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In-Class Real-Time Assessments of Students' Fundamental Vector and Calculus Skills in an Undergraduate Engineering Dynamics Course

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Abstract

Mathematics plays a significant role in solving engineering problems. This paper presents the results of in-class real-time assessments of students' fundamental skills of applying vectors and calculus to solve problems in engineering dynamics, a sophomore-year foundational undergraduate engineering course. In-class real-time assessments were conducted via a radio-frequency wireless technology called Classroom Response System (nicknamed clickers). The focus of this paper is not on the introduction to clicker technology because clicker technology has been well known and well documented. The focus of this paper is on detecting and assessing students' fundamental vector and calculus skills using clickers as a tool for collecting real-time data. This paper provides several examples to demonstrate how the instructor obtained immediate feedback from students in the classroom and then make just-in-time adjustments of the lecture to maximize student learning outcomes. Discussions are also made concerning the integration of engineering into mathematics classrooms.

Introduction

Engineering is a process of problem solving with the application of mathematics and science. The role of mathematics cannot be overemphasized in engineering, especially in cases where mathematical modeling is required for effective problem solving. Therefore, many engineering programs at institutions of higher learning share a common educational objective: to develop engineering students' fundamental skills in mathematics. In most recent 2019-2020 ABET (Accreditation Board for Engineering and Technology) criteria for accrediting engineering programs, Criterion 3 Student Outcomes explicitly states that programs under accreditation must demonstrate that students have "an ability to identify, formulate, and solve complex engineering problems by applying principles of engineering, science, and mathematics" [1].

The importance of mathematics is no exception in engineering dynamics, a sophomore-year foundational undergraduate engineering course. This course is often required in many undergraduate engineering programs, such as mechanical, aerospace, civil, environmental, mechanics, biological, and biomedical engineering programs. This course covers numerous learning topics in Newtonian mechanics, such as Newton's second law, the principle of work and energy, the principle of conservation of energy, the principle of linear/angular impulse and momentum, and the principle of conservation of linear/angular momentum [2] [3]. Students need to apply fundamental mathematical skills to solve problems in dynamics. Lacking fundamental mathematical skills has been a long-standing and significant challenge for students to succeed in learning dynamics [4] [5].

Two fundamental mathematical concepts are especially important in dynamics: vector and calculus. A solid understanding of these two concepts plays a significant role in developing student skills in analyzing and solving problems in dynamics. As we often use the term of "mathematical skills" to indicate student skills of applying mathematical concepts to problem solving, we use the term of "vector and calculus skills" in this paper to represent student skills of applying vector and calculus concepts to problem solving.

Numerous concepts involved in dynamics are vectors, for example, linear/angular velocity, linear/angular acceleration, force, moment, linear/angular impulse, and linear/angular momentum [6] [7] [8]. In their most recent study, Davishahl et al. [6] developed a pre-posttest assessment instrument to assess engineering students' understanding of vector representations in a mechanics course. They found "a positive correlation between students' accurate and effective use of [vector] representations and their score on the multiple-choice test." [6]

In cases where those concepts in dynamics are time-dependent variables, calculus is often involved during problem solving [9] [10] [11]. For instance, a time-dependent force (i.e., force is a variable rather than a constant) is applied to a block that is initially at rest on smooth ground. To determine the speed of the block after a certain specific time period, the work done by the force over the specific time period needs to be calculated first and then the principle of work and energy can be applied. Calculus is involved when calculating the work done by the force. Without fundamental skills in calculus, students will not be able to solve this problem even if they understand all concepts involved in this problem.

When teaching dynamics, it is important for the instructor to assess students' fundamental mathematical skills. The purpose of assessments is for the instructor to understand whether students have sufficient skills in mathematics for effective problem solving in dynamics. The instructor can then decide whether lectures should be adjusted or educational interventions should be developed or adopted to achieve maximal student learning outcomes.

