How Do You Teach Vibrations to Technology Students?

Dr. M. Austin Creasy, Purdue University (Statewide Technology)

Assistant Professor Mechanical Engineering Technology Purdue University

How Do You Teach Vibrations to Technology Students?

Abstract

Vibrations is an upper level mechanics course that is seen to require upper level math to understand. Technology students do not typically have the math background that most texts require in explaining the topic. The major concept inventories of the course (as submitted here) do not require the math background. Engineering technology is seen has a "hands-on," applied engineering education. The concept inventories can be taught to this group of students with the mathematical detail typically shown. This paper introduces the concept inventories needed for a vibrations class and provides a guideline for teaching these concepts to applied engineers.

Introduction

Engineering technology is seen as a "hands-on" engineering education. Classes are very similar to those found in any engineering curriculum with emphasis placed on the application and not necessarily the theory. Because of this emphasis, the technology curriculum does not always require that students take the same level of mathematics needed to understand all of the theory. Therefore the technology classes will introduce the typical assumptions applied to the theory for most types of application. The majority of mechanics classes can be taught in this fashion and allow technology students to become very proficient in the application of mechanics. Vibrations is one of those upper level mechanics courses that is seen to be heavily dependent on upper level mathematics courses, specifically differential equations, understanding complex numbers, and matrix theory. Some textbooks even start the topic of vibrations by introducing students to distributed-parameter systems that requires an in-depth understanding of partial differential equations and how you solve them. This type of analysis is straight forward for strings, rods, and simple beams without the complex geometries or varying material properties that a technologist encounters in applications.

What are the concept inventories that a student needs to understand in vibrations? A lot of work has been done with the typical introductory mechanics course (statics) in defining concept inventories that all students need to master or obtain a certain level of proficiency. Steif and Dollar[1, 2] created a concept inventory for statics with learning modules that assist students in becoming proficient in the material. Gray *et al.*[3, 4] created a concept inventory for dynamics and Richardson *et al.*[5] created a concept inventory for strength of materials. These inventories for these higher level courses have not received the recognition as seen by the statics course. Here I propose that in vibrations, the major concept inventories that students need to understand are damping, natural frequencies, and mode shapes. All other covered topics are related in some form to these three fundamental concepts. Vibration textbooks cover these three concept inventories and tend to start teaching the topic in one of three locations: 1) the distributed-parameter system, 2) simple harmonic motion (review from physics), 3) single degree of freedom systems (sdof consisting of a mass and spring that uses Newton's laws or motions or energy methods to obtain the equations), and/or a combination of the three.[6-10] A lot of emphasis is placed on modeling the exact motion of the system by solving the differential equations of the "textbook" example. Once a vibrating system leaves these "textbook" examples, finite element analysis, experimental methods, or numerical methods are used to analyze the system in terms of vibrations. The programming language Matlab is used in multiple textbooks as a means of analyzing these vibrations problems.[7, 10]

Why do technology students need to understand vibrations? Anything that is moving or having a force applied at a random or periodic frequency may experience resonance. These application engineers need to understand the variables that may be causing the resonance, how damping affects the vibration, and where nodes and peak amplitudes are located in order to dissipate the energy. Understanding these topics falls within the suggested concept inventories proposed for vibrations. Do technology students need a deep understanding of how to solve differential equations, understand complex mathematics, or matrix theory? No, but they do need to understand the basics of some of those topics to communicate with engineers and other technicians and understand where specific variables in vibrations are obtained. This paper will discuss this question in more detail and the following: where do you start, do you introduce technology students to differential equations, how do you introduce damping, how do you introduce mode shapes, and how and when do you introduce measurements.

Implementation

The first question for teaching vibrations to technology students: **where do you start**? This question is valid for engineering students as well in trying to teach the concept inventories. Cognitive theory of learning requires that students make associations between new and old information.[11] The starting point should be with material with which they have an understanding. Of the three typical starting locations for vibration texts, distributed parameter systems is eliminated because technology students do not have the background with differential equations. Many engineering and technology students with a background in mechanics have taken a dynamics course where oscillators and mass/spring systems are introduced. Motion of the systems is measured as snapshots in time using Newton's laws of motion and energy principles as introduced in many texts[12, 13]. Therefore these two starting locations will be reviewed as potential starting locations for technology students.

