Fluid dynamics dimensional analysis take-home experiment using paper airplanes

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Abstract

A dimensional analysis take-home experiment was developed for use in undergraduate fluid dynamics courses. This experiment requires students to construct and fly paper airplanes and then perform multiple dimensional analysis approaches on the collected flight data to obtain appropriate dimensional scaling. The data recorded includes time of flight and flight distance, which are used to calculate average velocities for each airplane. The airplanes are measured to obtain characteristic lengths, including average chord length, wingspan, and wing area. The students plot the measured data using different characteristic lengths and velocities and conventional non-dimensional numbers to obtain functional relationships and scaling between the planes. Common paper airplanes designs are used, including using the same geometry plane constructed from a full- and quarter-sheet of paper. A simple glider made from an index card and a paperclip is also used. The measured experimental data is supplemented with aerodynamic performance data for commercial aircraft, commercial gliders, birds, and insects. The activity highlights the importance of scaling and demonstrates how flight characteristics are similar across a wide range of flying objects. The plotting of data with different length scales helps students to learn that scaling requires the identification of the most important and characteristic scales in a problem. This take-home experiment was used as a project assignment in a fluid dynamics course for junior undergraduate students at New Mexico Tech in 2012. The homework assignment included a written introduction to scaling, an outline of how to perform the experiments, and a guided approach to developing the necessary scaling relationships. Students completed a survey after performing the experiment which showed an increased understanding of the importance and process of dimensional scaling.

Introduction

Undergraduate engineering curricula are typically heavily loaded with traditional classroom learning approaches and have a limited number of laboratory-based courses available to students. Many students, however, learn better from hands-on, laboratory-based courses and activities. Incorporating more laboratory courses into engineering curricula presents a separate challenge due to the cost of building and maintaining student laboratory facilities, space limitations, and the small class sizes that most laboratories are limited to. One potential solution to enhancing student learning in the traditional classroom environment is to incorporate take-home experiments as part of individual classes as homework assignments or projects.

Take-home experiments used as engineering course assignments are not a new idea, but they are also not widely used. Some early work on take-home experiments included the work by Bedard and Meyer [1] who developed two experiments investigating viscous properties of fluids. Scott [2] developed two fluid-statics experiments that were part of a laboratory class, but were assigned as take home experiments. Cimbala et al. [3] developed a successful take-home pump experiment that was well documented and implemented in large classes of students. More recently, Hertzberg et al. [4] have used art and fluid mechanics as take home projects to generate student interest in a wide range of courses and Jouaneh and Palm [5] have developed experiments for systems and controls classes.

This paper presents a new take home experiment for fluid dynamics classes that focuses on dimensional analysis and scaling. The experiment attempts to capture the imagination and excitement of the students by using a common and fun experimental group: paper airplanes.

Dimensional analysis and scaling, including the Buckingham Pi theorem, is an important topic that is typically only covered in fluid dynamics classes. Although the topic is clearly laid out by several textbooks, [6 7] this topic is difficult for students to grasp because of the vague idea of "important parameters" and the seemingly mysterious methods by which famous people such as Reynolds developed critical controlling parameters. To allow students to better learn this material, a simple experiment using paper airplanes was developed that students can perform to generate data which they then analyze using dimensional analysis. This activity was pioneered with a fluid mechanics class at New Mexico Tech in the 2012 fall semester.

The idea of using paper airplanes for a dimensional analysis experiment was inspired by the book *The Simple Science of Flight: From insects to jumbo jets* by Henk Tennekes. [8] In this book, Tennekes explores scaling relationships that are exhibited by flying objects from fruit flies to Boeing jets. The book is written for the lay-person, but explains and showcases several interesting scaling relationships. The most interesting relationship is Tennekes's "Great Flight Diagram" which shows a general scaling trend between all flying objects of weight versus cruising speed. Tennekes presents the idea of the simple index-card glider used here, but does not actually record data and place the glider on any of his plots.

The experiment presented here is an attempt to engage students in dimensional analysis through the use of simple paper airplanes. The experimental procedure and results are presented, along with student feedback and observations of the student work. Ultimately, this activity engaged students in a dimensional analysis project that they generally enjoyed and which improved their overall understanding of fluid dynamic scaling

Assignment objectives

The primary objective of this assignment is for students to learn more about the process of dimensional scaling through a hands-on-learning activity. The topic of dimensional analysis and scaling is typically difficult to teach to undergraduate students, and have them develop an understanding beyond the basic concept of matching given dimensionless numbers between models and prototypes. This assignment allows students to explore how dimensional data is converted into dimensionless numbers and to explore the importance of different dimensionless numbers.

Another objective is to have students perform a Buckingham Pi analysis to identify the important parameters in a problem and then create dimensionless numbers which they will then use. The students are given the opportunity to explore how dimensionless numbers are constructed and then have to figure out what the dimensionless numbers they created actually mean.

