Hardware Experiments in Feedback Control Systems Using a Geared Dc Motor

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Abstract

One of the difficulties in teaching control systems to engineering and technology students is to relate classroom theory and computer simulation to experimental results. Students tend to focus on analyzing feedback control systems without understanding where the transfer functions of real life systems come from. This effect is exacerbated by textbooks in control systems, where authors often assume that variables such as moment of inertia, damping coefficients and gear ratios are readily available when determining the transfer function of a system. The purpose of this paper is to present a series of experiments with a geared dc motor which is coupled to a rotary potentiometer. In the first experiment, the students look at the impulse response of the motor to determine the transfer function of the motor. Since an ideal impulse function cannot be provided, the students provide a short duration pulse to the motor to approximate an impulse. The results of experimentally determining the transfer function can then be used in a software package, such as Matlab, to compare simulation results to hardware results. In the second experiment, the students build a position control system for the geared dc motor and again compare results with simulation results. Non-linear effects such as saturation of operational amplifiers can be considered as part of both hardware and simulation. Students are able to gain a wealth of understanding from these labs, whereby the transfer functions in the textbook become more than just numbers and variables.

Introduction

Senior students in Electrical Engineering Technology (EET) at Penn State Erie, The Behrend College are required to take a course in feedback control systems. This three credit course includes two hours of lecture, along with a two hour lab section, each week. The intended course outcomes are for the students to be able to:

- Utilize prior knowledge of Laplace transforms to solve s-domain transfer functions in the time domain
- Determine open and closed loop transfer functions for a control system
- Determine the time domain response characteristics of a control system (e.g., rise time, settling time, % overshoot, etc.)
- Identify the different parts of a control system (e.g., actuator, sensor, controller, etc.)
- Utilize Bode and Nyquist diagrams to determine the stability of a control system
- Understand the difference between proportional (P), proportional + integral (PI) and proportional + integral + derivative (PID) control systems
- Build both parts of and entire control systems in the laboratory

A difficulty in teaching this course involves enabling students to relate the lecture and textbook material to the laboratory experiments. Specifically, textbooks[1,2,3] provide equations to model the dynamics of a system, assuming physical characteristics are available such as moments of inertia, spring constants and gear ratios. They also provide both classical analysis as well as (for [1] and [2] only) state space analysis techniques. However, problems at the end of each textbook chapter often focus on the analysis techniques without providing any information on the system itself. For example, the following problem is provided[1]:

'Sketch the Nyquist diagram for a unity negative feedback system with a forward transfer function of G(s) = K/[s(s+2)(s+10)]. From your Nyquist plot determine the range of gain, K, for stability.'

The solution to this problem is provided by using Matlab and its associated Control Systems Toolbox[4]. Students learn how to apply their knowledge of stability analysis to determine K. But the problem does not enable the student to learn how G(s) was initially developed. Often, engineers in the field will have neither a transfer function for the plant readily defined, nor an easy way of defining critical parameters to develop a transfer function for a plant. Finding the model of different components in the control system can be difficult, especially if data sheets are not available, or if the design of the component is highly complex. Yet they must still attempt to develop a model which is representative of the plant, if they wish to perform a dynamic analysis of the system. The ability to model the major elements of the control system can make the design of a controller much simpler.

The purpose of this paper is to present a series of laboratory experiments which will enable students to estimate the transfer function of a system and apply it to a control system design. The laboratory experiments require inexpensive components to construct the system. In the first experiment, students estimate the transfer function of a geared dc motor. In the second experiment, the students build a position control system for the geared dc motor. In both experiments, the students use their knowledge of control systems to simulate the system in Matlab and compare their simulated results with hardware results. The laboratory experiment to estimate the transfer function of the geared dc motor system is intended to take two (two-hour) lab periods. The laboratory experiment to build a position control system is intended to take one lab period.

