AC 2010-1069: FOUR FREE-VIBRATION LABORATORY EXPERIMENTS USING TWO LUMPED MASS APPARATUSES WITH RESEARCH CALIBER ACCELEROMETERS AND ANALYZER

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Four Free-Vibration Laboratory Experiments using two Lumped Mass Apparatuses with Research Caliber Accelerometers and Analyzer

Abstract

In 2004 a 3-credit engineering elective course in vibrations was created at the University of Southern Indiana. It consists of two hours of lecture and three hours of lab per week. One commercially available translational system and one rotational lumped mass system were purchased. Each turn-key system can be adjusted to study one, two, or three degrees of freedom systems in which the masses/inertial values can easily be changed. In addition, the translational system has three different types of springs and one variable air cylinder dashpot. Both systems come with an amplifier and motor which can optionally drive one of the masses in motion that is proportional to the voltage signal on the input. However, instead of using the optical sensors, accelerometers were procured that are more representative of what engineers use in industry and research, as well as provide instrumentation knowledge and skills. Likewise, instead of purchasing the computer board and software that accompanies the lumped mass apparatuses (which in this case was primarily developed for controls laboratory experiments), a world-class analyzer (that includes computer software for control) was purchased so that the sensors and analyzer can be used by students and faculty for research projects. This analyzer can also be used for acoustic measurements. A disadvantage is that the software that controls the analyzer is not user friendly, and requires substantial setup time by the instructor. The laboratory experiments that were developed include the study of free vibration, forced vibration, 1 DOF, 2DOF, and 3 DOF systems, dynamic absorber, modes of vibration, and the effects of damping. In this paper, only the free vibration experiments, four in all, will be described in detail, as well as their impact on the student learning outcomes for the course. These experiments were developed and refined over several years. Each laboratory workstation can accommodate two students at a time. Student surveys have indicated that the laboratory experiments were effective in understanding the theory and provide an increased level of intellectual excitement for the course. A subsequent paper is planned to describe the forced vibration experiments.

Introduction

There are two basic approaches to developing a vibrations laboratory for engineering students to study lumped parameter systems. One is to purchase a commercially available turnkey system complete with hardware and software. The other is to design and build a custom apparatuses to go with a research caliber accelerometers and analyzer, as well as potential software development. The laboratory experiments described in this paper use another approach which is a hybrid of the two.

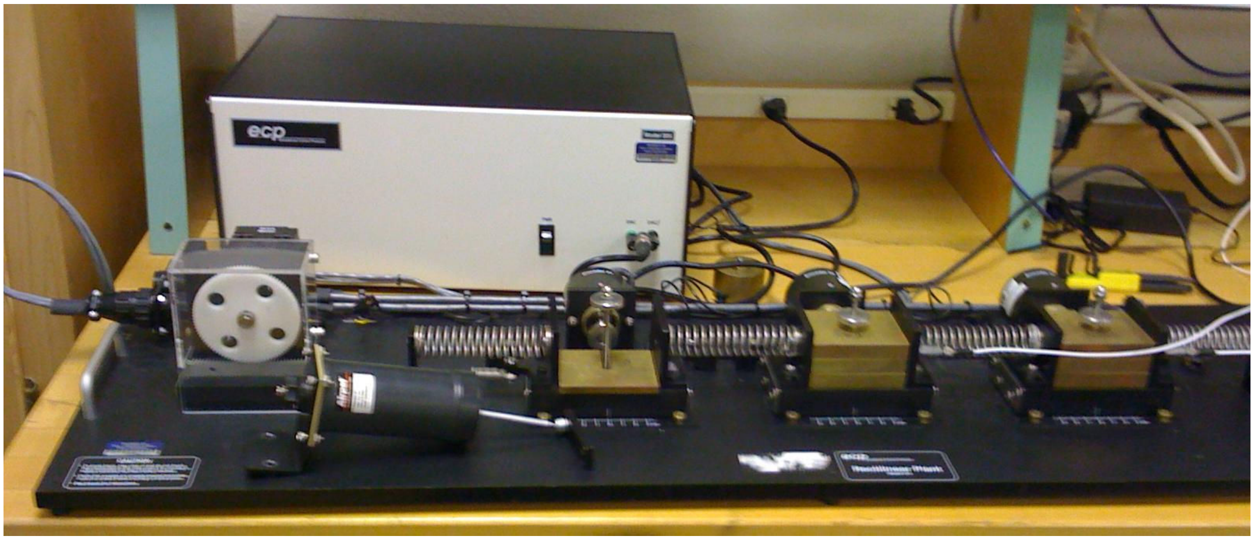

Figure 1: ecp Model 210 translational mass-spring-dashpot apparatus for evaluating one, two, or three DOF systems: forced or unforced.