A final objective of this assignment is to generate interest in performing experiments and analyzing experimental data. The simple experimental data that is generated in this assignment can be analyzed in a number of different ways, and students are encouraged to try different approaches and to find the best method for analyzing the data to allow accurate dimensional scaling.

Experimental data collection

This assignment was designed to allow students to obtain first-hand experience with performing dimensional analysis on a simple and entertaining experiment. The assignment description that was provided to students in an undergraduate fluid dynamics class at New Mexico Tech in the fall 2012 semester is provided in the Appendix of this paper. The assignment was used as part of the class project for the semester, and students were given two weeks to complete the assignment.

The assignment requires students to construct, fly, and record data for paper airplanes of three different designs, each at two different scales for a total of six airplanes. Two of the paper airplane designs represent "typical" paper airplane shapes. The shapes used here were obtained from the website "Fun Paper Airplanes" (www.funpaperairplanes.com).[9] These two airplanes are constructed out of a full- and a quarter-sheet of standard office paper. Two additional paper airplane gliders are constructed out of index cards (3x5 inch and 4x6 inch) and a paper clip. The index card glider design is discussed by Tennekes.[8] Examples of the three paper airplane designs are shown in Figure 1.

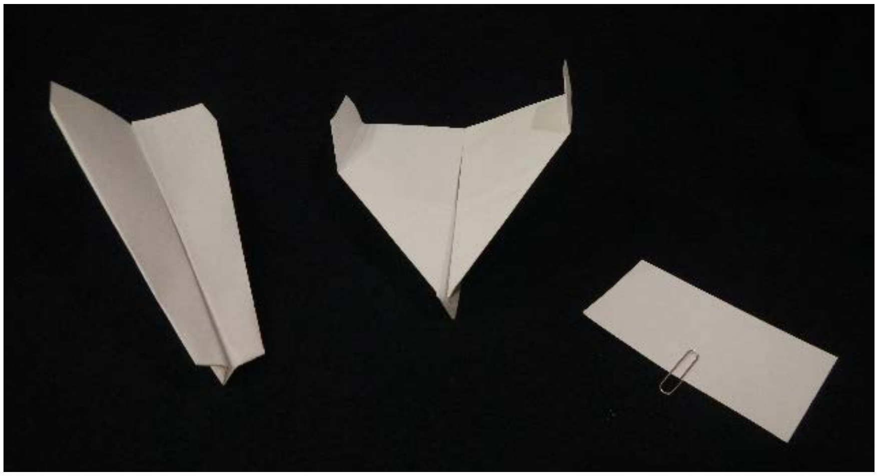

Figure 1: Image of the three paper airplane geometries used here: arrow, condor, and index card glider (left to right).

The students are instructed to throw each airplane from the same height above the ground, with the same motion and approximate force, in an attempt to obtain a similar initial velocity. The index-card gliders require that the students orient the glider at a slightly downward angle and release them with no initial velocity, so the glider will glide on its own. The gliders include a paper clip which is positioned along the midplane of the glider and hanging about one-third off of the index card to provide a forward center of gravity.

Each paper airplane is flown 15 times to obtain the data that is used in the dimensional analysis. For each flight students record the horizontal distance the plane flies and the time from release to touchdown. Additionally, the height the plane is thrown from is measured.

The flight distance and time are used to determine the velocity of the plane. Three distinct velocities can be identified: vertical, horizontal, and combined. The vertical velocity, which is created from the height at which the plane was launched from, represents a "settling speed" of the plane. The horizontal velocity, obtained from the horizontal distance the plane traveled, can be related to the forces acting on the plane, and is the velocity most easily identified by students. The combined velocity, formed as the vector magnitude of the vertical and horizontal velocities, is another important velocity for the analysis. This velocity may be more relevant to the forces experienced by the plane as it is a good approximation of the free-steam velocity the plane encounters, especially for the gliders. The students are encouraged to think of each of these velocities and what dimensionless numbers and ratios they are most appropriate for.

Addition geometric data is collected from the airplanes, including wing surface area, average chord length, wing span, and plane length. The weight of each plane is also measured using a laboratory scale, which was available in a campus laboratory the students could use.

Buckingham Pi analysis

The students perform a traditional Buckingham Pi analysis on the plane in flight as part of this assignment. The goal of the Buckingham Pi analysis is to use the method of repeating variables to generate the non-dimensional numbers that represent the problem. [6]

The Buckingham Pi analysis begins with the identification of the physical parameters that are important to the problem. For the paper airplanes in flight, typical parameters that students should be able to identify include: force of lift, force of drag, air density, air viscosity, gravitational constant, wing area, wing span, plane length, chord length, and plane weight. Most of these parameters are easily identified by the students and can be directly measured or looked up for the local atmospheric temperature and pressure while taking the data. The lift and drag forces are interesting parameters that students may identify and can lead to a more detailed analysis.

