AC 2004-1189: GUIDED TOUR OF HOUGH TRANSFORMS ON ELEMENTARY PATTERNS

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Guided Tour of Hough Transforms on Elementary Patterns [1]

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Abstract

Student motivation in elementary mathematics continues to be a major problem. The author recommends that one solution to this problem is through the integration of applications into the elementary courses that are consistent with student interests and experiences. This paper provides an introduction to problems in human vision research and provides applications of straight line slope concepts to problems in pattern recognition that are expected to be of general student interest. The notion of a vertical line having no slope in mathematics is substituted for an angle and distance using the Hough Transform.

I. The Hough Transform

The axiom in pattern recognition states that the essence of an image is contained in the edges of the image. This is, when for example we look at alphabets, E, F, H, L and N given in Figure 1, the edges contribute primarily to the recognition of the letters. The inside heavy black lines would not affect the recognition of the image. Therefore, the edges can be decomposed as a sequence of straight lines. However if we use the traditional description of lines given in all algebra courses implementing slopes, the problem of infinite slope presents itself when discussing a vertical line. Recall for vertical lines the change in the x-direction is zero giving us a meaningless slope.

One technique to overcome this problem is to use a parametric space using the angle of a normal line drawn from the origin to a given line of the image and the length of the normal measured from the origin to the given line. This is illustrated in Figure 2. The line segment denoted distance1 in Figure 2 represents the length of the normal line segment to the given line segment.

Figure1. The Alphabets, E, F, H, L, N

E F H L N

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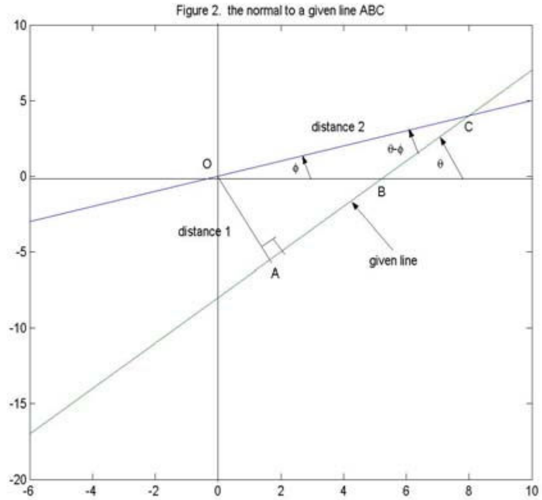
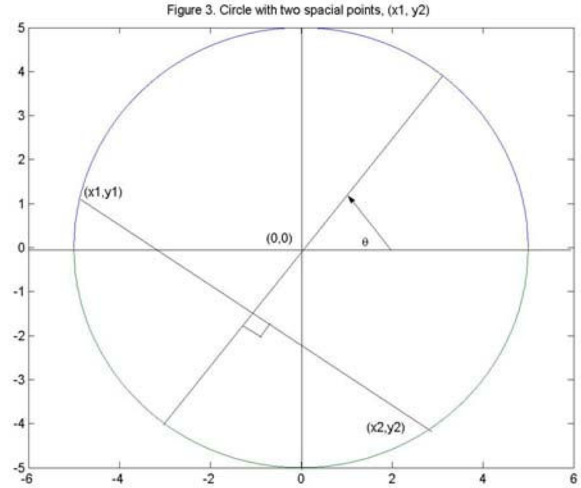
Figure 2. the normal to a given line ABC



The mathematical notation in our computations are given as distance1=d_1 and distance2=d_2..
If we implement trigonometry and use the notation defined in Figure 2, we have the following equations, $\frac{d_1}{d_2} = \sin(\theta - \phi)$ or $d_1 = d_2 \sin(\theta - \phi)$.

This prepares us for the Hough transform. Now consider two spatial coordinates, (x_1, y_1) and (x_2, y_2), on the circle. We denote this in Figure 3.