Turn-key systems that are intended for engineering laboratory course in vibrations, controls, dynamics, and similar fields provide an effective way for the instructor to implement and conduct the course. One system for the study of vibrations is the Model 210 by Educational Control Products[1], as seen in Figure 1. This apparatus can have up to three degrees of freedom along a linear path in which the masses and stiffness, and damping properties can be varied. Each DOF is a carriage supported by linear ball bearings and in which plates can be added to increase the mass. An air dashpot (seen unattached in foreground in Figure 1) with variable damping may be attached to any mass. One mass can be forced into vibration via an electric motor connected to a rack and pinion link (as seen on left side of Figure 1). (That is for forced vibration experiments, which are not described in this paper.) The translational position of each mass can be seen visually with a ruler, and more precisely, with an optical sensor, and each mass may be locked in place to reduce the DOF of the system to 2-DOR or 1-DOF. A computer board and software are used to control the frequency and amplitude of the force applied, and record the motion of each mass from the optical sensor. Force at another mass may be applied for the study of control theory. This system, as well as most systems by ecp, is designed primarily for the controls laboratory experiments, but are ideally suited for mechanical vibrations laboratory experiments.[2]

The Model 205 produced by Educational Control Products (ecp), as seen in Figure 2, is used to study the rotational vibration of up to three DOFs in a similar fashion. These systems are good at helping the students verify the theory of vibration with less time learning how to do the measurements. Up to four brass elements may be attached on each disk, and at varying radial positions, to change the polar moment of inertia for each disk. Like the Model 205, each disk may be locked to the frame to reduce the system to a 2-DOF system or a 1-DOF system. Unlike the Model 210, stiffness and damping cannot be varied without custom alterations.

Figure 2: ecp Model 205 rotational mass-spring-dashpot apparatus for evaluating one, two, or three DOF systems: forced or unforced.

One disadvantage of most turnkey systems is they do not use instrumentation - accelerometers and FFT analyzers – typically used for vibration analysis of vehicles and machines in industry or research. However, much more time is required if one designs and builds custom apparatuses, such as a rig for 2 DOF torsional system developed by Souza et al.[3] Also, with a custom apparatus, custom instrumentation and transducers are required – which may or may not be research caliber instruments. One unique apparatus that the author experienced as a graduate student at The Pennsylvania State University in the 1990's used an air-hockey like track to connect mass elements with springs. It worked well, but a leaf-blower like device was required to produce enough air flow, which was noisy and could break down. When parts break down on custom apparatuses, repair or replacement is usually more difficult than a commercially available apparatus.

The hybrid approach developed by the author uses the "plant only" option of the ecp Model's 205 and 210 (which omits the computer card and software). The optical sensors are disconnected and replaced with PCB uniaxial accelerometers as necessary. The PC hardware and software for the turn-key systems are replaced with a Brüel and Kær Sound and Vibration Analyzer, Model 3560C and the accompanying PULSE software to design and use virtual instruments on a PC to control the analyzer and process the data. This particular analyzer module has two outputs and four inputs. This author has found that although the PULSE software is powerful and flexible, it is also not straight-forward to create custom experiment templates. (However, Brüel and Kær's technical support has been very helpful.) Other high caliber transducers and analyzers should be suitable for the experiment described here, such as National Instruments with LabVIEW software, as described in Reference [3].

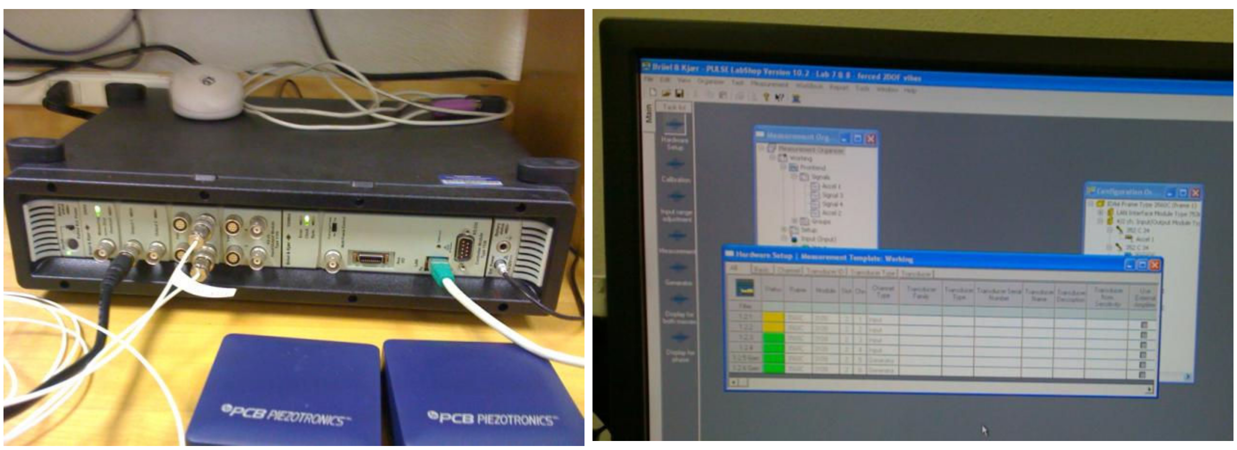

Figure 3: Brüel and Kær Sound and Vibration Analyzer, Model 3560C (left) and the accompanying PULSE software, Version 10.2, showing the hardware setup task (right).