Lift force for these planes in a gliding flight can be estimated from a simple force-balance on the plane in flight. From the force balance the lift force can either be equated to the weight if no vertical acceleration is assumed for the plane during the steady-state period of the flight or can be related to a fraction of the weight and an average acceleration calculated from the time-of-flight data.

The drag force can be estimated from the geometry of the plane and the average velocity. A good first approximation for the drag force can be found by assuming that the plane is a flat plate with a given size and surface area. The traditional flat-plate laminar (or turbulent) friction coefficient relationship can then be used with the experimental velocity measured to obtain an estimate of the drag force on the plane. The use of this activity after external flow and flat plates have been introduced in the class will help students to be able to more accurately identify how to estimate the drag.

When the method of repeating variables is applied to the set of parameters for this problem, the students should typically find that the number of primary dimensions will be 3 (mass, length, time). The students then will then have to identify several Pi numbers relative to the number of parameters they included in the analysis. The assignment explicitly stated that a minimum of 4 Pi numbers must be found, which requires a minimum of 7 parameters identified in the first step.

Students are guided through the Buckingham Pi analysis by using their textbook and class notes. The fluid dynamic textbook by Cengel and Cimbala [6] includes an example problem that performs a Buckhingham Pi analysis on an airfoil, which can be followed almost exactly by students.

Ultimately the Buckingham Pi analysis will yield the dimensionless numbers that the students will use to scale their data and explore functional relationships. The typical dimensionless numbers that students should be able to identify include the Reynolds number, lift and drag coefficients, Froude number, dimensionless velocities, and dimensionless lengths. The Froude number is particularly appropriate for this analysis and students are explicitly told to include the gravitational constant in their list of parameters so they will obtain a relationship that is or is similar to the Froude number.

Experimental results and analysis

The final part of the assignment asks students to use their data and the dimensionless parameters they developed to find interesting and useful relationships between dimensionless numbers. The students are given limited guidance in this step intentionally to force them to explore their data and find meaningful relationships. They are given hints that some of the relationships should result in the data collapsing for different plane geometries or that the data will yield fundamental physical relationships between parameters.

A few of the relationships that can be identified include: Froude number versus Reynolds number, drag coefficient versus Reynolds number, and drag coefficient vs. Froude number.

The Froude number versus Reynolds number graph, as shown in Figure 2, is one graph that students generally create. The Reynolds number is defined as:

$$Re = \frac{\rho v L}{\mu}$$

and the Froude number is:

$$Fr = \frac{v}{\sqrt{gL}}$$

where v is the measured velocity of the planes and L is a characteristic length, which can be the total length of the plane or the chord length. The graph of Reynolds number versus Froude number seems interesting to students because the data for each plane exhibit a linear relationship and are spread out in what seems like a characteristic fashion: each large-scale plane has a lower Froude number for any Reynolds number.