Figure 1 shows an example of a harmonic system used in a dynamics course. The pendulum is used to understand energy principles and rotational acceleration. The first analysis is to calculate the velocity of the ball at any point along the range of motion with a known release point. This exercise relates kinetic and potential energy[12, 13] and the oscillation of the pendulum can be related to the natural frequency of the system, but is typically not made here. The second analysis of the pendulum in a dynamics course is to calculate the rotational acceleration,[12] which is also used in introductory vibrations texts to introduce vibrations[7]. The problem is that as a vibrations system, it is nonlinear and requires small angle assumptions before the math is introduced to calculate the natural frequency. This additional assumption may confuse students when learning the foundations of the topic. Also, this system does not introduce the topics of spring/material stiffness into the problem which is ultimately used with finite element analysis to understand vibration of larger systems. Therefore in the author's opinion, this does not seem appropriate as a starting point and has not been used with technology students.

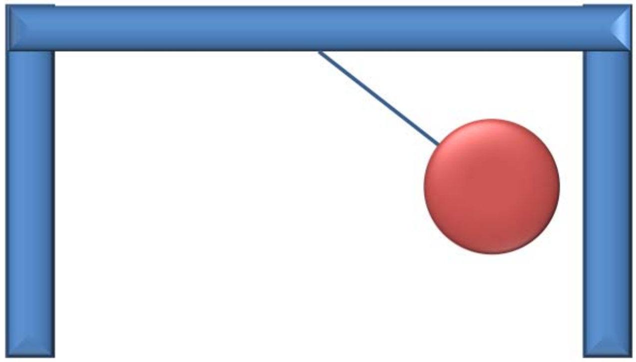

Figure 1. Pendulum system used in a vibrations course to introduce the concept of harmonic motion.

Figure 2 shows a spring with a mass and is used in dynamics to introduce Hooke's law, understand potential/kinetic energy principles, and understand Newton's laws[12, 13]. These problems deal with setting equilibrium locations for the spring and defining stretch or unstretched equilibrium positions once the mass is added using Hooke's law and Newton's laws. Potential/kinetic energy principles can be reviewed assuming initial conditions for the location and velocity of the mass. These problems assume the spring is linear, but only solves for the snap shots of time with the energy balance. Students can follow the energy through the system, but cannot account for the time of the vibration. This system does introduce the concept of stiffness due to Hooke's Law and therefore can be used to build larger systems and introduce the concept of finite element methods into the topic. Of the three typical starting locations of vibration texts, this location appears the most suited for technology and engineering students. Reviewing the dynamics of spring and mass systems should be a requirement for teaching vibrations and will ensure that the students understand energy principles and Newton's laws.

Figure 2. Spring/mass system used in vibrations course to introduce the concept of stiffness and mass and its relationship to the natural frequency.

The second question for teaching vibrations to technology students: **do you introduce the students to differential equations**? In the author's opinion, yes you do introduce them to differential equations. Figure 3 shows a mass spring system and a free body diagram (FBD) of the forces acting on that mass in the horizontal direction (assuming rollers without friction). Through the dynamics course, the students should be able to obtain the following equation using Netwon's Second Law and Hooke's Law

$$\sum F = -kx = ma \tag{1}$$

where F is the force, k is the stiffness of the spring, m is the mass, a is the acceleration, and x is displacement. Now, the students can be taught to take the next step and algebraically manipulate Equation1 in order to get the differential equation

$$m\ddot{x} + kx = 0 \tag{2}$$

This procedure builds on previously learned information[11] and in the author's experience, the students typically struggle the most with reassigning the acceleration as a second time derivative of the position with the symbology shown. Students can be introduced to the characteristic equation of the differential equation, shown how to find the natural frequency of the system, and the solution for this differential equation. The students now understand how time is involved in the oscillations of the system and how the system constants are associated with that time. This procedure also leads well into introducing damping forces and forcing functions in the system.