Four different experiments for the study of free vibration are presented in this paper. Each experiment correlates to material covered by the lecture to help reinforce the course learning outcomes. Additional experiments not describe here can be done using the research caliber equipment to evaluate real world vibration of structures and machines, which are desirable to help students make a connection to real-world engineering problems. A second paper is planned for 2011 to describe several forced vibration experiments that have been developed using the same equipment here.

Course description and learning outcomes

ENGR 363 – Vibrations – is a three credit elective engineering course which has 2 hours lecture and 3 hours lab each week. (Most labs are broken into two 1.5 hour session to minimize the number of idle students. They should be preparing for lab or performing calculations during the other 1.5 hours they are away from the equipment, but close to the vibrations laboratory.)

The course is an introduction to vibration theory, including the modeling and analysis of oscillatory phenomena found in linear discrete and continuous mechanical systems. The two prerequisites courses are Dynamics and Differential Equations. This course will also introduce noise and vibration control as an application of vibrations theory. A hands-on laboratory should enhance the learning experience and bridge the gap between theory and practice.

Topics include undamped harmonic oscillator, natural frequency, mechanical resistance, damped natural frequency, torsional vibrations, forced vibrations, multiple degrees-of-freedom systems, control of vibrations, vibrations of strings, beams, membranes and plates, and a brief introduction to acoustics and noise control.

The student learning outcomes and performance criteria for this course are:

1. Students will have the ability to apply knowledge of mathematics, science, and engineering. (ABET Criterion a)

   Performance Criteria

   i Compute the natural frequency and predict the response for a one-degree-of-freedom system undergoing translational vibrations, with or without damping.

   ii Compute the natural frequency and predict the response for a one-degree-of-freedom system undergoing torsion vibrations, with or without damping.

   iii Compute the natural frequency and predict the response for a machine with a rotating unbalance.

2. Students will have the ability to design and conduct experiments, as well as to analyze and interpret data. (ABET Criterion b)

Performance Criteria

   iv Practice vibration measurements on a structure using state-of-the-art equipment, rigor and documentation.

   v Analyze the data from an experiment appropriately.

   vi Assess the validity of the experimental results and compare with theoretical results when possible.

3. Students will have the ability to identify, formulate, and solve engineering problems. (ABET Criterion c)

Performance Criteria

   vii Compute the natural frequencies and illustrate the mode shapes of a two-degree-of-freedom system;

   viii Sketch the first several mode shapes for a string, bar, or membrane, and compute the natural frequency for each.

The laboratory component of this course directly supports ABET Criterion b, and indirectly supports ABET Criterion a and c.

Experiment One – Free vibration of mass-spring systems

The two objectives are:

- To measure the natural frequency of several one-degree-of-freedom (DOF) vibration systems using experiments and compare with theory.
- To learn how to use precision grade accelerometer with a vibration analyzer to measure the acceleration and natural frequency of a 1-DOF mass-spring system.

The equipment consists of the ecp Model 210 (translational system) set up with one mass (sliding carriage without plates – 0.59 kg) and one soft spring (200 N/m), alternative stiff spring (770 N/m), additional plates (0.5 kg), piezoelectric accelerometer (PCB model T352C34), Hand-Held Shaker (PCB model TC352C34), Brüel and Kær Sound and Vibration Analyzer (Model 3560C), stopwatch, cables, and PC (with PULSE software to control the analyzer). See Figure 4. The center or right carriage is the preferred one to use for this experiment since the right carriage has a link to the motor. The other carriages not in use are decoupled and locked to the frame. In the case that the center carriage is free to move, the carriage that it is connected to must be locked to the frame, as shown in Figure 4.