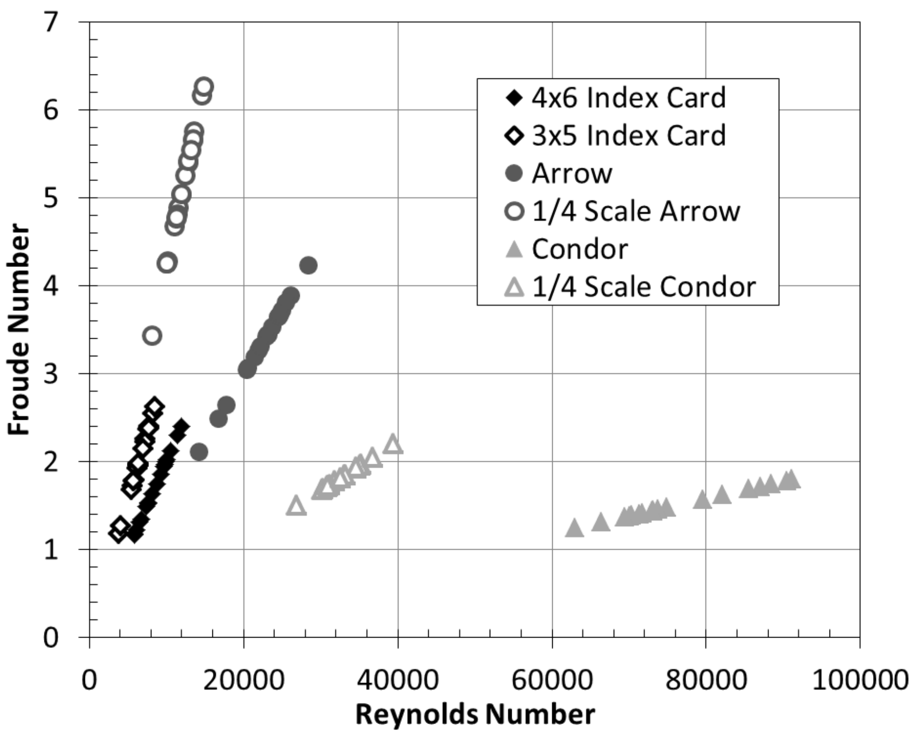

Figure 2: Froude number versus Reynolds number

Students are encouraged to think about what the relationships they plot mean. Through class the students are taught that the Reynolds number is a primary dimensionless number for fluid dynamics and can be thought of as the ratio of the intertial forces to the viscous forces in a flow. the Froude number is the ratio of intertial forces to gravitational forces. With this approach the students could analyze the graph in Figure 2 to discuss the relative importance of the gravitational and inertial forces as the plane sizes change. For the geometry change from the larger to the smaller plane, the effect on Reynolds number is not significant except in the case of the Condor, but the change in the Froude number is always significant. The smaller planes generally have a higher Froude number, or an increase in the intertial force relative to the gravitational force.

The students are also encouraged to consider what the slopes on various graphs mean. The graph in Figure 2 can be modified to look at the Froude number squared versus the Reynolds number, as shown in Figure 3. The slope of each curve on this graph is the velocity of each plane, multiplied by physical constants. This is the type of analysis for the students to strive for.

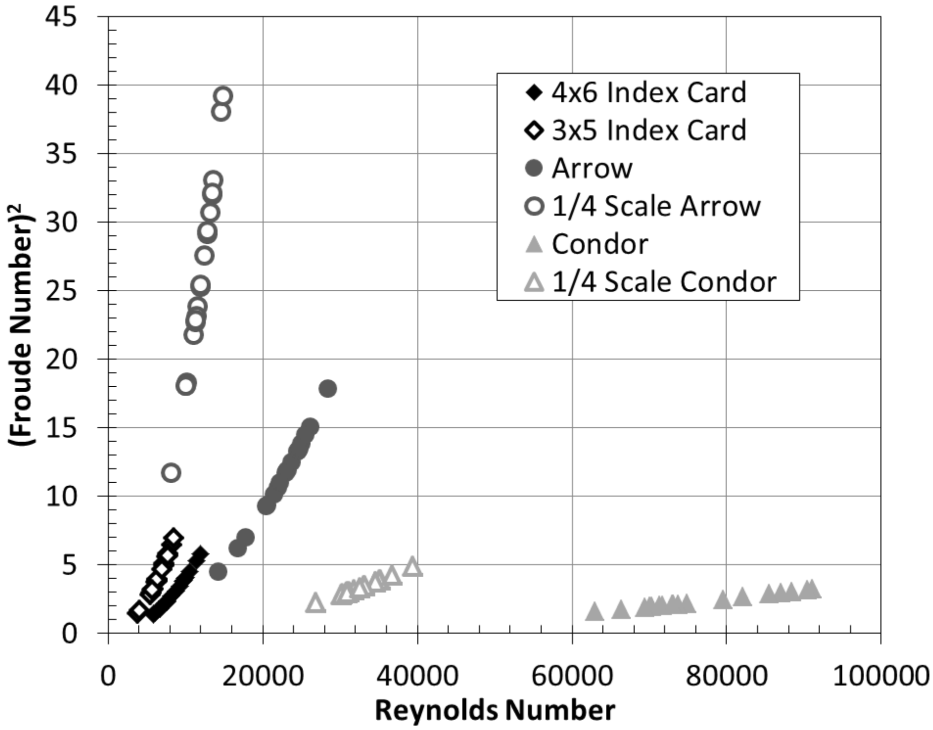

Figure 3: Froude number squared versus Reynolds number. The slope variation between each data set on this graph is essentially the velocity variation among planes.

The drag coefficient versus Reynolds number is also an interesting plot for students to create. If the students create the drag coefficient from assuming the planes can be approximated as a flat plate, the graph will look as shown in Figure 4. This graph is essentially showing that the friction coefficient varies with Reynolds number to the one-half power. The students can then look at a drag force, created from the friction coefficient, versus the Reynolds number, which is shown in Figure 5. The graph in Figure 5 shows that the planes have general trend lines for the shapes, showing increased friction for the increased size of each plane.