System Construction

The system to be studied is provided in Figure 1. A geared dc motor is mounted to a wooden base via a metal bracket. The motor should be geared so that rotation can be easily observed and timed. The shaft of the motor is connected to a piece of Tygon tubing using an aluminum coupling. A set screw is provided with the coupling to ensure a tight fit onto the motor shaft. The Tygon tubing is sized to enable a snug fit with the aluminum coupling. A second aluminum coupling connects the Tygon tubing to the shaft of a 50kΩ, 10-turn potentiometer. This shaft is directly connected to the wiper arm of the potentiometer. The Tygon tubing fits snugly enough around the aluminum coupling of the potentiometer to allow the potentiometer wiper to turn with the motor shaft. However, the tubing is also loose enough such that when the wiper arm reaches the end of its travel, the motor will continue to turn while the wiper arm does not. The Tygon

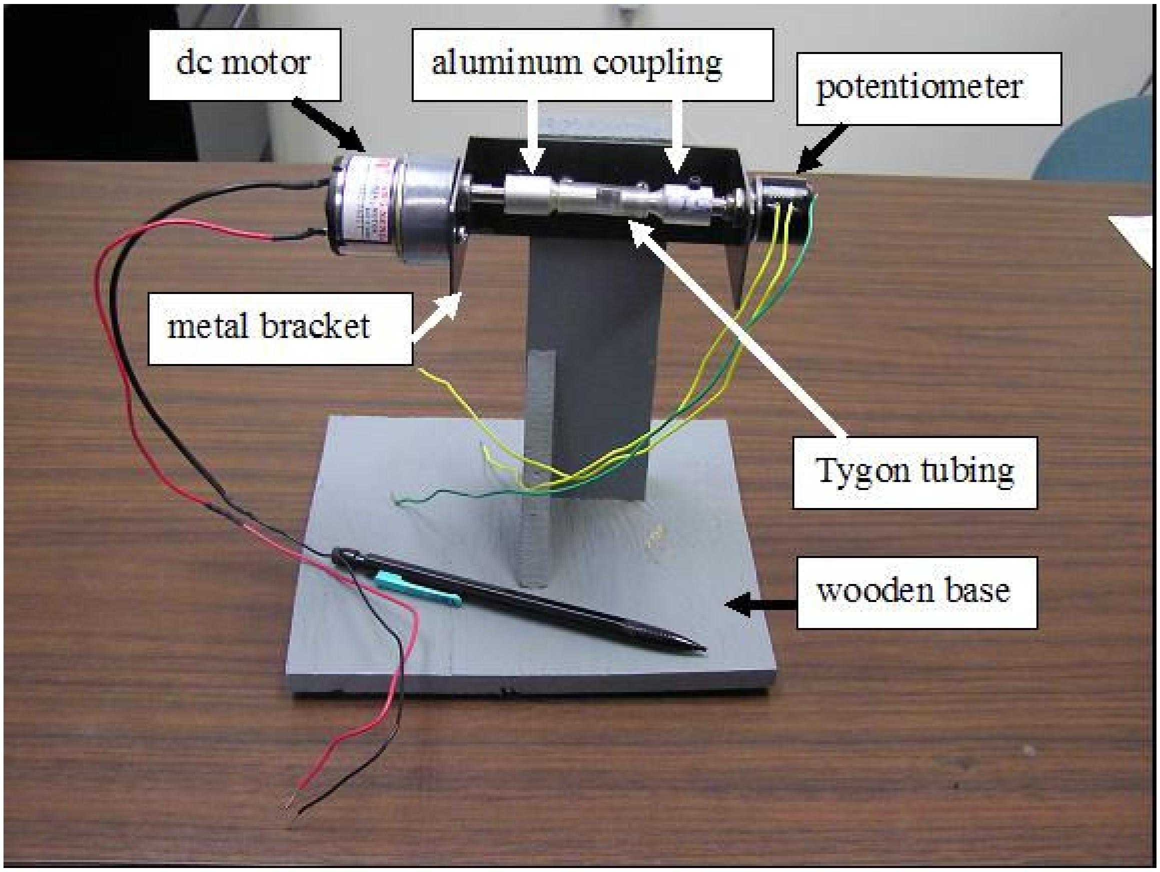


Figure 1. Constructed laboratory system for a geared dc motor.

tubing “slips” on the aluminum shaft connected to the potentiometer wiper, preventing damage to the potentiometer which would occur if the wiper were forced past its travel limit. To enable this slippage to occur, the Tygon tubing needs to be especially snug around the aluminum coupling to the motor shaft. This ensures that when slipping occurs, the side of the tubing which is mounted to the aluminum coupling for the potentiometer will begin to slip first.

