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Fostering Spatial Visualization through Augmented Reality in Calculus learning

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Abstract

We are part of a team of educational innovation that aims to transform the teaching and learning of Calculus through the integration of digital technologies. We are looking to foster a visual and tangible learning of Mathematics. As a team of educational research we care for developing mathematical cognitive skills that are not explicit in curriculum but have been taken for granted. Most of them is basic to the understanding of mathematics and are useful in the process of problem solving. Spatial visualization, for example, has been taken as an innate skill in students, however, experience with teaching solids of revolution, may question whether this is really the case. We identify spatial visualization as a cross-curriculum content and take the task of designing an educational technological resource to improve this skill. We have investigated the benefits of Augmented Reality technology for learning Calculus. The purpose of this paper is to present the AR Application we have developed to promote spatial visualization. Elements of its design as educational resource will be discussed in order to situate its use in ordinary Calculus I, II and III courses at college. In this paper we want to share aspects about application design as a whole, but will deepen in Calculus II level, where the visualization of solids of revolution takes place. Finally we share the kind of experience we have had with students and looking for better learning experiences for current and future generations.

Introduction

Introducing visual aspects in the learning process makes technology a powerful ally. Augmented Reality (AR) provides one of the best opportunities to see objects in a realistic way. If these objects are graphical representation of mathematical ideas (concepts or processes), then we provide students with a visual and tangible Math that their minds can deal with.

This has been our goal since 2012, when we started working on the design of an AR application, that could foster Calculus learning in undergraduate students[1, 2]. The app is now completed and it is time to share the way it has been designed for its use, to promote the learning goal of the spatial visualization skill.

In this paper we will discuss about AR emergent technology, and its use especially in education, as several authors have enlightened us. We will discuss our approach for the learning of Mathematics, which arises from our research experience in Mathematics Education. We will also comment about the production expertise in AR in our institution, in order to build a teaching resource for the learning of Calculus. Describe the entire application considering its three levels. We will also deepen in the second level, intended to the visualization of solids of revolution, explaining how they expect to help students understand this topic usually in the second year of Calculus in college, when the concept of Integral is used to calculate the volume the solid generated. Finally, we will discuss the challenges we face in deciding to engage on the integration of digital technologies in the educational process. To make a difference in the way we can teach and learn science, we are bound to develop ourselves as teachers, designers and educational researchers.

Augmented Reality and Education

Augmented Reality is an emergent technology located in the middle of two extremes in what is called the *virtual continuum*, which describes all the ways in which a human can interact with a computer. At the end of the continuum are the Real Environment and the Virtual Environment, and they cover from Tangible User Interfaces (TUI) to Virtual Reality (VR). Augmented Reality (AR) is predominantly a real-world space in where virtuality is inserted in real time, and the enrichment of the real scene is done with virtual targets[3].

AR interfaces enable "ubiquitous computing" models, because students carry wireless mobile devices can reach virtual information overlaid on physical landscapes in the real world. It is a type of mediated immersion that infuses digital resources through the environment. This enhances the experience and interaction of students, and researchers begin to concern how these learning models can help students[4].

The interview with researcher Craig Kapp, from University of New York, is highly illustrative of the potential impact of AR in Education. You can enhance traditional textbook with simulations that pop right out of the book to the world around you, or can think on the ability to put new content in the textbooks; material that can be updated on demand. Next level of AR means adding interactivity, possible by the creation of "digital manipulatives" that can support hands-on learning. There is a lot of work done finding ways to use AR to simulate the physical world. For example, the creation of SimSnails offers an interactive visualization of a 3D snails colony that lives, breeds, and is eaten by predators. It could be used in museums, but also as a teaching tool, because it can be easily carried to classroom[5].

Assistive technology is another area where AR has been playing a role. One of its goals is to turn the processes of rehabilitation in a game, reinforcing exercises physical therapy, helping with basic cognition and bringing patients some fun. Also noteworthy is its application in children with autism to manage their attention. The Mobile Object Identification System (Mobis) allows teachers to overlay digital content over physical objects, increasing the sustained and selective attention of patients, and eliciting positive emotions during therapies[6].