Figure 3. Circle with two spacial points, (x1, y2)



We implement trigonometry and similar triangles to give us the equations,

$$\cot\theta = -\tan(\theta + \pi/2) = \frac{-(y_2 - y_1)}{(x_2 - x_1)}$$

implying

$$\frac{\cos\theta}{\sin\theta} = -\frac{(y_2 - y_1)}{(x_2 - x_1)},$$

implying,

$$x_1\cos\theta + y_1\sin\theta = x_2\cos\theta + y_2\sin\theta.$$

Clearly this last equation is valid for each pair of distinct points on the circle and the associated θ in the interval, $0 \le \theta \le 2\pi$. The Hough transform is now defined as

$$r = x\cos\theta + y\sin\theta$$

where $-5 \le r \le 5$, $0 \le \theta \le \pi$. From this analysis we see that

$$\theta = \text{Arc cot}\frac{-\Delta y}{\Delta x}.$$

II. Hough Transform of a Line Segment

We continue our development using the circle approach. Let us compute the Hough transform of a normalized line segment, which can be considered to be an edge in an image. The equation of the line, y=ax+b, and the unit circle, $x^2+y^2=1$ will intersect at $x=\frac{-ab \pm \sqrt{a^2 - b^2}}{a^2+1}$. Therefore substituting our coordinates into the Arc cot formulas of the previous section gives us

$$\theta = \text{Arc cot}(-a).$$

Writing the Hough transform formulas as

$$\frac{r}{\sin\theta} = x\frac{\cos\theta}{\sin\theta} + y$$

and substituting the value cos(θ) = -a, 0 for x, and b for the y ordinate gives us r=bsin(θ)=bcos(π/2-θ)

or

r/b=cos(π/2-θ).

Example: Let y=x+1/4 be the equation of the line and the Hough transform then gives θ=Arc cot(-1) = 3/4π and r=1/4sin(3/4π)=$\sqrt{2}$ /8. Thus the line y=x+1/4 is transformed to one point given by the parametric coordinates, ($\frac{\sqrt{2}}{8}$, 3/4π). The line and point is illustrated in Figures 4 and 5 respectively.

Figure 4. The line y=x+1/4 intersecting a circle

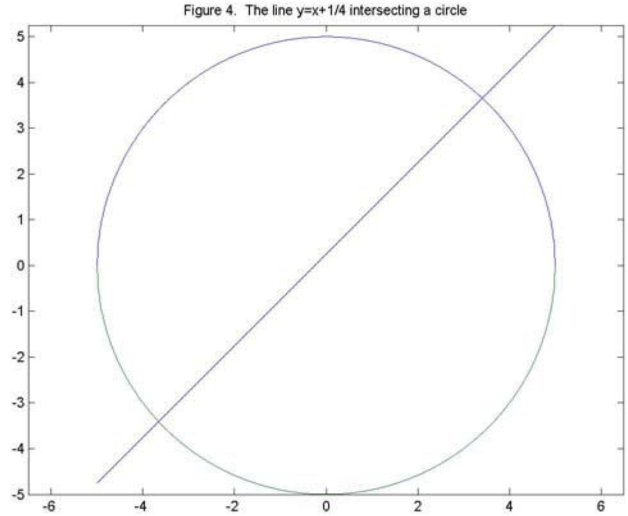
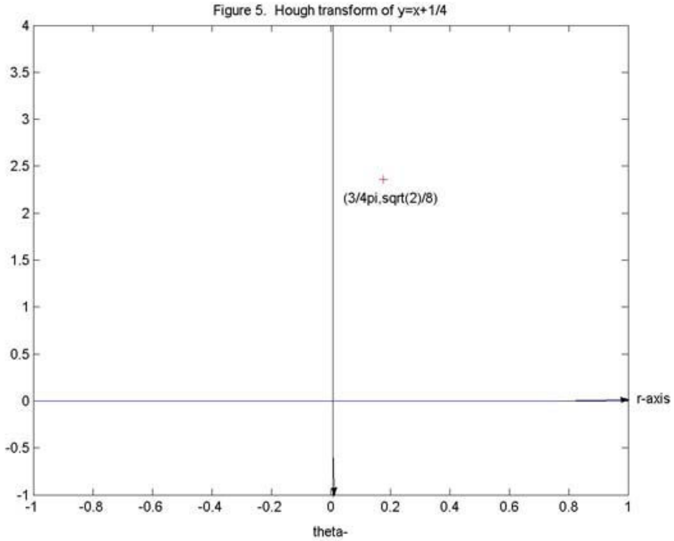