Laboratory Experiment 1 Setup

In this laboratory experiment, students experimentally estimate the transfer function (model) of the geared dc motor by observing the impulse response of the motor. The students look at the time domain characteristics of the motor and rotary potentiometer, and then build their model in Matlab/Simulink and compare simulated results with the experimental response.

Figure 2 represents the block diagram of a basic system with one input and one output. In this system,

- R(s) represents the input signal in the Laplace domain.
- G(s) represents the transfer function of the system to be analyzed
- C(s) represents the output signal due to the input signal and transfer function

Figure 2. Block diagram of a basic input/output system.

In the Laplace domain, the output signal is related to the input signal via the equation:

$$C(s) = R(s) \cdot G(s) \qquad (1)$$

If the input to the system is an impulse function, it can be shown that the Laplace transform of the impulse function is one[5]. Thus, for Equation (1) above, the output of the system C(s) will be equal to the transfer function G(s) if an impulse function is applied to the system.

The characteristics of an impulse function are:

1. Infinite height,
2. Infinitely small width, and
3. Area of one.

In real life, such a function cannot be realized. However, it can be reasonably approximated if a pulse can be applied and shut off with a short duration. For this experiment, the students practiced turning on and off a Hewlett Packard E3631 Triple Output DC Power Supply. The dc voltage was modified based on the time required to apply and shut off the power supply, such that the area of the pulse was approximately one. The pulse that was generated tended to have the form of Figure 3. Due to the somewhat slow slew rate, students were told to estimate the end of the pulse to be halfway between the pulse peak and when it reached zero. After a few trials, most students were able to apply a 5Vdc pulse for 200ms, achieving the third characteristic of the ideal impulse described above.

A functional diagram for testing the impulse response of the system of Figure 1 is provided in Figure 4. A separate power supply, providing ±15Vdc is supplied to the end leads of the potentiometer. This enables the voltage pulse (Vpulse) to be applied without affecting the voltage on the potentiometer. The wiper of the potentiometer should be adjusted using the input power supply (Vpulse) until Scope Probe 2 reads approximately zero volts. By adjusting the voltage supply of Vpulse to 2Vdc, one can get very close to zero volts. When the voltage Vpulse is applied to the system of Figure 4, a response similar to that shown in Figure 5 is obtained on Scope Probe 2 (this result was obtained by the instructor in one of his initial trials). The plot rises up to a value of approximately 74.4mV.

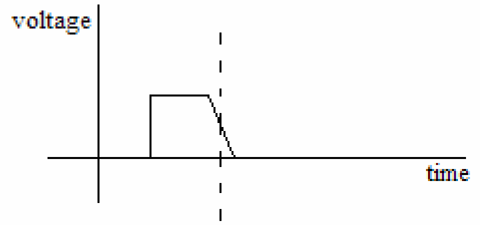


Figure 3. Typical waveform created by turning on and off the dc power supply.