As an innovation in education, the educational value of this technology should be considered not only based on their use, but closely related to the design, implementation, and how they integrated in formal and informal learning settings. Wu, Lee, Chang and Liang[7] identified five features and affordances that AR systems could exploit for educational purposes. First, AR could enable learning content in 3D perspectives; second, its ubiquity, collaborative and situated learning. Third feature relates to the sense of presence of learners, immediacy and immersion. Fourth and fifth are the affordances to visualize the invisible and bridge formal and informal learning.

The effects of AR systems in learning, relate to the development of skills and acquisition of knowledge in a more effective way. Present lessons in a 3D format bring to learners the opportunity of virtually manipulate a variety of learning objects, and manage information in an unusual and interesting way, increasing student motivation. Besides that, the potential of AR in learning and education is increasing due to the emergence and widespread of mobile devices; the use of computing in recent years across the world is leading to a subset of AR: mobile AR[8].

Related to this feature of having virtual 3D object within reach, the development of spatial abilities has a great opportunity to be part of our daily learning. Particularly, in Mathematics, this skill is meaningful. Martín-Gutiérrez, Navarro Trujillo y Acosta-González[9] point out the consequences of spatial abilities for all scientific and technical fields. They are fully award of the low level of development in engineering students. Their work includes the development of a didactic toolkit AR_Dehaes that aim to improve spatial ability in freshmen engineering students. These authors state that spatial ability is something that cannot be taught but instead needs training (development and improvement). Within these considerations, testing of tool promise its release.

Our perspective in Mathematics Education, always grounded in the classroom as a college teachers, makes us aware of the difficulties when dealing with spatial visualization. The teaching of solids of revolution in Calculus II has been a crucial issue in this reflection. When teaching in Calculus I the graphs of functions of a single real variable, graphs visualization stays in a 2D plane perception. These curves, compelled in 2D, could be a starting point for the development of spatial skill we expect students have earned in Calculus II. Also, having taught in Calculus III, the graphs of functions of two real variables, the visualization of a surface in a 3D space should contain the visualization of 2D curves in parallel planes, in order to understand the variation offered by the partial derivatives of the function. We noticed it is important to emphasize this change of view from 2D to 3D and vice-versa as an important feature of the visualization skill related to Mathematics. This skill of spatial visualization inspired the design of an AR Application, as we describe below.

Elements in the design of the AR App

We conceived the AR application to foster the skill of spatial visualization related to the learning of Mathematics. This includes some learning objectives of Calculus I, II and III; graphics of functions of one real variable, solids of revolution, and graphics of functions of 2 real variables.

AR display of the content is through recognition of images, with printed and stuck *markers* on acrylic paddles for free manipulation. The Graphical User Interface (GUI) has the same design for the three levels. On the main display you can access these levels with names: From 2D to 3D, Solids of Revolution, and Surfaces in 3D. To access each level, there is a menu to interact with different "forms", which means considering different mathematical functions that include parabolic, circular and sine forms.

Once inside each of these forms, you get an image that states the actions that the application can do. In the figure 1 we can see an example of three different levels, indicating the possibilities of interaction that the application provides, as shown in the level of solid of revolution.

Fig. 1. Three levels of interaction in the AR app.

When you get to the zone of simulation AR, the interface shows several elements, one of them includes a short video with an explanation of the AR simulation. Another zone, above the video, is to show the mathematical information that is being considered, and below the video is the zone of the operable buttons to display the Simulation AR. Figure 2 shows this organization.

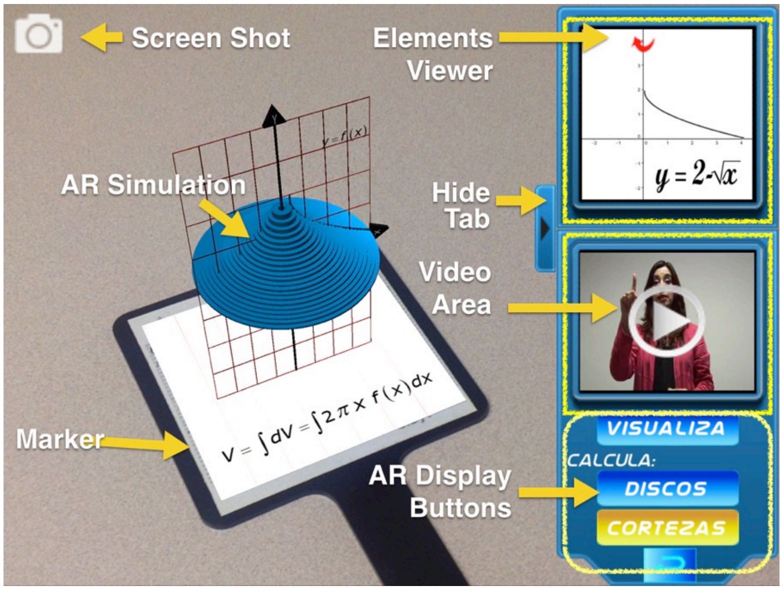

Fig. 2. Elements in the interface AR application.