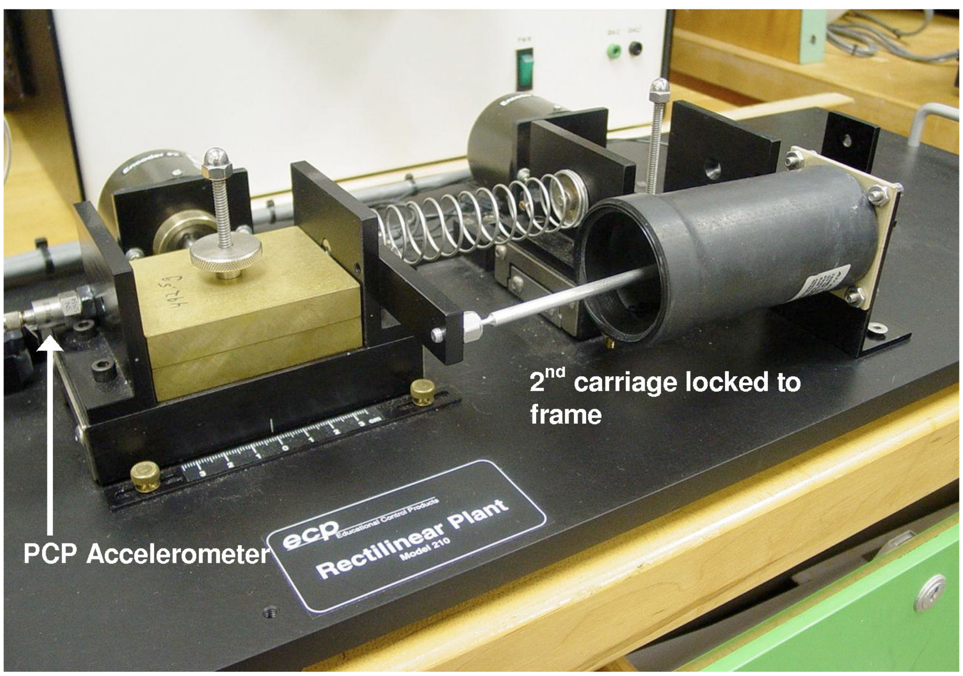

Figure 4: Setup for Experiments 1 and 2. In Experiment 2, the dashpot is disconnected.

Because the natural frequencies of the three systems are less than 5 Hz, oscillations can be visually observed and counted using a stopwatch. This is done first in order for the student to gain confidence with the analyzer and software. Given an initial displacement of 2 cm, and using a stopwatch to measure the time for a set number of oscillations (of their choosing), the students can determine the period and frequency of oscillation. Comparison with theoretical calculations should be within 10% difference.

Next the mass is increased by adding a plate and the experiment is repeated. The findings should demonstrate that the natural frequency is inversely proportional to the square root of mass in a 1-DOF system with light damping. Then the stiffness is increased by replacing the soft spring with a stiff spring to demonstrate that the natural frequency is proportional to the square root of stiffness.

The students then begin using an accelerometer and analyzer to measure the period of oscillation and the natural frequency. Calibration is introduced by first verifying the transducer's sensitivity using a hand held shaker that produces a known acceleration (1 g at 1000 rad/s). Then the accelerometer is attached to one side of the carriage (via screw thread intended for the dashpot connection or the threaded plastic base bonded to the carriage– as shown in Figure 4) and the three different mass-spring configurations are reevaluated. The natural frequency for each system is measured using the acceleration verses time plot (most precise method), and also using the analyzer's Fast Fourier Transform (FFT, the most accurate and fastest method, but precision is limited due to the length of the FFT record and thus the time it takes for the system to return to static equilibrium).

Each student is required to document all findings, calculations, observations, and answers to specific questions in their laboratory notebook.

An alternative approach to experiment one is to have the students determine the spring stiffness or carriage mass, and then compare with the values supplied by the manufacturer (ecp).

Experiment Two – Free vibration of mass-spring systems with variable damping

The objectives are:

- To obtain the key properties of one-degree-of-freedom (DOF) freely vibrating system using experiments and theory.
- Get more familiarized with using the Fast Fourier Transform (FFT) analyzer.
- Understand how varying amounts of damping affect the free vibration of a structure.
- Work with one or two classmates to conduct the experiment and write the report together.

The same equipment is used as in experiment one, with the exception with the air dashpot attached part of the time, as seen in Figure 4.

First, the undamped natural frequency is determined by a mass-spring system by using the FFT analyzer without the dashpot attached. (Of course all systems have some damping, but the ball bearings provide such low side-to-side damping that the effects on the natural frequency are negligible.) A different mass/spring combination from the first experiment is recommended. A representative result is shown in Figure 5.