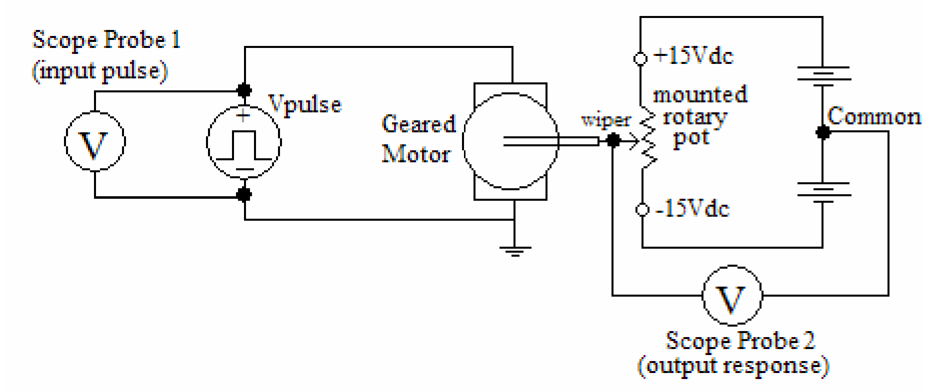
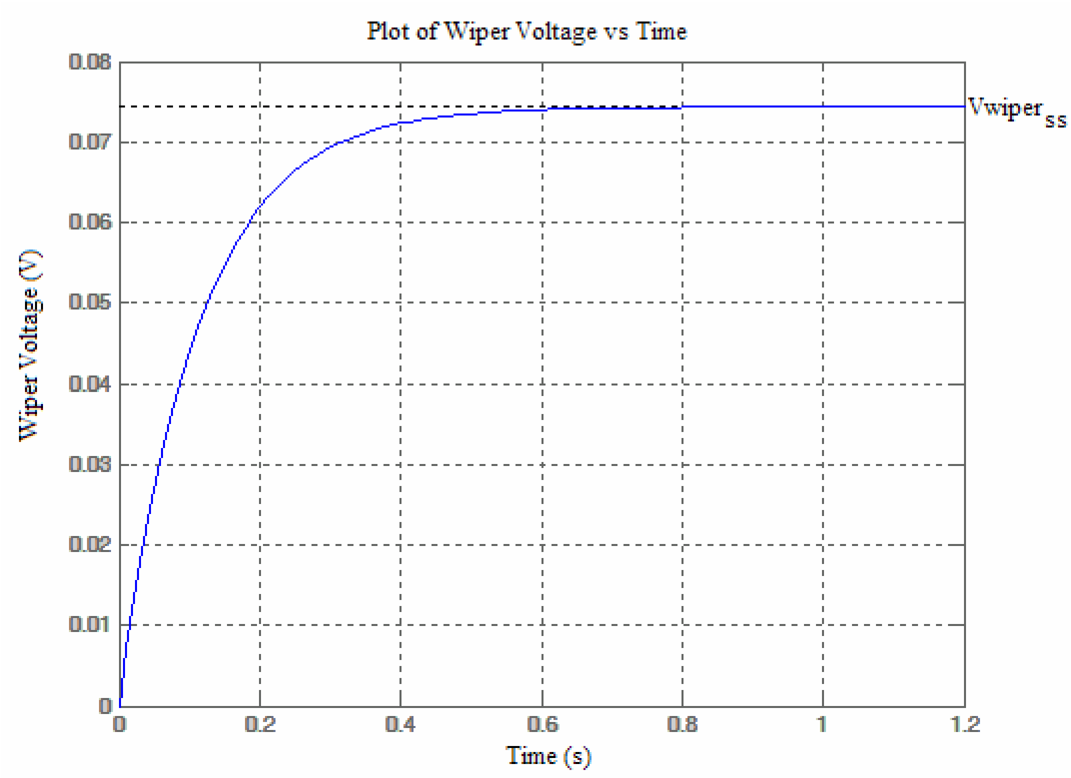


Figure 4. Functional diagram of the geared dc motor test system.



Laboratory Experiment 1 Analysis

The block diagram of the system is shown in Figure 6. By working backwards from the wiper voltage, the transfer function of the motor can be obtained. First, note that the potentiometer has a full range of 10 turns (which corresponds to 3600 degrees). Since 30 volts is applied across the two end leads, the transfer function of the potentiometer, in volts/degree, is 30V/3600 degrees, or 0.008333 V/degree. By working backwards, the change in angular position (θ(t)) can be obtained by multiplying Vwiper(t) by 120. For the example in Figure 5 above, the steady state change in angular position (θ_{ss}) would be approximately 8.93 degrees.

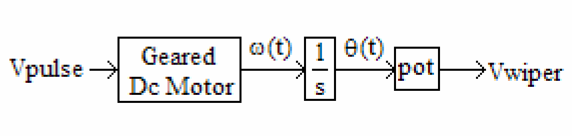


Figure 6. Block diagram of the geared dc motor test system.

From textbooks on electric machines[6], the speed of a shunt dc motor ($\omega(t)$) is proportional to the voltage applied. Hence, the voltage supplied to the motor will result in a rotational velocity. However, the application of Vpulse resulted in a shift in the motor position $\theta(t)$, as seen in Figure 5. To convert from motor speed to motor position, an integrator (1/s) must be provided to the system. However, since $\theta(t)$ is already available, one can go backwards and obtain $\omega(t)$ by differentiating $\theta(t)$. Despite the complexity in the design of the dc motor, the "impulse" response tends to yield a first order response. By looking at Figure 5, the time constant τ of the system[5] (where the wiper voltage reaches 63% of its steady state value of 74.4mV) is approximately 100ms.