This paper presents the results of in-class real-time assessments of students' fundamental vector and calculus skills in an undergraduate engineering dynamics course. The in-class real-time assessments were conducted via a radio-frequency wireless technology called Classroom Response System (nicknamed clickers). It must be noted that the focus of this paper is not on the introduction to clicker technology because clicker technology, as well as many other classroom technologies such as tablet PCs, have been well known for many years and well documented as well [12] [13] [14] [15]. The most significant scientific contribution that the present study makes is the development of an unconventional method of implementing clickers in the classroom. In this method, clicker technology is employed in class for *multiple times* to detect and assess student understanding of *the same learning topic until student misunderstanding is corrected.* To the best of our knowledge, this unconventional method has not been reported in existing clicker-related literature.

In the present study, clickers were used as a tool to collect real-time data, as computers are used as a tool to write papers and essays or pianos are used to play music. Through the use of clickers, the present study reveals that many engineering sophomore students still do not have sufficient fundamental vector and calculus skills even if they have taken Calculus courses before. The findings reported in this paper would help the engineering mechanics education community to think more about how to improve students' mathematical skills, so we can teach mechanics in mechanics courses, rather than teach or re-teach students mathematics in mechanics courses.

In the remaining sections of this paper, student participants are described first, followed by a brief description of how in-class real-time assessments were conducted via clickers. Then, the assessment results are presented, followed by discussions. Concluding remarks are made at the end of the paper.

Student participants

A total of 70 students who took an engineering dynamics course taught by the author of this paper in a recent semester participated in this study. The majority of students were from two departments in the College of Engineering at Utah State University: Department of Mechanical and Aerospace Engineering (MAE) and Department of Civil and Environmental Engineering (CEE). Prior to taking engineering dynamics, students had taken prerequisite courses including engineering statics, college physics, calculus I, and calculus II. These prerequisite courses had covered topics involving vectors and calculus.

A brief description of in-class real-time assessments via clickers

To assess students' fundamental skills in vector and calculus, in-class real-time assessments were conducted during lectures in engineering dynamics. A radio-frequency wireless technology called Classroom Response System (CRS) [12] or Audience Response System [13] was employed. The CRS consists of transmitters (nicknamed clickers) and a base. A clicker often has five buttons labeled as A, B, C, D, and E.

During a lecture, each student pushes a button (A, B, C, D, or E) on their clicker to respond to multiple-choice questions the instructor poses and displays on a projector screen in the classroom. The collective response from all students is immediately displayed on the projector screen. The students and the instructor can see the number or percentage of students who choose A, B, C, D, and E, respectively. The clickers provide immediate feedback and real-time assessments of student learning during the lecture. After the class, the instructor can also use the clicker-collected data to analyze in detail the performance of each individual student.

Results

Students' fundamental skills in vectors

After teaching the concept of normal acceleration, the instructor assessed whether students understood the scalar and vector forms of an equation to determine normal acceleration based on angular speed and radius in rigid-body rotational motion. The following paragraph shows the multiple-choice clicker question the instructor posed and displayed on a projector screen in the classroom.

The disk shown in Fig. 1 has radius r and rotates about its center with angular speed ω. Point A locates at the edge of the disk. The center of disk is also the origin of the coordinate system. The relationships between normal acceleration a_n of point A and angular speed ω of the disk are expressed as

Scalar form: $a_n = -\omega^2 r$ (1)
Scalar form: $a_n = \omega^2 r$ (2)
Vector form: $\boldsymbol{a_n} = \omega^2 \boldsymbol{r}$ (3)
Vector form: $\boldsymbol{a_n} = -\omega^2 \boldsymbol{r}$ (4)

where bold letters indicate vectors. Which of the following statement is true?

A) Equations (1) and (3) hold
B) Equations (1) and (4) hold
C) Equations (2) and (3) hold
D) Equations (2) and (4) hold

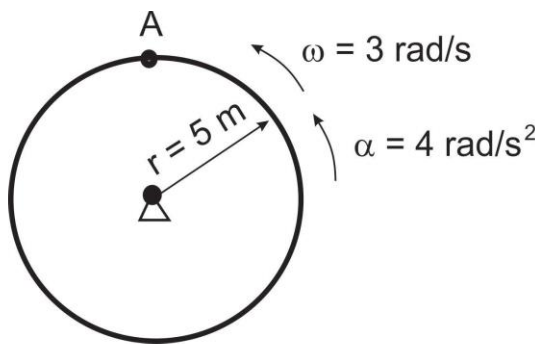

Figure 1. A disk rotates about its center

Students' initial responses are shown in Fig. 2. As can see from Fig. 2, only 57% of the students chose correct answer D. In other words, 43% of the students chose wrong answers.