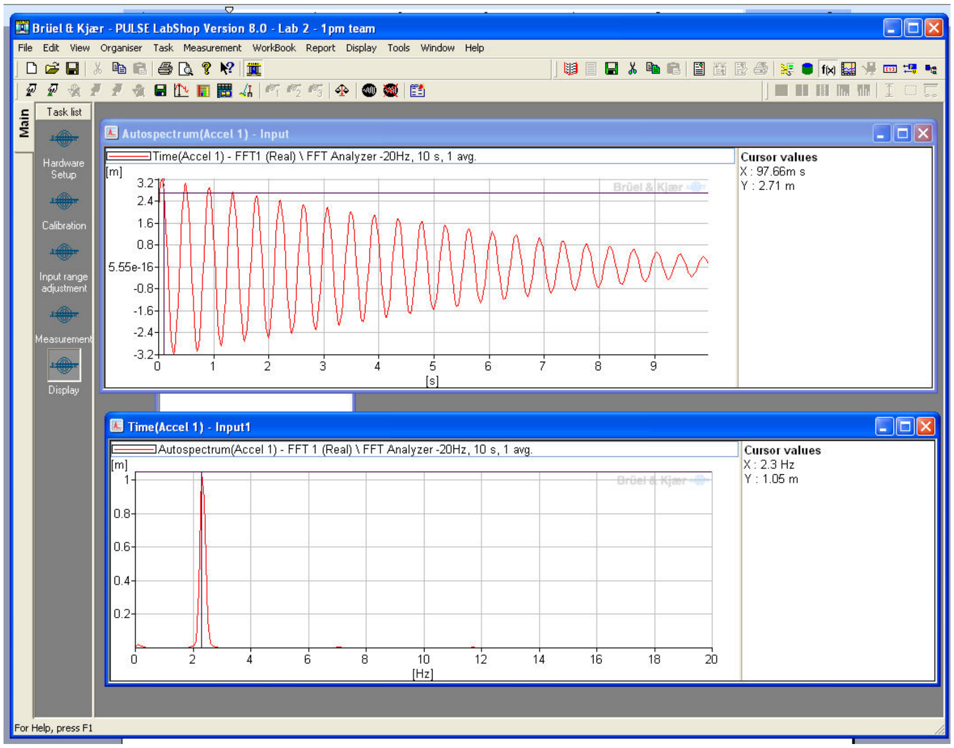

Figure 5: Analyzer results of a 1-DOF free vibration experiment with light damping.

Next, three cases of increasing damping are evaluated: underdamped, critically damped, and overdamped. The air valve on the dashpot is varied until the system comes to rest without any oscillations for the case of critical damping. With the underdamped system (with dashpot attached and adjusted to minimal damping), FFT results should reveal a damped natural frequency that is slightly lower than the undamped natural frequency previously measured. From this data, the damping constant and the damping ratio can be calculated. Another way to estimate the damping properties with an underdamped system are to measure the successive peaks from the time domain plot, then use the logarithmic decrement method.

FFT post processing is used (using the PULSE software) to quickly convert the acceleration vs. frequency graph to velocity vs. frequency, and then displacement vs. frequency. The displacement magnitude should be between initial value of 20 mm and the final value of 0 mm (and closer to zero if the time period is very long or taking a large number of spectral averages). This should again provide confidence to the students that what they are measuring with the FFT correlates to what they are observing. The students are then shown that to get time plots in dimensions of velocity or displacement, the experiment needs to be repeated using single, then double time integration. Unfortunately, due to low frequency errors from the accelerometer, which are greatly amplified with each integration, the time domain plots often turn out wavy.

A stopwatch is used to measure the time it takes the system to come to rest. This can be a rough approximation for five times the time constant of the system, providing an alternative method to find the same damping properties.

Unfortunately, the accelerometer procured is not intended to measure the very low and constant accelerations that occur with critical and overdamped systems, and the transient motion occurs too fact to accurately measure using a stopwatch. The recommended fix would be to purchase a transducer to measure frequency motion under 1 Hz. Another way around that problem is to use a high speed camera aimed at a mark on the sliding carriage next to the fixed ruler, which was implemented during the Fall 2009 semester somewhat successfully. The logarithmic decrement method can also be sued to find the damping constant and ratio when using the high speed camera.

An optional element of experiment one or two is to have the students predict what error the additional mass of the accelerometer has on the natural frequency (which should be less than 5% error). They can determine if this agrees with the rule-of-thumb that the accelerometer should weigh no more than one-tenth of the moving mass.

Technical report for this lab should improve students' technical writing abilities and improve learning through critical thinking that usually accompanies an engineering report.

Experiment Three – Free vibration of a 1 DOF system with torsional motion

The three objectives are:

- To obtain the key properties of one-degree-of-freedom (DOF) system undergoing torsional vibrations using experiments and theory. (natural frequency)
- To demonstrate the effect of inertia on a freely vibrating torsional system.
- Understand the similarities and differences between torsional systems and translational systems undergoing vibrations.

The ecp Model 205, as seen in Figure 2, is a different but analogous apparatus as compared to the ecp Model 210. As stated in the introduction, the Model 205 has up to three disks in which the inertia can be changed by adding brass cylindrical weights at various distances from the center of rotation. The three disks are connected together by two thin steel rods that act as torsional springs. The lowest disk also contains the inertia of a sheave/belt/motor system. The belt also adds significant damping, but the system is still underdamped. (If desired, the belt may be removed.)

In this experiment, the top and middle disks are locked to the frame. The lowest disk is free to rotate approximately ±10°, limited by the torsional elastic deformation of the lower steel rod. (Optionally a different disk may be selected. However, when placed on a lab bench top, the higher disks are more difficult to reach.)