The *hide tab* allows a better visualization of the simulation process with AR. Design promotes learning Mathematics ideas, not only trying to show an AR object, but also emphasizes the visual process that take place when constructing the object. We reinforce this statement at all three levels as follows.

In Calculus I, we integrate the idea of introducing a parameter *k* in the algebraic representations of functions, and visualize their effect on the corresponding graph. Sometimes it shifts up or down, left or right, sometimes its shape is compressed or expanded. We use this to promote a visual effect putting curves close together but in parallel planes, in order to generate a surface in the space showing these features. Once the surface is generated, the student could interact with it, through a small paddle containing the marker to display a yellow plane. Using the yellow plane handled with the paddle, the student can virtually cut the surface, and capture different curves made by the intersection between the surface and the plane. This is a powerful feature of AR, which we believe will have much use in the future. In the Figure 3 we can observe students using the paddles as we have described above.

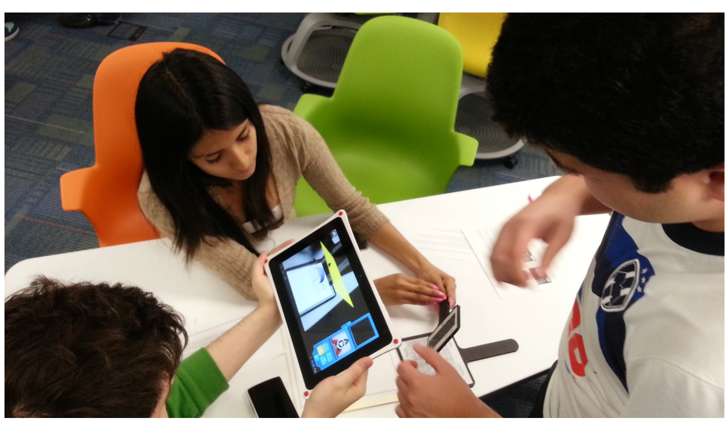
Fig. 3. Students using the AR paddles.

In this paper we look deeper in the Calculus of the II level, but before that, we seek to emphasize how the action of interacting with the small paddle and AR surface, it is an opportunity to reconstruct the process of generation of the surface, and at the same time, the opportunity to find different curves of intersection which do not match the original. Because our purpose is to develop in students the skill of spatial visualization, we give much importance to the deliberate action to cut the surface in different ways. This will be widely evoked when we have to deal with Calculus of level III, to deconstruct the surface, cutting it, with parallel planes. The AR App offers the simulation of this action performed with planes parallel to the coordinate planes XY, XZ and YZ. In Figure 4 we can see images of this simulation, representing reversible thought which we consider is as key element for the visualization process.

Fig. 4. Simulation that fosters the visualization process.

The intersection of the curves will be projected on the XY coordinate plane, and a simulation performed with another button also reverse this effect to reconstruct the surface.

About solids of revolution

The second level of our AR App includes the visualization of solids of revolution. Traditionally the subject of calculating the volume of a solid of revolution is in the chapter on applications of Integral Calculus. In this chapter two methods are taught, commonly identified by discs and cylinders. Our outlook for the design of the AR App includes an extra feature, which allows the visualization of the solid early on that those methods are taught.

Visualization of the solid includes a mental process that AR can perform realistically. Once the solid is conceived in our mind, is the time to introduce the mathematical idea of looking for an Integral to capture the numerical value of its volume. Our proposal for learning associated with this topic, is to take care of the differences between the solid and visualization of how we should view it, in order to understand the method to calculate its volume. This is the reason to include a button that allows the AR simulation of constructing a solid of revolution. In the Figure 5 we can see the steps you go through the App to visualize the solid of revolution, we consider the rotation curve to build a surface and then rotate the area under the curve, in order to build the solid.