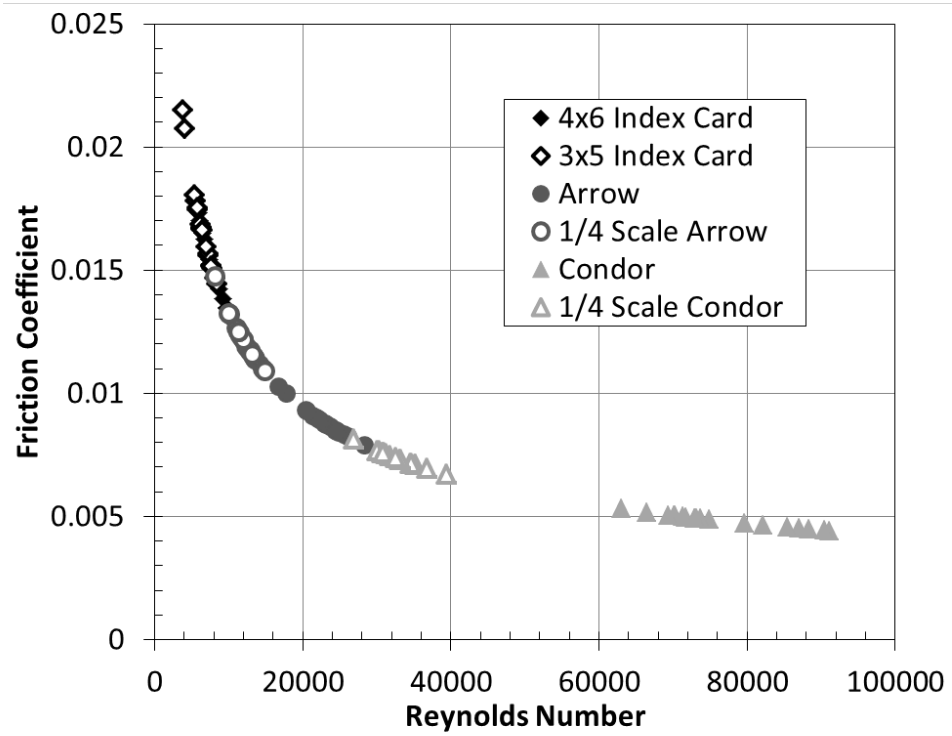

Figure 4: Friction coefficient versus Reynolds number

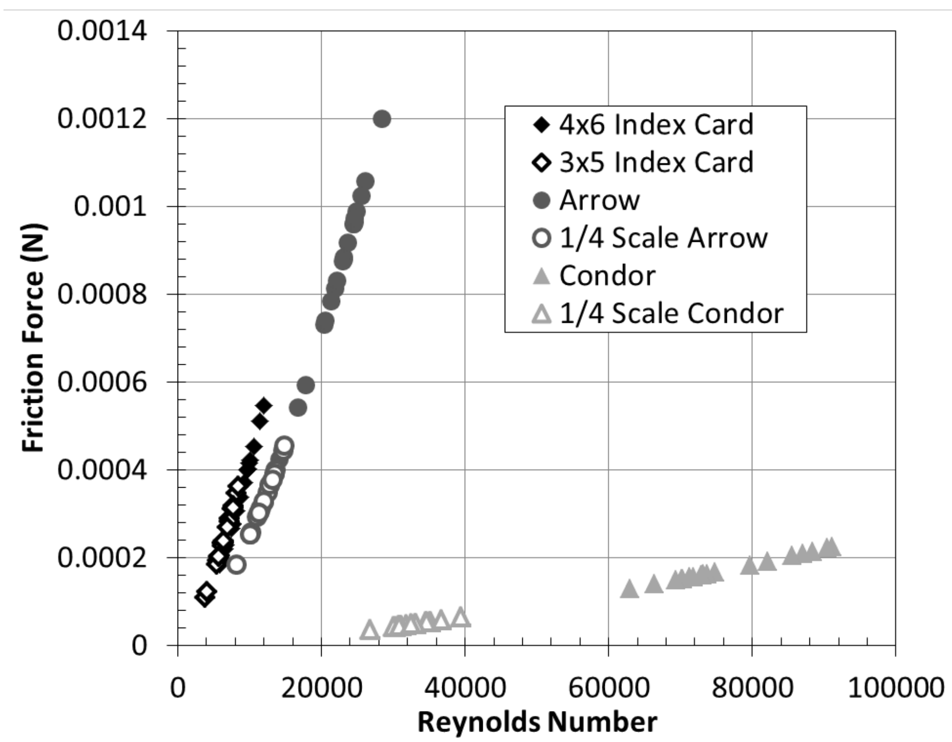

Figure 5: Friction force versus Reynolds number.

The friction coefficient versus the Froude number is another interesting graph for the students to consider, and is shown in Figure 6. This graph is a combination of an inverse square of Reynolds number versus Froude number, which can be compared to Figure 2.