There is one key difference between theory and experiments for rotational vibration. The variable in the equation of motion is angular displacement, but the most common vibration transducers available are for the Cartesian coordinate system (translational motion). The experiment can be successfully conducted using the same unidirectional accelerometer in the experiments 1 and 2, but attached tangential near the edge of the disk using a small light weight aluminum bracket as seen in the lower disk in Figure 2. Even if a coordinate transfer from x to θ is not done with measured acceleration in the PULSE program, the natural frequency will be the same. A tri-axial accelerometer also works here, with only the acceleration tangential to the edge of the disk needed.

The procedure is similar to experiment one. Three 1 DOF systems are evaluated. One is just the lower disk without added weights. The second system differs by adding two brass cylinders symmetrically mounted as close to the center as possible. The third system moves the two brass cylinders as far from the center as possible. Natural frequency for each system is measured using the analyzer. The dimensions of the steel rod are needed to calculate the torsional spring constant.

Due to the attached sheave/belt/motor system attached to the lower disk, the inertia is not calculated directly. Instead, it is calculated from the measured natural frequency and the torsional spring constant.

A review of rotational dynamics is helpful in guiding the students to calculate the inertias for the other systems. The inertial system of the second system is calculated by adding the inertia of each brass cylinder to the inertia of system one, using the parallel axis theorem. This has the added benefit of reinforcing this important dynamics concept.

The students should see that the natural frequency of a 1 DOF rotational system is inversely proportional to the square root of its inertia, and that this system is analogous to the 1 DOF translational system.

Experiment Four – Free vibration of a 2 DOF symmetrical system

The three objectives are:

- To determine the natural frequencies for a two-degree-of-freedom (2-DOF) system.
- To visualize the mode shapes for a symmetric 2-DOF system.
- To see how the initial displacement of each mass affects the free vibration for a system with two degrees of freedom.

The ecp Model 210 Translational system is again used, but now two carriages (each not connected to the rack-and-pinion) are used with three springs. Each carriage holds up to three large plates. The additional mass of the accelerometer (only 5.8 grams), and the effective masses of the springs may be added if increased accuracy is desired. The dashpot is not used and damping is neglected. Two stiff springs connect each carriage to a wall. Another stiff spring is placed between the two. One piezoelectric accelerometer is attached to on carriage 1 (mass 1) while a second accelerometer on carriage 2 (mass 2). The accelerations should be measured simultaneously using two inputs on the analyzer. This configuration is shown in Figure 1 – note that the carriage on the right is fixed to the frame.

Because the system is symmetrical, vibration mode 1 can be achieved by giving both masses an initial displacement in the same direction and approximately equal in magnitude. Vibration mode 2 can be achieved by giving both masses an initial displacement in the opposite direction and approximately equal in magnitude.   Several trials may be needed to get the feel for exciting only one mode of vibration. Any other initial condition, such as holding one mass and displacing the other will excite both natural frequencies.

In addition to the FFT plots, the frequency response should be measured to help students better understand what phase means. The phase for mode one will be very close to 0º at the lower natural frequency, while the phase for mode two will be close to 180º at the higher natural frequency. Typical results for this experiment are seen in Figures 5—7. Again, low frequency distortion amplified by double integration caused some waviness in the time domain plots. However, it is still possible to see that the mass are in-phase in Figure 5 while 180-degrees-out-of-phase in Figure 6. In Figure 7 it is not possible to see the two modes of vibration in the upper plot of time domain, but with the FFT, one can observe both natural frequencies.