Figure 5. Hough transform of y=x+1/4



The density of the line will contribute to the "shape" of the Hough transform. We have illustrated 5 alphabets in Figure 1 and now we illustrate their Hough transform in Figures 6 to 10. The advantage of the transform is that a complete line segment is transformed to one point in the Hough space. This enables the reader to see a clearer view of the Hough transform.

Figure 6. The Hough Transform for the letter E

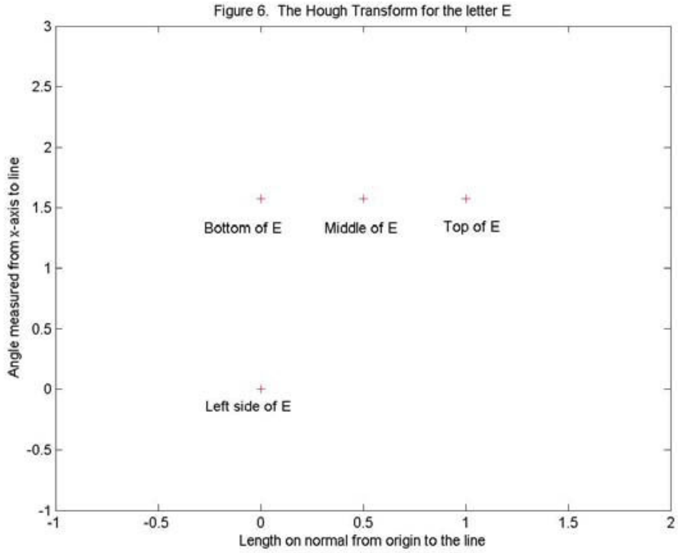


Figure 7. The Hough Transform for the letter F

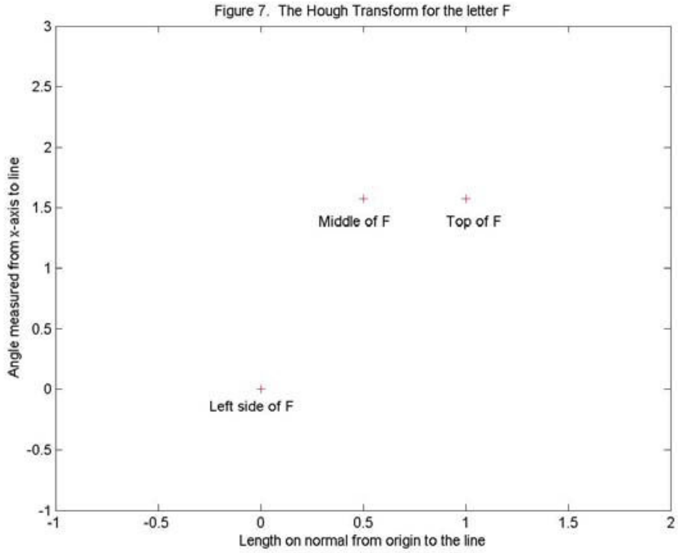


Figure 8. The Hough Transform for the letter H