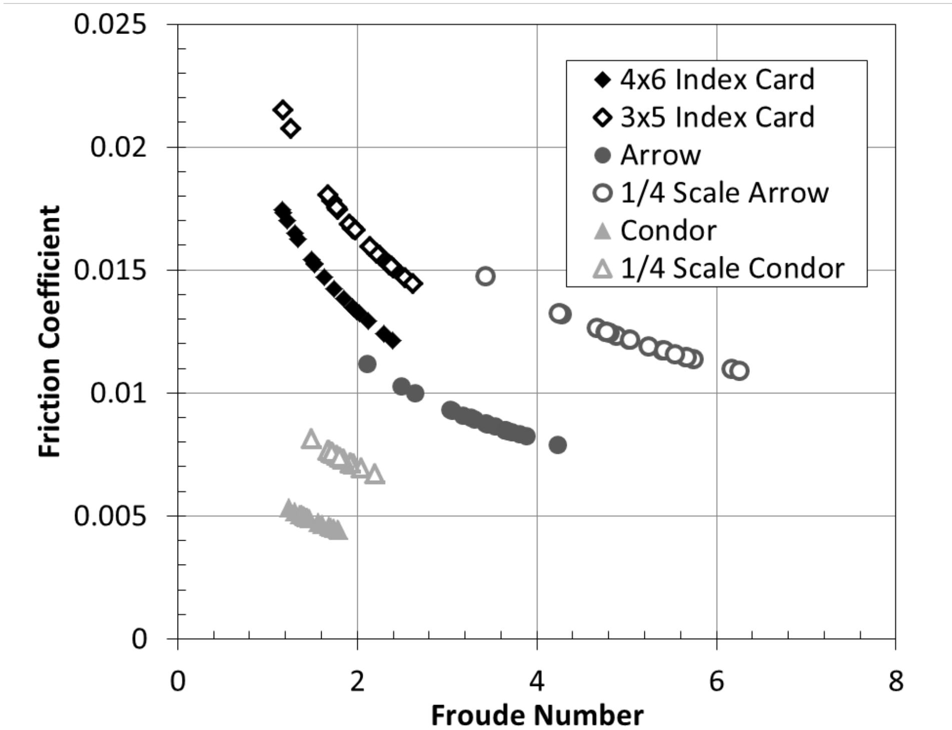

Figure 6: Friction coefficient versus Froude number.

The average performance for each plane can also be plotted on the Great Flight Diagram as described by Tennekes [8.] The result of this is shown in Figure 7. Students can use this to compare with the other planes and make comparisons. This is particularly useful if Tennekes's book is read by students as part of the project, which was not done here. In the future it could be assigned as part of the project and would allow students to enhance their analysis of the data. Of particular interest is that the paper airplanes do deviate from the general curve and arguments can be made that the deviation shows that paper airplanes are not ideal flight geometries. More analysis could be done to determine what paper airplane shapes are more "ideal" and how this compares to what plane shapes students are familiar with.