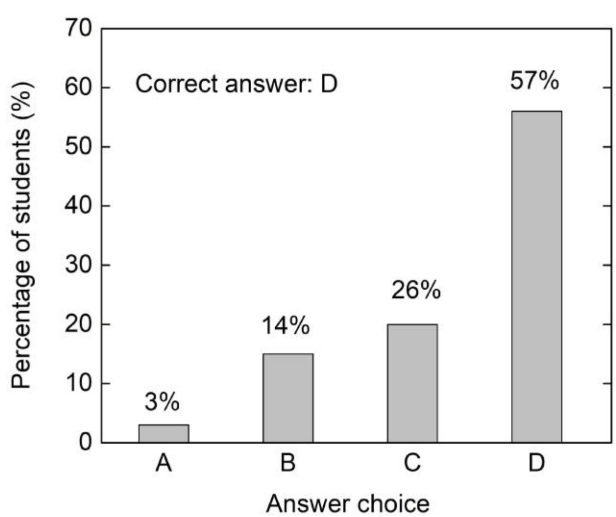

Figure 2. Students' initial responses

Therefore, the instructor provided further instruction on the difference between scalars and vectors, emphasizing the parallel but opposite directions of normal acceleration vector $\boldsymbol{a_n}$ and position vector ***r***. Moreover, students were asked to use numerical numbers (r = 5 m and ω = 3 rad/s) shown in Fig. 1 to calculate the values of $\boldsymbol{a_n}$ and ***r*** and then determine the relationship between $\boldsymbol{a_n}$ and ***r***.

After the above activities, the instructor posed the exactly same multiple-choice clicker question and asked students to respond to it again. Students' final responses were shown in Fig. 3. As seen, 97% of the students chose the correct answer. This result demonstrates the importance of immediate student feedback and just-in-time adjustments of the instructor's lecture.

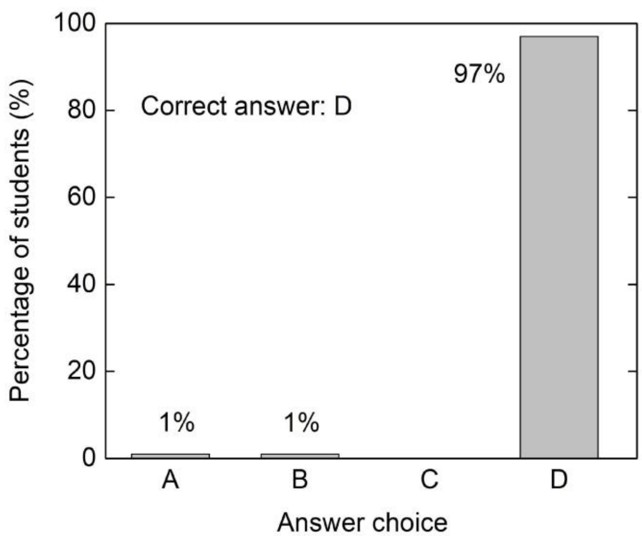

Figure 3. Students' final responses

Students' fundamental skills in calculus

Engineering dynamics is a calculus-based undergraduate course. When teaching rigid-body kinetics, students' fundamental skills in calculus were also assessed in class via clicker technology. All of the three examples presented in this section of the paper focus on students' skills in applying chain rule to solve problems in dynamics. By the chain rule, the derivative of f(g(x)) is f '(g(x))·g'(x).