The bandwidth of the geared dc motor (α) of Figure 5 is related to the inverse of the time constant ($1/\tau$), or α=10 rad/s. This corresponds to a cyclic frequency bandwidth of approximately 1.59 Hz. Although students may not be able to determine if this bandwidth is correct, they should be able to comprehend whether such a result makes intuitive sense. For example, the instructor can ask the students what would happen if a signal with a frequency of 1/30Hz was supplied to the geared dc motor. The students should be able to visualize the geared dc motor "following" the slow sinusoidal input voltage by first spinning in one direction, then slowing down and beginning to spin in the other direction. The instructor can then ask the students what would happen if a signal with a frequency of 30Hz was supplied to the geared dc motor. The students should respond that the motor will not respond, or will vibrate slightly, because the geared dc motor cannot adequately respond to a signal which is changing so rapidly. This approach should enable the students to get an intuitive feel for the bandwidth of the geared dc motor.

The system of Figure 6 can now be completely characterized. An approximate equation describing the wiper voltage of Figure 5 is

$$\text{Vwiper}(t) = \text{Vwiper}_{ss} - \text{Vwiper}_{ss} e^{-\alpha t} \tag{2}$$

The equation for the motor position in degrees can then be found by multiplying the wiper voltage in (2) by the *inverse* of the transfer function of the potentiometer:

$$\theta(t) = (\text{Vwiper}_{ss} - \text{Vwiper}_{ss} e^{-\alpha t})*120 = \theta_{ss} - \theta_{ss} e^{-\alpha t} \text{ (degrees)} \tag{3}$$

where $\theta_{ss} = (120)(\text{Vwiper}_{ss}) = 8.93$ degrees for the example of Figure 5.

Differentiating $\theta(t)$ in (3) yields the equation for the motor velocity in degrees per second:

$$\omega(t) = (d/dt)(\theta(t)) = \theta_{ss} \alpha e^{-\alpha t} \text{ (degrees/second)} \tag{4}$$

Note that even though the equation for the motor rotational velocity is in degrees per second, the bandwidth of the system (α) is still in radians per second. Since the input to the geared dc motor is approximately an impulse function, the output is due entirely to the motor itself. Taking the Laplace transform of (4) yields the transfer function of the motor:

$$G(s) = \mathcal{L}\{\omega(t)\} = (\theta_{ss}\alpha)/(s + \alpha) \tag{5}$$

In the sample result in Figure 5, the transfer function for the motor is approximated to be

$$G(s) = 89.3/(s + 10) \tag{6}$$

Laboratory Experiment 1 Transfer Function Validation

To test whether the transfer function G(s) obtained for the geared dc motor is accurate, the students first needed to observe the response of the geared dc motor system. With the potentiometer wiper at one end of its travel, A steady dc supply voltage of 15Vdc was supplied by Vpulse to the system of Figure 4, and the time for the wiper to traverse the entire 10 turns (3600 degrees) of the potentiometer was recorded. This time tended to be approximately 22 seconds for the geared dc motor used most frequently in the experiment. The time to go half the distance (having the wiper go from the center of its travel to one end) was approximately 11 seconds.

The students then constructed the system of Figure 6 in Matlab/Simulink, and looked at the step response of the system. A step input of 15V was supplied to represent half the travel time of the wiper, and a saturation block (saturating at ± 15V) is included to represent the limiting travel of the wiper. The configured system in Simulink is provided in Figure 7. The Simulink results are provided in Figure 8, and show that the wiper reaches the end of its travel after approximately 13.5 seconds, yielding an error of approximately 17.4%.

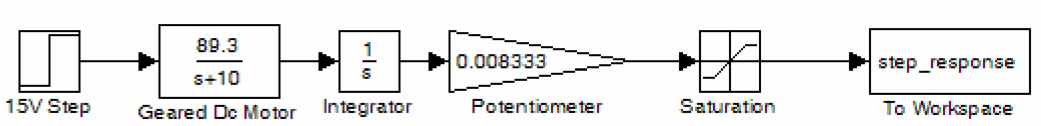


Figure 7. Simulink block diagram for testing the transfer function of the geared dc motor system.