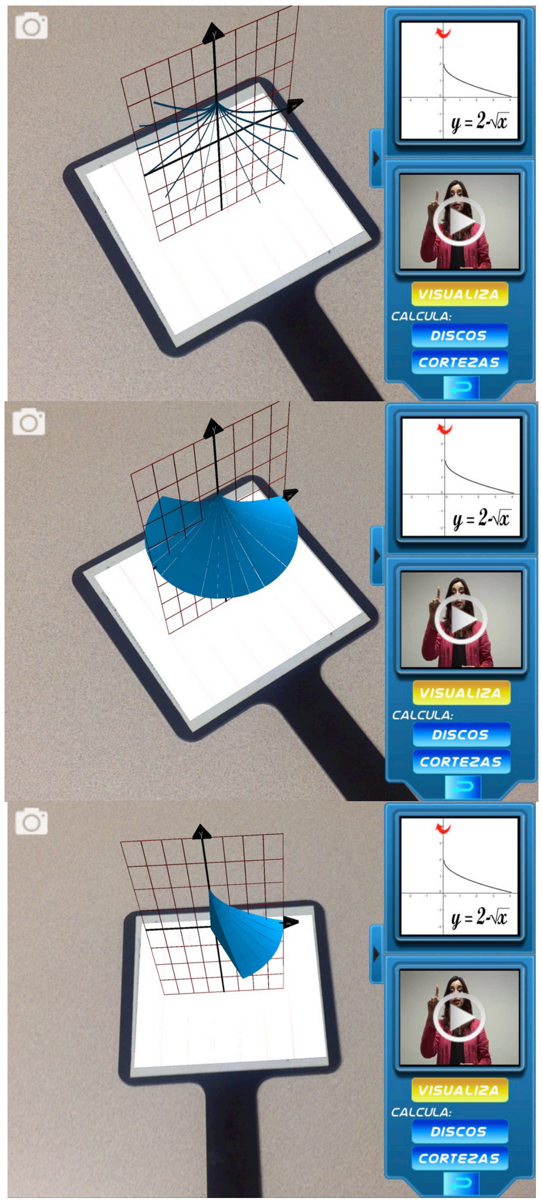

Fig. 5. Some steps to visualize the solid of revolution.

Another main feature of the AR App in this issue, is that we take some advantage of how to present the solids of revolution in relation to the curve that is being considered. We selected some curves as the graph of a function that intersects the coordinate axes x-y, and having only positive values of these variables. We chose 4 functions that have a similar behavior as part of a parabola, but their differences are stated in terms of behaviors, like increasing (decreasing) graphs, or with concave up (concave down) behavior. Combining these characteristics we deal with 4 cases: increasing and concave upward, increasing and concave downward, decreasing and concave upward, and decreasing and concave downward, as the GUI presented before. For each one of this curves cases, we add a button to choose the generated solid when the rotation is performed around the x-axis, or about the y-axis.

We illustrate this with the case of a decreasing and concave upward graph. In figure 6 we can observe the situation when the area under the graph of the function is rotated around the y-axis, and the method of discs is applied to identify the integral to calculate the volume. This is possible if the limits of the integral are considered.

Fig. 6.Steps to present an integral to calculate the volume with the method of discs.

The last image shows the accumulation of volume *differentials,* which are represented by the infinitesimal thickness *dy* discs. The differentials are spliced and their radii become smaller as we move alone the y-axis. Finally, AR simulation smooths the contours of the figure and the solid is finally visualized.

In figure 7 we can see the visualization of the same solid but now when the cylinders method is applied to calculate its volume. Is a good opportunity for students to verify that the same value for the volume is obtained using both methods.