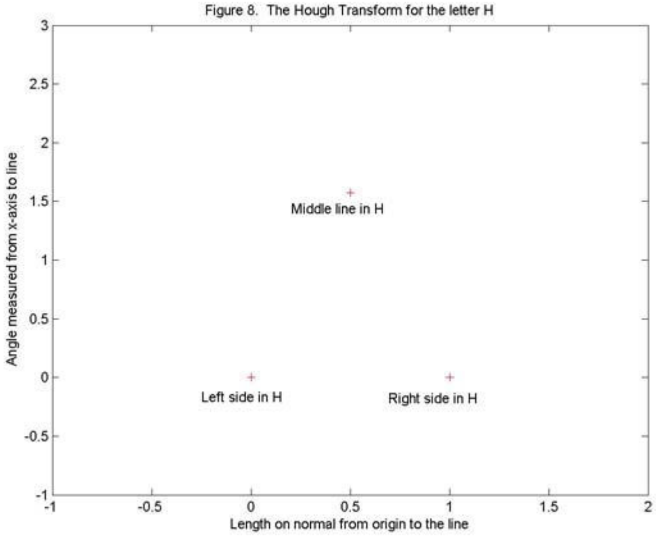
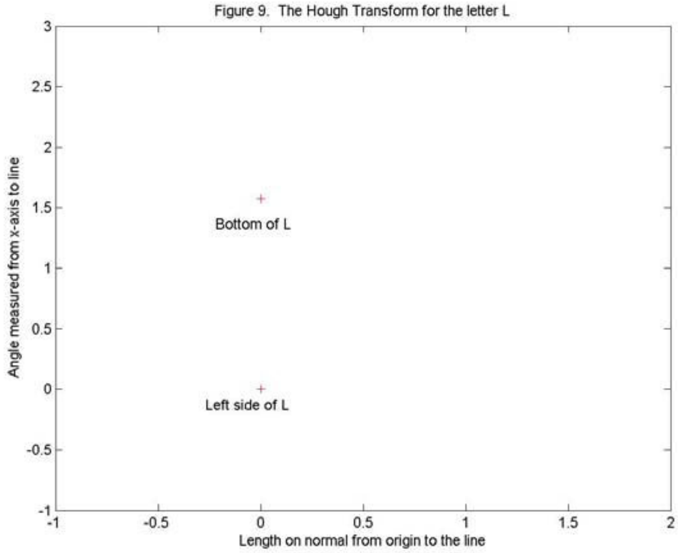
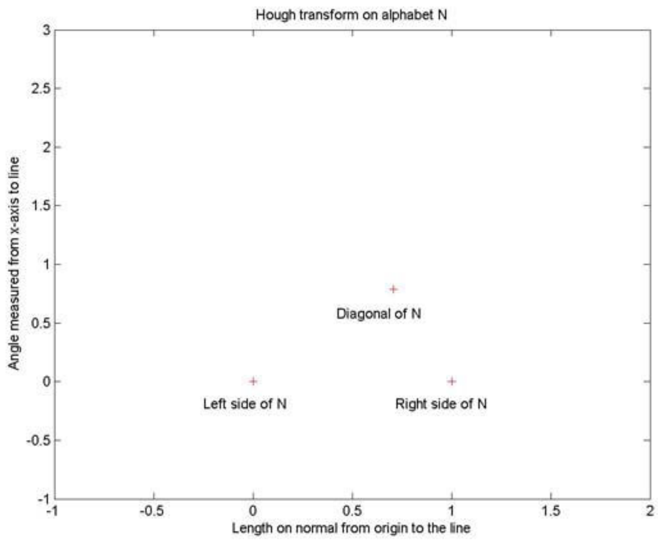


Figure 9. The Hough Transform for the letter L





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Dr. John Schmeelk is a Professor of Mathematics at Virginia Commonwealth University and is currently teaching at VCU-in Doha, Qatar. He has spent summers of 1986 through 1993 at Fort Rucker, Alabama where he implemented procedures in generalized functions. He was invited to the Fourteenth Conference on Differential Equations in Plovdiv, Bulgaria during the summers 2003.