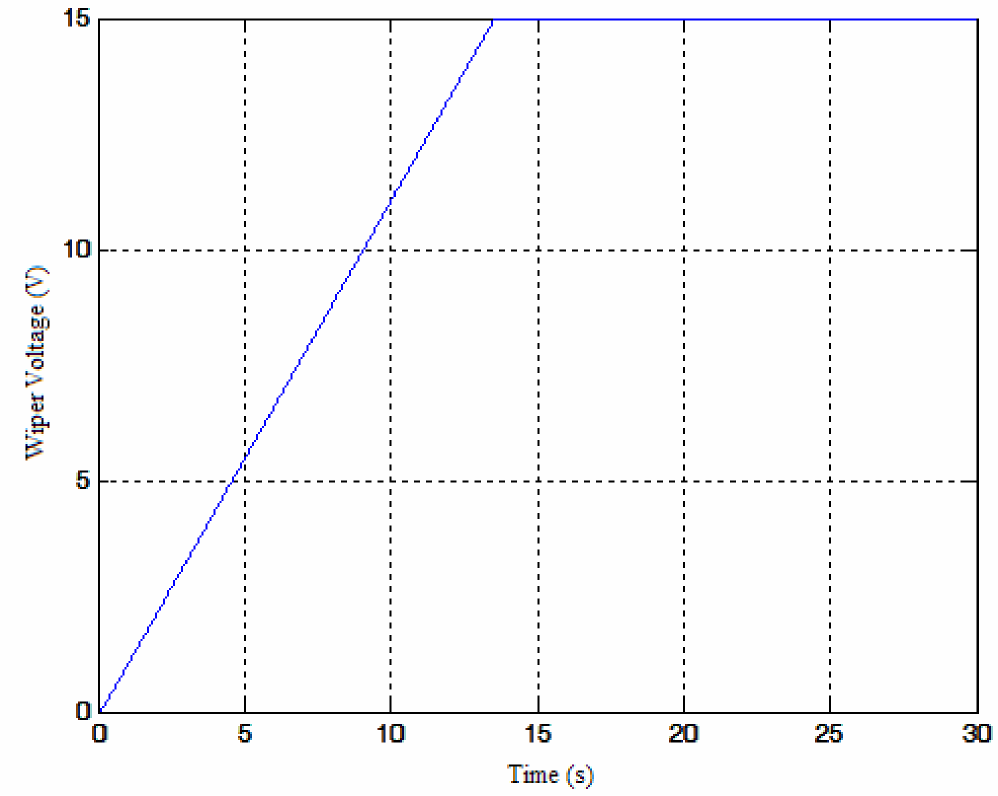


Figure 8. Plot of simulated response based on the developed system transfer function.

Laboratory Experiment 1 Results

Each lab team consisted of two senior students in EET. More than half of the student teams were able to develop a model of the geared motor system which resulted in a simulation time within 50% of the actual time for the motor to rotate either the full 10 turns or 5 turns (halfway). This level of accuracy is reasonable considering the variability in supplying the impulse, the non-ideal nature of the impulse itself, and non-linear effects related to the Tygon tubing coupling.

Laboratory Experiment 2 - Development of a Position Control System

Position control systems are used heavily throughout industry in areas such as robotics and radar systems. The students were able to use their transfer function obtained in the previous lab experiment to simulate the response of their position control system in Matlab/Simulink. A possible block diagram of the position control system is provided in Figure 9. In this example, a simple proportional controller is implemented with a gain of 10. The rotary potentiometer is used as a sensor. A saturation block is included at the output of the controller due to the railing of the operational amplifier (op-amp) used to implement the controller. A circuit schematic used to implement the position control system is shown in Figure 10. An LM675 power op-amp[7] is used to drive the geared dc motor. The students were provided with the power op-amp but were not given guidance on how to implement the control system. Thus, a number of different approaches were used, with widely varying results. However, all of the student groups were able to successfully implement the position control system. Simulated results in Figure 11 show that when a 15V step voltage is applied to the system, the system reaches steady state after approximately 16 seconds, versus approximately 12 seconds in the actual system. It was also noteworthy that some teams were able to implement a proportional-plus-integral (PI) controller and see the system overshoot the set point while still remaining stable, consistent with simulation results.