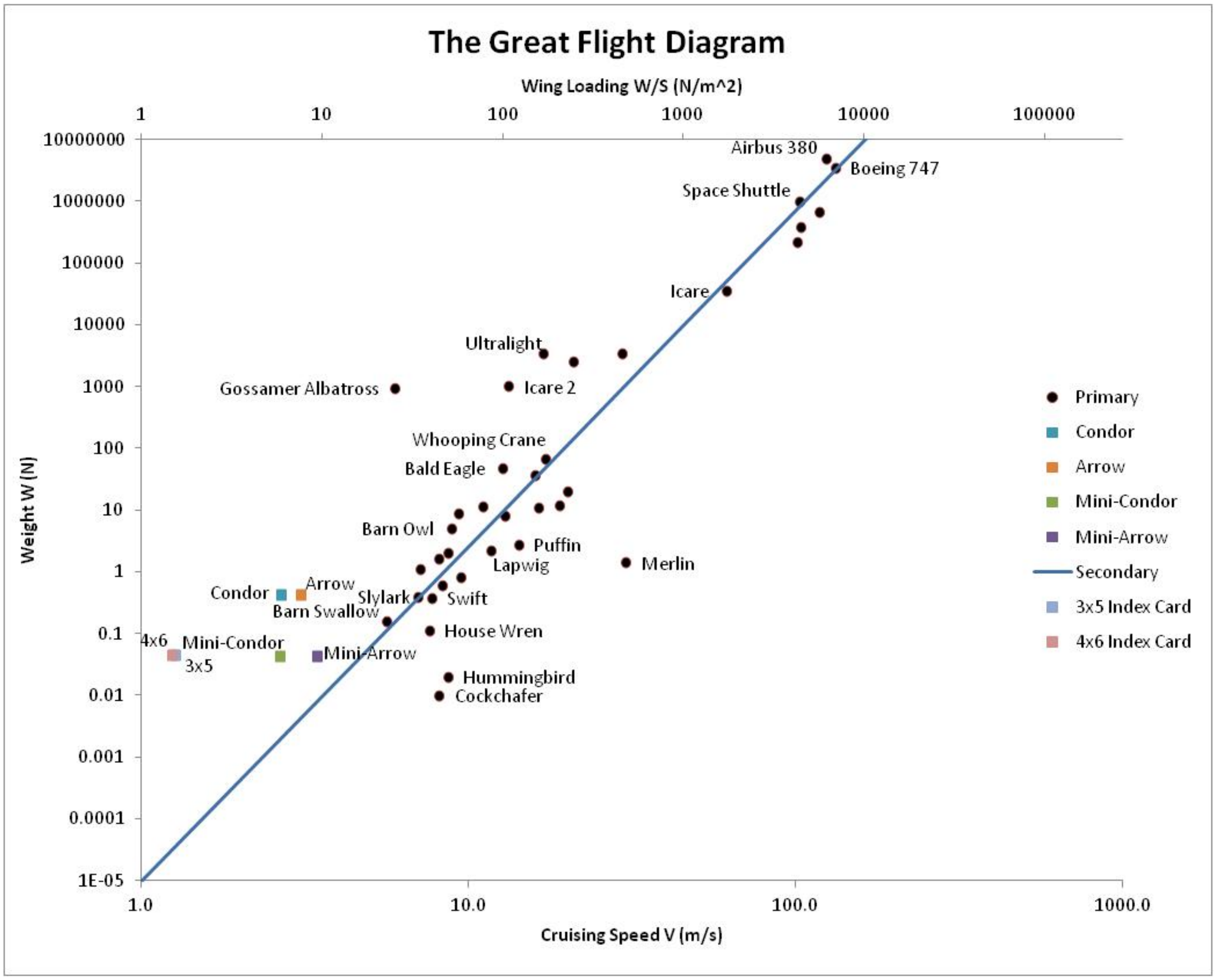

Figure 7: The Great Flight Diagram

Discussion of student learning and feedback

This activity was given as a project for an undergraduate fluid dynamics class. The students were given two weeks to complete the project and were encouraged to work in groups of up to three students. The students were given a survey after they completed the project to gage their perception of the assignment and its benefit to their learning. The survey was collected anonymously during a class period after the assignment had been returned to the students. Students were asked to rate their response to each question on a Likert scale from 1 (strongly disagree) to 5 (strongly agree). The results of the survey, which were collected for 30 of the 40 students registered in the class, are presented in Table 1.

Table 1: Student responses to survey questionnaire

Question		Average	Standard Deviation
1	The paper airplane activity helped me to better understand non-dimensional numbers	4.03	0.76
2	After this activity I better understand the Buckingham PI theorem and process	3.97	0.76
3	This activity helped me to better understand the importance of dimensional and non-dimensional scaling	3.83	0.70
4	Adequate information was presented in class and in the book (Cengel and Cimbala 6) to help me perform the Buckingham Pi analysis in this activity	4.30	0.70
5	This activity has made me more interested in experimental testing and scaling	4.03	0.89
6	The instructions for performing the experiment were adequate	4.00	0.74
7	This activity was more helpful for learning dimensional analysis than a traditional homework problem	4.00	0.74
8	I feel I learn better with hands-on experiments or activities relating to coursework	4.23	0.68
9	I wish I had more time to perform these experiments and data analysis	4.63	0.76
10	This paper airplane activity was fun	4.20	0.89

The results of this survey show that the students in general feel they improved their understanding of dimensional analysis, scaling, dimensionless numbers, and the Buckingham Pi theorem. The students also felt strongly that the activity was fun.

The students did feel strongly that they needed more than two weeks to complete this assignment. This conclusion may be affected negatively because this activity was assigned at the end of the semester and over the Thanksgiving holiday. Many students expressed personal frustration due to the lack of time and said they wished they had more time to work with the data more and explore more dimensionless numbers.

Of particular interest here are questions 7 and 8 which show surprisingly different results. In question 8 the students felt strongly that they learn better with hands-on activities, but in question 7 they did not strongly feel that this assignment was better than a traditional homework. This feedback is attributed to the open-ended nature of this project and that there were no “right answers” to the dimensional scaling. Students were asked to find dimensionless relationships that they felt scaled the data and that were “interesting” to them. This is an atypical goal for an engineering assignment, where students are regularly asked to find the “right” answer. Some of this feedback may be tied to students feeling worried that their grade is dependent on their creativity or on nebulous grading criteria. A more structured grading scheme could be delivered to students with the project assignment in the future.

Questions 2 and 3 also show that the students likely need more structure to the assignment to help them to better learn the importance of the dimensional scaling. Future implementations of this assignment could include more steps to help guide the students and include intermediate calculations such as the drag force on each plane which would be graded.

In addition to the survey, the student reports that were turned in for the assignment also revealed a high level of interest in the project and an improved general understanding of the dimensional analysis concepts. Several students identified unique dimensionless numbers, and enjoyed naming their numbers after themselves. Some students even named the airplanes, and discussed how well the condor performed relative to the condorito (-ito diminutive suffix meaning small in Spanish).

One group of students performed a significant amount of extra work, which included building planes of a larger size made out of poster paper and also recording accelerometer data on these larger planes. This group was interested in trying to measure the forces on the planes in flight, and are still performing some additional experiments and data-reduction at the time this paper was written. This group also posted a video of their test flights online, which shows them and several volunteer helpers they recruited who were not registered in the class. [10]

Conclusions

This project demonstrated the ability to involve students in a hands-on experiment to learn about dimensional analysis. The project used paper airplanes to generate student interest and to show how to perform a complex dimensional analysis on a simple problem. The experiment was used in an undergraduate fluid dynamics class in the fall 2012 semester at New Mexico Tech.