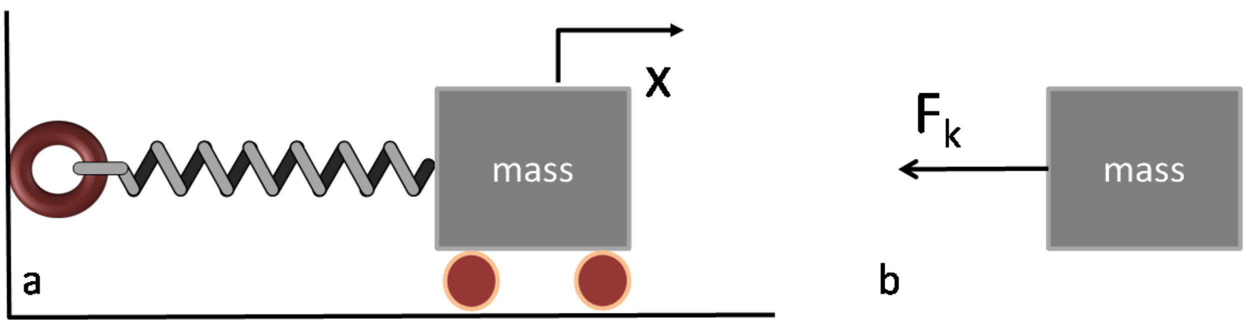

Figure 3. a) Spring and mass system on frictionless rollers and b) free body diagram of the horizontal forces acting on the mass.

Students can also be introduced to numerical methods for solving these systems. Multiple texts introduce Matlab[7, 10] as a tool to solve the position as a function of time for these problems. Matlab is a good tool that can be used to introduce students to numerically solving differential equations. There are multiple types of solvers inherent within the program. Therefore, the students do not need to spend a lot of time reviewing numerical methods and/or creating a numerical solver. Students and/or teachers that are interested in programming a separate solver can take that route. In the author's opinion, a detailed understanding of numerical methods is not necessary for applied engineers as long as the students understand how to check the simulated response back to the physical system constants. Therefore the student should understand the components of the equation to check the time response of the numerical simulation. The student should be able to answer what part is the stiffness, what part is the mass, and how do you get the natural frequency from that information? A student that can answer these questions will be able to see how changes to the system will change the natural frequency of the system and see the harmonic oscillation of the system. That information is learned by algebraically manipulating Equation 1 to Equation 2 and shows the importance of introducing students to the differential equation.

Long, soft springs and masses can be used as hands on examples to introduce this topic to students and show how the math relates to a real system. The spring should be sufficiently stiff as to negate the spring weight but should have a natural frequency of less than 1 Hz when combined with the mass. Initial displacements of the mass will cause oscillation that can be measured with a ruler and stop watch for long, soft springs. Figure 4 shows an example of one of these springs used with an attached mass. The natural frequency of the system can be measured with a stop watch. Over time, the damping in the spring will stop the vibratory action, but a spring with a small damping ratio gives the students sufficient opportunity to measure the displacement and associated natural frequency and compare the values to the theoretical values calculated from the mass and string stiffness. The students can see how changing the mass or the stiffness will change the natural frequency as well. Changing the mass on the bottom of the spring is straight forward and Figure 4 shows that with these long, stiff springs, the stiffness can be easily changed by tying part of the spring together.

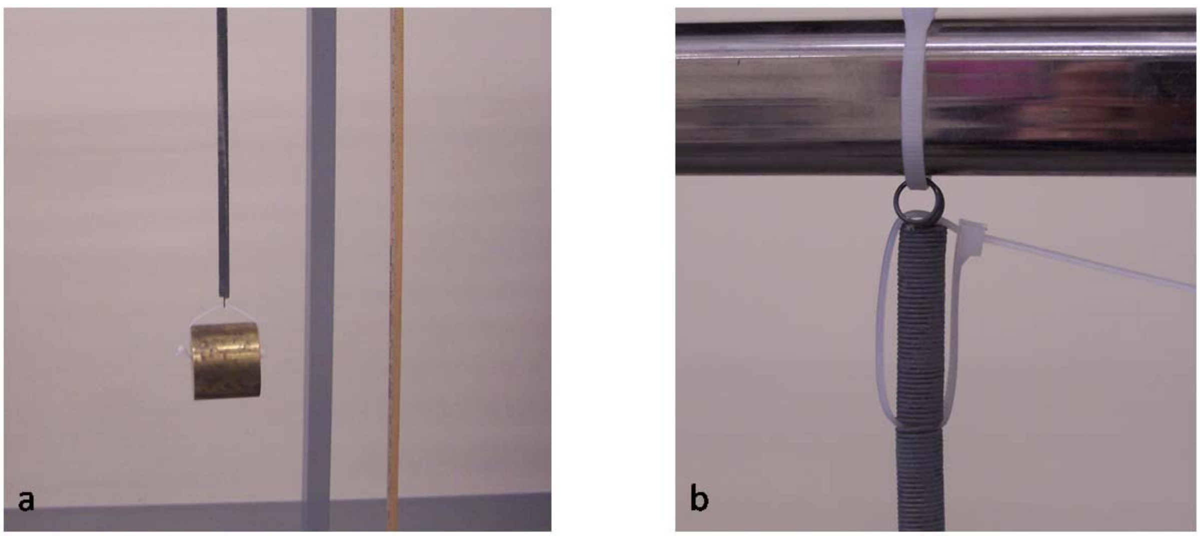

Figure 4. a) Spring and mass system used to introduce students to a single degree of freedom vibrating system and b) an example of a zip tie used to change the stiffness of a string.