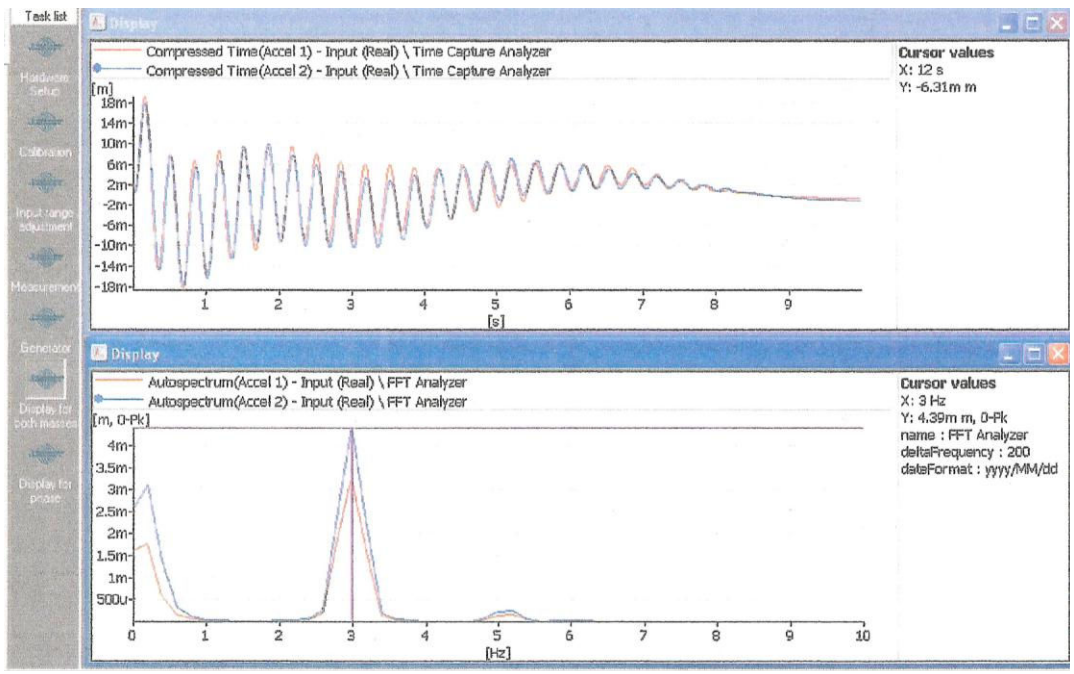

Figure 6: Measurement of a 2-DOF, free-vibration, symmetric system when both masses are given the same initial condition which excites mode 1.

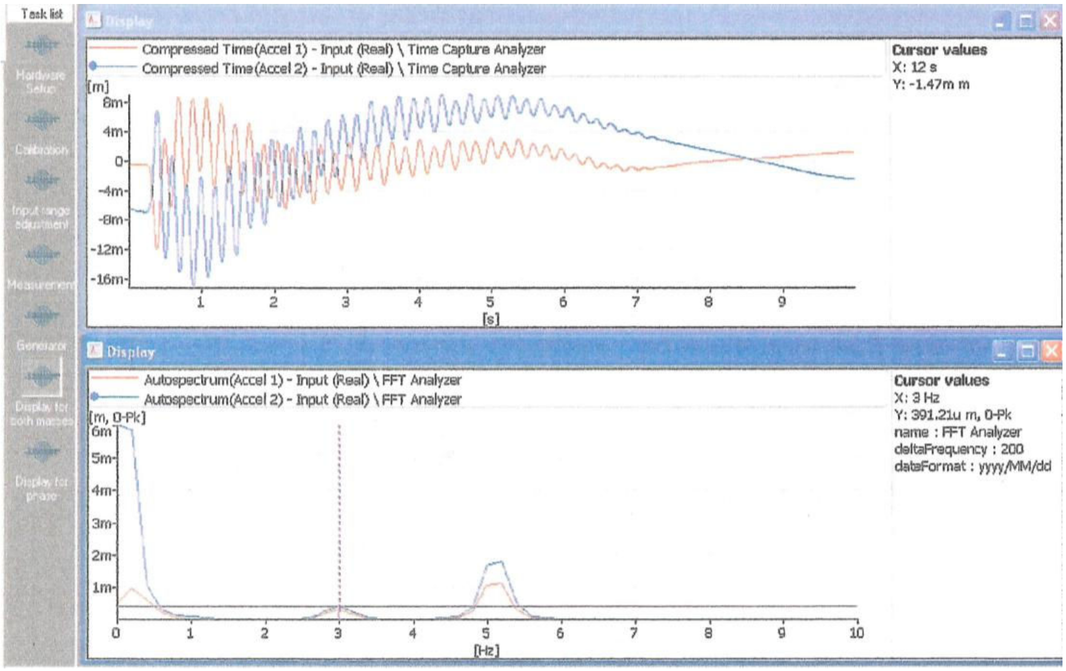

Figure 7: Measurement of a 2-DOF, free-vibration, symmetric system when both masses are given the same initial displacement, but in opposite directions, which excites mode 2.