The results of the project showed that students were very interested in the work and were able to come up with interesting and unique data analysis. The students performed a Buckingham Pi analysis on data they recorded and then generated plots to discuss the physical data and make comparisons between the paper airplanes tested. The most common relationships involve drag coefficients, Froude numbers, and Reynolds numbers plotted against each other. From these graphs the students can obtain several general trends between the numbers, and the different airplane geometries and scales. Of particular interest is that the drag coefficient is essentially a pure function of Reynolds number based on assuming the planes act like flat plates in a crossflow. Plots of the Froude number versus Reynolds number shows that decreasing the size of the planes increases the Froude number, thus decreasing the gravitational force relative to the inertial force.

Overall, the hands-on experiment helped reinforce ideas taught including experimental techniques, data analysis, and the reason for nondimensionalizing data to create functional relationships. It also teaches students that techniques they learn in the classroom can be useful and applied to research. Additionally, it brings an aspect of fun into a typical engineering classroom.

Based on a survey given to the students after the project, the project was helpful in improving their understanding of dimensional analysis, they enjoyed the hands-on learning, and they had fun performing this assignment. They did feel that more than 2 weeks was required for a thorough analysis of the data.

References

1. Bedard, A. J.; Meyer, D. G. Hands-on Engineering Homework: A new approach to out-of-class learning. *ASEE Annual Conference*, 1996.
2. Scott, T. C. Two "Take Home" Experiments in Fluid Mechanics. *ASEE Annual Conference*, 2000.
3. Cimbala, J. M.; Pauley, L. L.; Zappe, S. E.; Hsieh, M. Experiential learning in a fluid flow class via take-home experiments. *ASEE Annual Conference and Exposition*, 2006.
4. Hertzberg, J.; Leppek, B. R.; Gray, K. E. Art for the Sake of Improving Attitudes Toward Engineering. *ASEE Annual Conference*, 2012.
5. Jouaneh, M.; Palm, W. System Dynamics and Control Take Home Experiments. *ASEE Annual Conference*, 2010.
6. Cengel, Y. A.; Cimbala, J. M. *Fluid Mechanics: Fundamentals and Applications;* McGraw Hill: Boston, 2010.
7. Stephan, E. A.; Bowman, D. R.; Park, W. J.; Sill, B. L.; Ohland, M. W. *Thinking like an engineer: An active learning approach;* Pearson: New Jersey, 2011.
8. Tennekes, H. *The Simple Science of Flight;* MIT Press: Cambridge, Massachusetts, 2009.
9. Bailey, K. Fun Paper Airplanes. http://www.funpaperairplanes.com (accessed Dec 18, 2012).
10. Kupchella, K.; Runnels, J. YouTube. http://www.youtube.com/watch?v=d3CVg0UD-lI (accessed Dec 18, 2012).

Appendix – Assignment used in Fall 2012 at New Mexico Tech

Dimensional analysis is an important tool for examining experimental data to identify similarities between scenarios. To better explore the importance and usefulness of dimensional scaling, you will perform some experiments with paper airplanes and attempt to scale them.

Procedure:

1. You will test 3 different plane geometries, each with 2 different sizes. The planes to be tested are the "condor" and "arrow" from http://www.funpaperairplanes.com/ and a simple glider made from an index card. For each plane you will have a large version made from a full sheet of 8.5x11" paper and a small version made from a quarter-sheet of paper. For the index card you will use a 3x5 card and a 4x6 card.
2. For each plane you need to perform 15 flights. Launch the planes from the same height, and with roughly the same throwing velocity. Record the horizontal distance the planes travel and the time from release to touchdown.
3. Your primary data will be the length the planes traveled, both horizontally, vertically, and along the diagonal and the time that the planes flew for. You can use the distances and time to also calculate the velocity of the plane.
4. Perform a Buckingham Pi analysis on the experiment of the plane flying to identify the important Pi numbers. Identify any of the Pi numbers you generated as conventional dimensionless numbers (if one of your numbers is the Reynolds number, state that). You should generate at least 4 unique Pi numbers. Also make sure that g (gravity constant) is one of your parameters.
5. Plot your data in dimensional and non-dimensional forms to identify important scaling relationships which collapse your data. (Hint: try things like plotting lift coefficient versus Reynolds number) You must submit at least 3 different, unique, and interesting graphs.
6. Provide a few sentences describing the relationships you generated and what the scaling tells you.
7. An interesting book by Hank Tennekes titled *The simple science of flight: from insects to jumbo jets*, discusses the importance of scaling for insects, birds, and airplanes. One of the important scaling arguments he makes is in his "great flight diagram", which is a plot of weight versus cruising speed. Use the data provided on the course website to create this diagram and add your data to it. You may also want to consider adding the birds and airplanes to your other scaling graphs for comparison.

Submit all of the work and analysis that is required as part of the above procedure.