The third question for teaching vibrations to technology students: **how do you introduce damping**? This topic is probably the most difficult to introduce to technology students mathematically without introducing them to the differential equation for second order systems, but is easily viewed in spring/mass systems as the vibration amplitude is reduced through each vibration cycle. Using Newton's Second Law and some algebraic manipulation, the differential equation for a system with damping is

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{3}$$

where c is the damping coefficient. Most vibrations text view damping in terms of underdamped, critically damped, and overdamped. Students can be taught with the characteristic equation of Equation 3, how to determine if a system is underdamped, critically damped, or overdamped from the system properties (mass, stiffness, and damping coefficient).

The system shown in Figure 4 can be used to introduce students to damping and can be used to measure the damping coefficients. This measurement procedure can be found in this reference.[7] Measurements can be made of the amplitudes and time of vibration and used with the logarithmic decrement to determine the damping ratio. In order to maintain linearity, the damping force should be modeled as a force proportional to the velocity. Most problems in dynamics do not have forces that are proportional to the velocity and is the part that will be new to the students. The numerical solvers can also be used to account for damping and used to simulate the time response of the system. This simulation will allow the students to approximate what they see happening with a spring/mass system over time.

The fourth question for teaching vibrations to technology students: **how do you introduce mode shapes**? The first thought would be to use distributed parameter systems because they have been analytically solved with known natural frequencies and mode shapes. The second thought would be to use multiple degree of freedom (mdof) systems made of multiple springs and masses. The author would argue that the students should be introduced to mdof spring/mass systems. If the students have learned about vibrations with a sdof system, the mdof system will look as an extension of what they have been doing and have learned. A two spring/two mass system will allow the student to see that systems have multiple natural frequencies and that the systems can vibrate in different ways. Figure 5 shows how the system from Figure 4 can be adapted with the springs and masses to have a mdof system. In application, most parts cannot be easily analytically modeled as distributed parameter systems such as strings, bars, or beams. So the progression of using mdof systems can be ultimately related to the finite element method (FEM). The progression can be introduced at this point in the course, but has not yet been implemented by the author.

Figure 5. A mass with a secondary spring that is used to introduce students to multiple degree of freedom systems.

The fifth question for teaching vibrations to technology students: **how and when do you introduce measurement**? Students should be introduced to basic measurements with sdof systems as previously mentioned in other sections, but most vibration measurements of systems consist of frequency response functions (frf). Most real systems have multiple natural frequencies and time data provides you with vibration amplitude, but frequency information may be difficult for introductory students to see in the data. At the same time, introductory students will look at a frf and not understand what the plot is telling you. Therefore in the author's opinion, these types of measurements should be held until after the students understand the concepts of natural frequency, damping, and mode shapes. The students should be introduced to the frf of a sdof system first where the natural frequency is the only peak shown in the frf. This step can be performed by numerically analyzing a sdof system with a forcing function and seeing how the peak amplitude changes as a function of the frequency. The students can also be introduced to the algorithms that perform transformation from the time domain to the frequency domain to obtain frfs of data. Do technology students need to know the details of the transformation? No because there are numerous types of algorithms that perform the transform from the time data in numerous different programs. The students only need to know about these transforms and any limitations that are associated with that specific transform. Once the sdof system is reviewed, the students should be introduced to the measurement of mdof and/or distributed parameter systems. This step allows the students to understand what the measurement is related to the natural frequencies and mode shapes of the part. The "text book" examples of strings, bars, or beams would be ideal for this step. Figure 6 shows an example of a beam in a clamp that is used for this type of measurement. Different materials can be used to reiterate the damping measurements associated with different types of material as well.