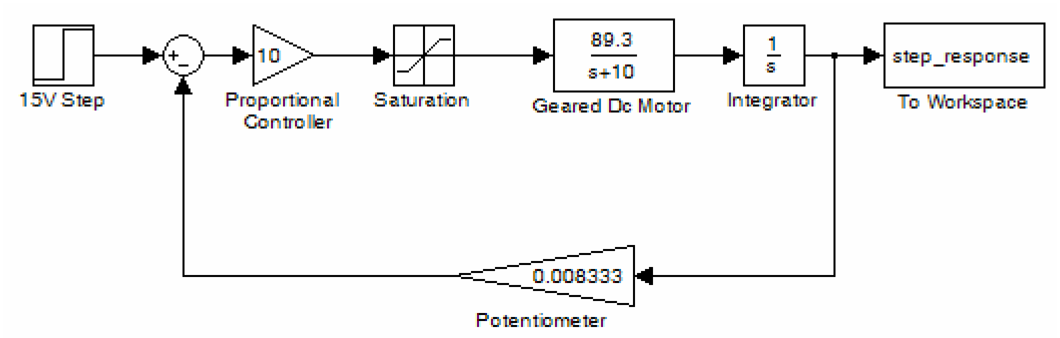


Figure 9. Block diagram of a position control system with proportional controller.

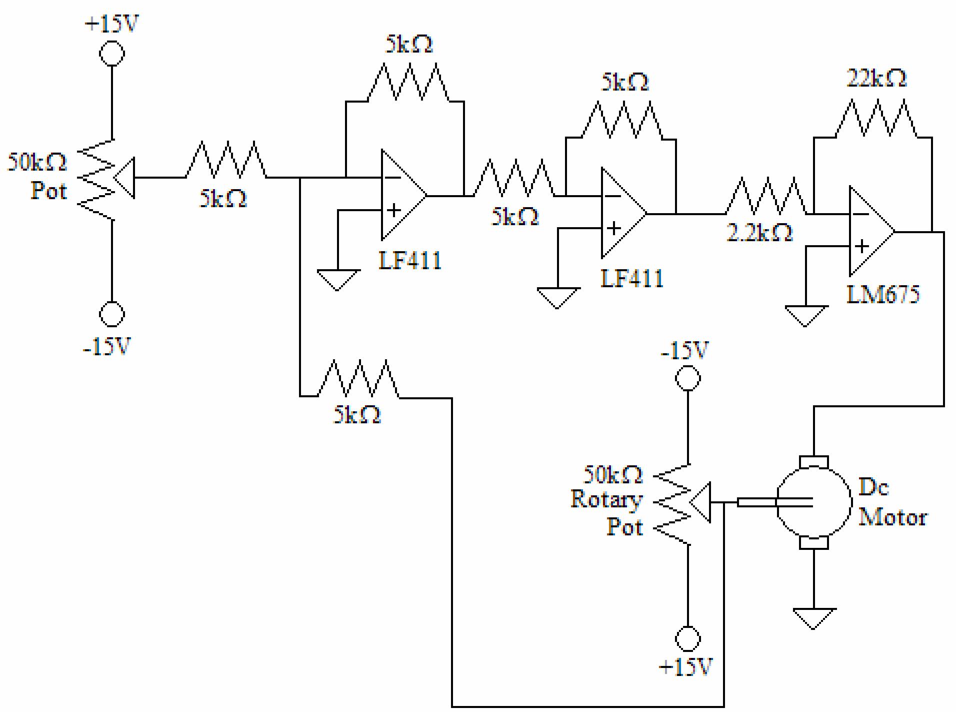


Figure 10. Schematic diagram of a position control system with proportional controller.