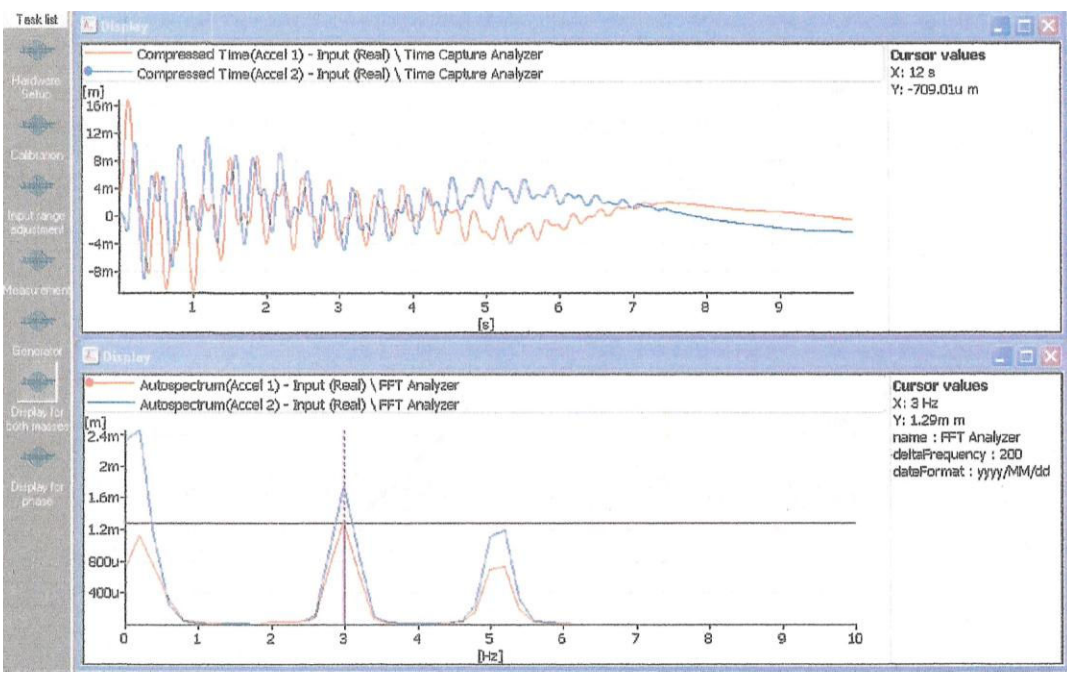

Figure 8: Measurement of a 2-DOF, free-vibration, symmetric system when one each mass is given a different initial displacement. This excites both modes 1 and 2.

This lab is relatively short, but effective. This author has found that it works well before or after introducing the rigorous solution to find natural frequencies and mode shapes for an undamped 2-DOF system. Natural frequencies are be straight-forward to predict and compare with the experiments.

This experiment could be repeated using either different weights, adding a third mass to make it a 3 DOF system, or as 2-DOF or 3 DOF system on the torsional apparatus.

Student evaluations of learning objectives

At the end of fall semester 2009 a survey was conducted to evaluate how well each student agrees that they met each laboratory learning objective. The results of this survey for the four free vibration experiments described herein are displayed in Table 1. Note that additional objectives for the forced vibration and other experiments are not shown in this table. Although most students did agree on each learning outcomes, the objective ratings associated with torsional vibration were slightly lower. It is also important to note that the instructor of this laboratory left the course in the middle of the semester due to another faculty position opportunity. This may be a reason why these scores are lower than expected.

Prior to an ABET visit in 2006, direct and indirect assessment was performed for ENGR 363 Course Learning Objectives. One objective that year that related directly to the vibration lab was, to use teamwork and rigor in conducting engineering experiments. The result of the indirect survey was that five students strongly agreed and one agreed with achieving that learning outcome. Unfortunately results of the course learning outcomes described in this paper are not available.

Table 1: Assessment of some of the laboratory learning objectives as evaluated for the 2009 fall semester. Each student evaluated each objective as 1 for strongly disagree, 2 for disagree, 3 for neutral, 4 for agree, and 5 for strongly agree. The average score is based on 7 students who completed the anonymous survey at the end of the semester.

	Lab Learning Objective	Average Score
1	Measure the natural frequency of several one-degree-of-freedom (DOF) vibration systems using experiments and compare with theory.	3.9
2	Obtain the key properties of one-degree-of-freedom (DOF) freely vibrating system using experiments and theory.	3.9
3	Use an accelerometer (a type of transducer) and an analyzer to measure the vibration of a structure in both time and frequency domains.	3.9
4	Understand how varying amounts of ***damping*** affect the free vibration of a structure.	3.9
5	Work with one or two classmates to conduct an experiment and write a report together.	4.3
6	Obtain the key properties of one-degree-of-freedom (DOF) system undergoing **Torsional** vibrations using experiments and theory.	3.3
7	Understand the effect of inertia on a freely vibrating torsional system.	3.3
8	Understand the similarities and differences between torsional systems and translational systems undergoing vibrations.	3.3
13	Determine, experimentally, the natural frequencies for a two-degree-of-freedom (2-DOF) system.	3.7

Summary

Four free-vibration experiments are described in this paper intended to use in an upper-level laboratory to study engineering vibrations. The unique aspect of these experiments is that it is a hybrid of a turn-key educational hardware and measurement systems that are often used in research labs and industry. One apparatus is the ecp model 210 and the other is the ecp model 205. The former provides translational vibration and the later provides rotational vibration. One, two, or three degrees of freedom can be easily setup. Also, it is easy to change key parameters of mass, stiffness, damping, and inertia. The optical sensors provided by ecp are disconnected and replaced with PCB uniaxial accelerometers. The PC hardware and software for the turn-key systems are not purchased and are replaced with a Brüel and Kær Sound and Vibration Analyzer, model 3560C, and the accompanying PULSE software to design and use virtual instruments on a PC to control the analyzer and process the data. PULSE templates were created by the author for each experiment to minimize the time students have to spend learning the software program. Yet the students do gain insight to real word instrumentation and calibration that they would not if using the sensors and software provided in a complete turn-key system. The research caliber analyzer and transducer may also be used for advanced student projects and for research. Other experiments for forced vibrations will be described in a future paper.

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