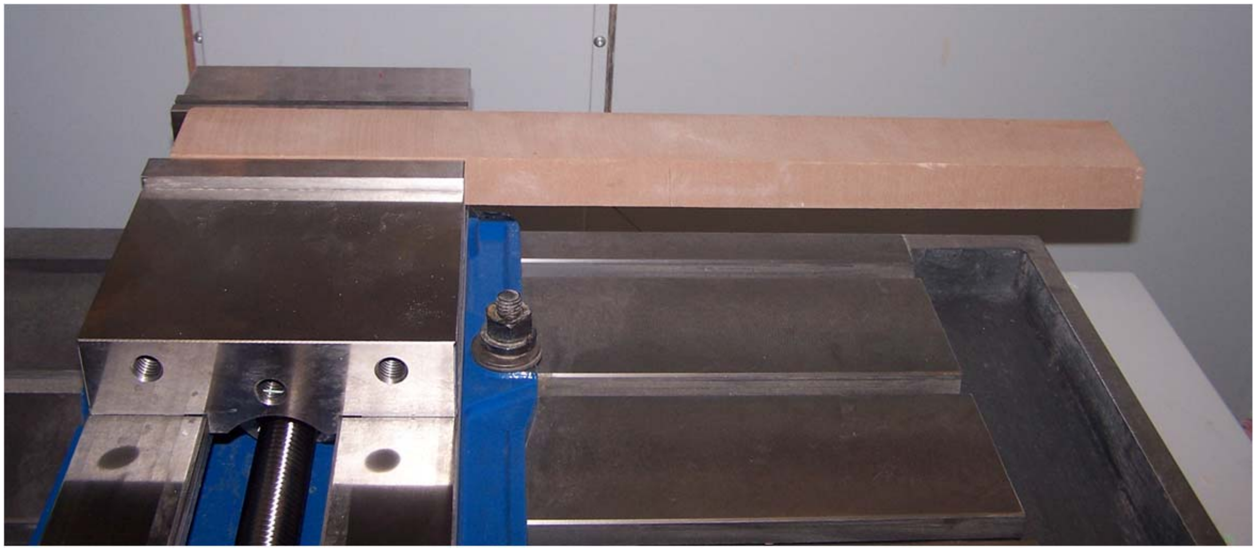

Figure 6. A plastic beam that is used to show students measurements of mode shapes for vibrating systems

Conclusion

Engineering technology is seen as a "hands-on" education and typically vibrations courses are taught with emphasis on theory. This paper has provided a framework for teaching vibrations to engineering technology students that builds on information they have already learned. This step is essential for technology students because they do not necessarily have the math background to understand what is happening with the physical system and need additional examples and time to grasp the concepts. Numerical simulations will provide sufficient results as long at the student understand the limitations and expectations that come from the problem. As long as the students understand the major concept inventories of the course, the student will be able to apply the applications in practice.

References

1. Dollar, A., and Steif, P. (2006) Learning Modules for Statics, *International Journal of Engineering Education 22*, 381-392.
2. Steif, P., and Dollar, A. (2005) Reinventing the Teaching of Statics, *International Journal of Engineering Education 21*, 723-729.
3. Gray, G., Evans, D., Cornwell, P., Costanzo, F., and Self, B. (2003) Toward a Nationwide Dynamics Concept Inventory Assessment Test, In *Proceedings of the 2003 American Society for Engineering Education Annual Conference & Exposition*, American Society for Engineering Education.
4. Gray, G., Costanzo, F., Evans, D., Cornwell, P., Self, B., and Lane, J. (2005) The Dynamics Concept Inventory Assessment Test: A Progress Report and Some Results, In *Proceedings of the 2005 American Society for Engineering Education Annual Conference & Exposition*, American Society for Engineering Education.
5. Richardson, J., Steif, P., Morgan, J., and Dantzler, J. (2003) Development of a Concept Inventory for Strength of Materials, In *33rd ASEE/IEEE Frontiers in Education Conference*, IEEE, Boulder, CO.
6. Bottega, W. (2006) *Engineering Vibrations*, CRC Press, Boca Raton, FL.

7. Inman, D. (2013) *Engineering Vibrations*, 4 ed., Pearson Education, Upper Saddle River, New Jersey.
8. Rao (2003) *Mechanical Vibrations*, Pearson Education.
9. Norton, M., and Karczub, D. (2003) *Fundamentals of Noise and Vibration Analysis for Engineers*, 2 ed., University Press, Cambridge, United Kingdom.
10. Palm, W. (2007) *Mechanical Vibration*, John Wiley.
11. Svinicki, M. (1999) New Directions in Learning and Motivation, *New Directions for Teaching and Learning 80*, 5-27.
12. Hibbeler, R. (2013) *Engineering Mechanics Dynamics*, 13 ed., Pearson, Upper Saddle River, New Jersey.
13. Meriam, J., and Kraige, L. (2012) *Engineering Mechanics Dynamics*, 7 ed., John Wiley.