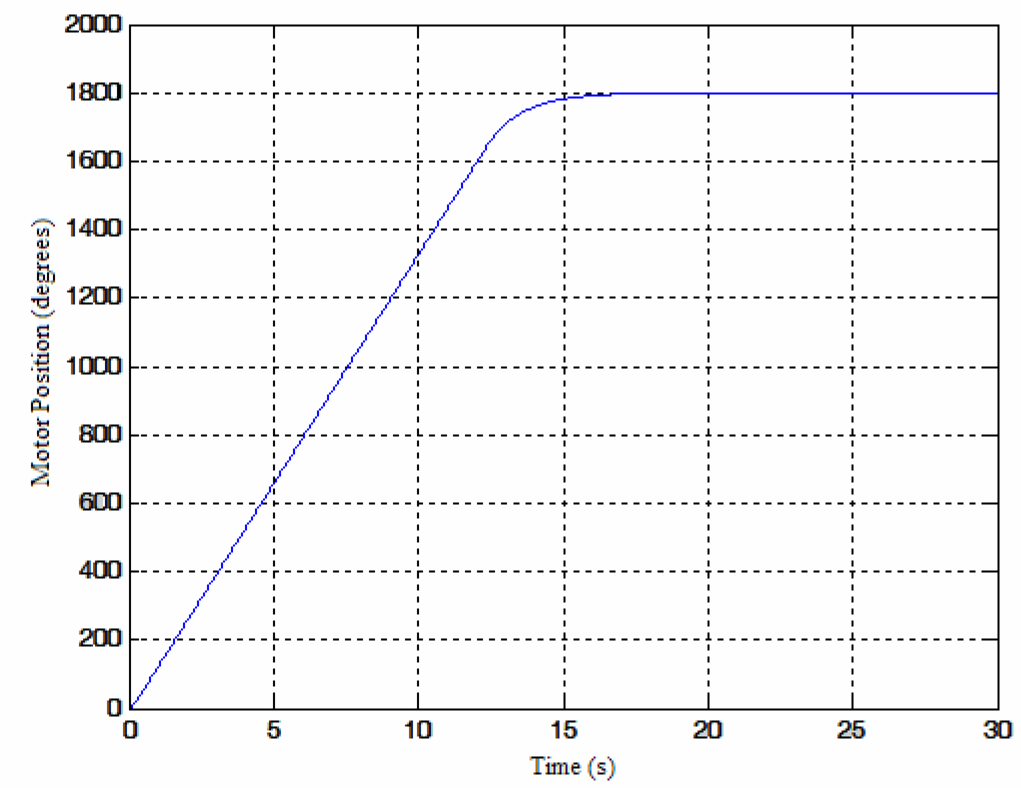


Figure 11. Matlab/Simulink results of the position control system.

Conclusion

Students in EET enrolled in a control systems course at Penn State Erie, The Behrend College, were given a hands-on laboratory where they were able to experimentally determine the transfer function for a non-trivial system. A majority of the teams were able to positively validate their results by comparing simulation results with hardware results. The students were then able to use the same system to build a position control system. The major advantage of this experiment comes when students realize that the axes on their Matlab simulations really do represent variables such as time, voltage and position, because the students were the ones who provided all of the transfer functions for the system simulation. Then, the transfer functions provided at the end of the chapter in control systems textbooks are no longer mysterious and arbitrary. Students are able to fill a void that is apparent in most textbooks, namely how transfer functions can be determined, and thereby get a more thorough understanding of control systems.

Acknowledgements

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References

[1] Control Systems Engineering, by N. Nise, 4th Edition, John Wiley & Sons, 2004
[2] Modern Control Systems, by R. Dorf and R. Bishop, 9th Edition, Prentice Hall, 2000
[3] Introduction to Control System Technology, by R. Bateson, , 6th Edition, Prentice Hall, 1999
[4] Matlab, Release 13, The Mathworks Company, Natick, MA 01760, www.mathworks.com
[5] Transform Circuit Analysis for Engineering and Technology, by W. Stanley, 5th Edition, Prentice Hall, 2003
[6] Electric Machinery Fundamentals, by S. Chapman, 3rd Edition, McGraw Hill, 1999
[7] National Seminconductor Corporation website for the LM675 data sheet: http://www.national.com/ds/LM/LM675.pdf

Biography

ROBERT S. WEISSBACH received his Ph.D. in electrical engineering from Arizona State University in 1998. Since August 1998, he has been an assistant professor of engineering in the Electrical Engineering Technology department at Penn State Erie, the Behrend College, where he is currently the program chair. His research interests are in power electronics, power systems and multidisciplinary education.