ & \text { easured Delay } \end{aligned}$ |  |  |
|  |  |  | Max $\underset{396.29}{\text { Lifteff Weight }}$ |
|  | $\begin{aligned} & 0.00 \\ & 239 \end{aligned}$ |  | ${ }^{36629}$ |
|  | 4.25 5.75 | 45.29 44.9 |  |

TYPICAL
CURVE


## Estes D12-5 Rocket Engine Specifications:

| Engine Type: | D12-5 |
| :--- | :--- |
| Total Impulse - N-sec: | 20.0 |
| Time Delay - Sec.: | 5 |
| Maximum Lift Mass - g: | 283 |
| Maximum Thrust - Newtons: 32.9 |  |
| Thrust Duration - Sec.: | 1.6 |
| Initial Mass -g: | 43.1 |
| Propellant Weight -g: | 24.93 |

Note that the average thrust is 12 N and this results the engine labeled as " 12 "
The engine has a max lift mass of 283 grams. We could load up about 40 grams more.


## Jolly Logic AltimeterTwo

Jolly Logic's AltimeterTwo is a rechargeable digital altimeter for model rockets. The AltimeterTwo analyzes and reports flight altitude, acceleration, top speed, flight duration and six other important flight statistics. The AltimeterTwo retains the rugged design, easily-readable LCD, small size, and rechargeability of the AltimeterOne. With the addition of a three-axis accelerometer and a 4X speed increase in processing speed, it can provide a full suite of important flight statistics that can help you analyze and improve your rocket's performance. The Jolly Logic AltimeterTwo has been approved by the National Association of Rocketry for use in rocketry competitions.

- 3-axis, 24 g accelerometer can detect launch, acceleration, speed, ejection, and landing
- Accurate 19-bit barometric pressure sensor sensitive to altitude changes of less than one foot
- Daylight-readable LCD display clearly displays all flight statistics-no computer needed
- Rechargeable Lithium Polymer battery lasts for hundreds of launches, no batteries to buy
- Recharges in less than 2 hours from any standard USB port (no cable necessary)
- Samples pressure over 30 times/second, and acceleration over 200 times/second
- Power button turns device ON/OFF and RESETs the display between flights
- Automatically powers down to conserve power
- Handy tether point allows secure attachment
- Rugged fiberglass and ABS construction to survive crashes
- Approved for use in official contests
- Displays results in either English or metric units

Specifications:

- Size: 0.57" x 0.71" x 1.93" (14.5mm x 18mm x 49mm)
- Weight: 0.36 ounces ( 9.9 grams)
- Max Altitude (above sea level): 29,500 feet (9000 meters)
- Max Acceleration: 23G to 40G (depending on mounting)
- Altitude Precision: Nearest foot below 10,000; Nearest 10 feet above 10,000 feet; Nearest meter
- Acceleration Precision: Nearest 0.1G
- Speed Precision: Nearest MPH; Nearest m/s
- Timing Precision: Nearest 0.1 second


## Sample Data Collected in Fall 2015






Coast to Apogee Time. $\qquad$ ser. Total flight time $\qquad$ sect.

Peak Acceleralın: $\qquad$ G Average Acceleration: __ G where: $C=9.81 \mathrm{r} / \mathrm{sec}^{2}$

Angles at $\qquad$ feet: $\qquad$ Average Angle: $\qquad$ degrees

Angie based apogee :alculation: Tangent h x $\qquad$ $\times 6.304 \mathrm{~B} \mathrm{~m} / \mathrm{ft}=$ $\qquad$ meters

OR5ERYATIONS: \{weather, นnumbal ovens, etc..)

ATTACH †'HIS SHEET IN THZ APPLNDIX SECT DN D: -HE TENM RFPORT.


## Sample Data Collected in Spring 2016



MロDEL ROCKET PRDJECT LAUNCH RECORD JT QLD COMINION UNIVER\＄ITY－NOIfolk，VA
NAMIES


ROCKET ND： 代 0
Model N：Mi』 BSD
Pay＇ual：Alt meter2；Mass： 0.0099 kg
Lingira： $212-5$ suragevasr： 0,04 tikej


The VAR tereifed：！mpulse is 17 ．Thrust diuldimis i．65；Propel artemas is 0.921 －
 Launch filass： $\qquad$ ｜kji！includires plugs，ignizer，＝rigine，rir．j．screws，fire retardant，ër＇d bipes．「xillides p\＃yload．
 $\qquad$
$\qquad$
 Top spesd： 39 rass $\quad$ Destart meed 5
 const to Apogee lime 3,5 seconds．Total j！ight lirme $2 Z$, Escarnis
 Amples at 200 fuet：＿＿＿＿＿i＿＿i＿＿＿Me：atuc Angle： $\qquad$ degroes．
 Arale basud apngee alculation：Tangerilic $1 \times 20 \mathrm{C} \times 0.304 \mathrm{Bm} / \mathrm{fi}-$ $\qquad$ meters

Launch No．2：OBSERVA＇ION5：；weather，unsubial events，etc．．．j： $\qquad$
$\qquad$ Alyoger ： $\qquad$ ：me：Lrs：Ejention at $\qquad$ （metars）


Coast to spugee Time $3 \quad L_{y}$ reconds．Total flight tirnu 2.4 .2 seconds

Ariglc based apocee calculation：Tangenti
$1 \times 200 \times 0.3048 \mathrm{~m} / \mathrm{ft}=$ $\qquad$ meters


> From the technic report by
> J.S. Demur $(1997)$. See our reference list.

Appendix B: Test Model Drawings and Measured
Drag Coefficients This project is in metric although Notes:

This page has the standard units.

1) All models are finished with average smoothness, primed and sprayed with Krylon, unless
noted.
2) All models have no launch lug unless noted.
3) All dimensions are in inches. Dimension $X$ is 0.75 unless noted.


Note that model 8 resembles our B\&D, but ours has a diameter of almost 2 inches. With launch lug of $1 / 8^{\prime \prime}$ dias \& 2 "length, the $C_{0}=0.88$ for this smaller model. our launch lugs are $1 / 4^{\prime \prime}$ dir on $3^{\prime \prime}$ long. Withal length of 30.5" mir nroileit model is twice Th sizer il model?.

Some Pictures from Spring 2016 Projects