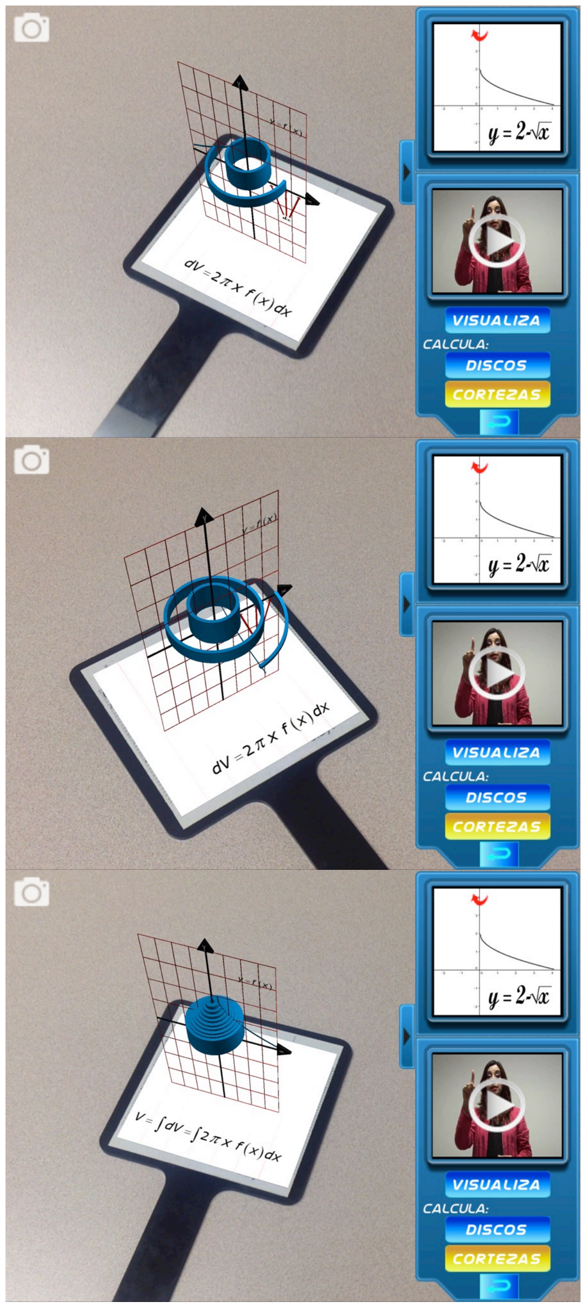

Fig. 7. Steps to present an integral to calculate the volume with the method of cylinders.

In the last picture we observe the accumulation of the volume *differentials* represented by the cylinders infinitesimal thickness *dx,* coming together to build the solid of revolution, from inside out. Finally, the AR simulation smooths the contours and the solid is finally visualized.

Discussion and Conclusions

The purpose of this work is to present an AR Application designed and developed for the learning of Calculus. The conjunction of Education of Mathematics and expertize in the production of AR have led to an opportunity to evaluate the benefits of this emergent technology for learning purposes. In the two years dedicated to the design and creation of the App, it has always been our explicitly concern with the transformation of the process of teaching and learning of Mathematics. In this matter, we promote mathematical skills, rather than focusing only on learning a specific mathematical content. This is why we got to the production of a visual and tangible approach to certain themes that will foster the development of spatial visualization skill.

It is very simple to accept today that our students are part of this digital age, and the means by which access to knowledge is very different from the past. This increasingly global, technological and online world, therefore requires responses to a new kind of challenges involving significant skills that combine visual information. Education has to be reinvented, and technology itself won't do this. We require new ways to deal with teaching delivery, enhance the approach to science students, to promote their active participation in their own thought processes.

The App that we share here intends to give this kind of interaction, for students with their own thoughts. Based primarily on our own reflection, we outlined a way to interact with the process of thinking, conceiving the production of a solid of revolution. We identify a natural process to visualize the solid. We distinguish this of two different processes, not as natural, but necessary to visualize the solid through *differentials accumulation* leading to calculate its volume.

We have sharing the experience gained with the elaboration of our App AR and two pilot studies performed when the first level of it was ready[1, 2]. Reflecting in the usability of the App to impact the motivation of students in these experiences, we feel a great opportunity for this technology to improve education. We agree with Lee[10] on the undeniable availability of AR technology, anytime, anywhere, and with this, the potential to promote efficient and effective learning experience.

We have realized that this technology has come to transform our perception of what is possible in the teaching of Mathematics, and consequently, the traditional perception of learning Mathematics cannot stay without change. The question of the experience of efficient and effective learning, should be accompanied by the appropriate method that deepens in the kind of learning event that is taking place in the minds of our students. We cannot hold into the old ways of evaluating the achievement of our students'. New ways to assess should come to allow us to improve our work.

Somehow we have been scaffolding the process of visualization, and our AR App aims to offer students an enhanced learning experience context for visual perception. Yin, Song, Tabata, Ogata and Hwang[11] have developed a framework of Participatory Simulation for Mobile Learning using Scaffolding. We find in their approach the opportunity to explore the new learning events that are arising with the use of digital technology in Mathematics Education.

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