During the lecture, the instructor presented the <u>*first*</u> problem shown in Fig. 4. Bar AB slides along ground. The displacement of point B is indicated by x. Because $x = h \tan\theta$, velocity of point B, V_B, can be calculated by taking the time derivative of x. We get $V_B = h \sec^2\theta \cdot \omega$, where ω is angular speed. The chain rule should be applied because angle θ is a function of time also.

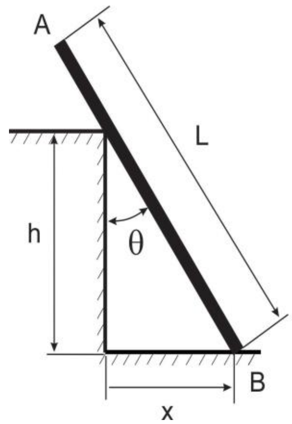

Figure 4. Problem #1 (reproduced from a figure in [2])

The instructor posed clicker question #1 as follows:

Is $V_B = h \sec^2\theta$ correct?

A) Yes, it is correct
B) No, it is wrong

Students' responses were shown in Fig. 5. As seen, 43% of students chose the wrong answer. The instructor then explained why A is the wrong answer and emphasized the application of the chain rule.

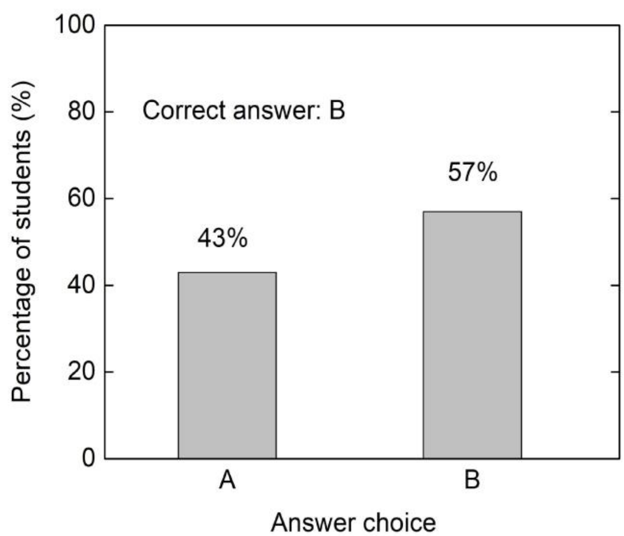

Figure 5. Students' responses to clicker question #1

The instructor presented the <u>*second*</u> problem shown in Fig. 6, where the displacement x at point B is given as a function of angle θ : $x = 2r \cos\theta$. Velocity at point B, V_B, can be calculated by taking the time derivative of x. We can get $V_B = -2r \sin\theta \cdot \omega$, where ω is angular speed. The chain rule should also be applied because angle θ is a function of time.

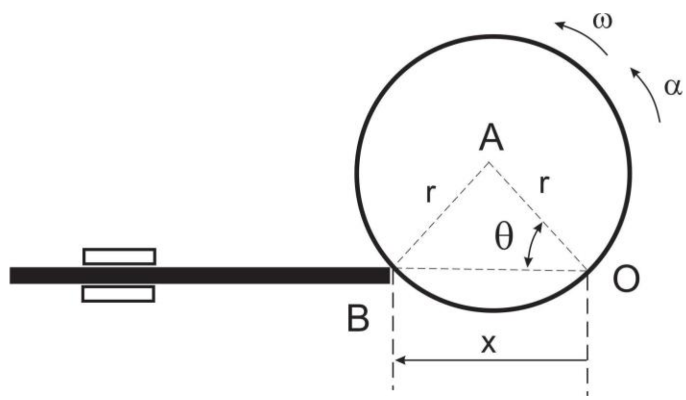

Figure 6. Problem #2 (reproduced from a figure in [2])

The instructor posed clicker question #2 as follows:

Is $V_B = -2r \sin\theta$ correct?

A) Yes, it is correct
B) No, it is wrong

Students' responses were shown in Fig. 7. As seen, the percentage of students who chose the wrong answer dropped from 43% to 27%. The instructor then explained why A is the wrong answer and emphasized the application of the chain rule again.

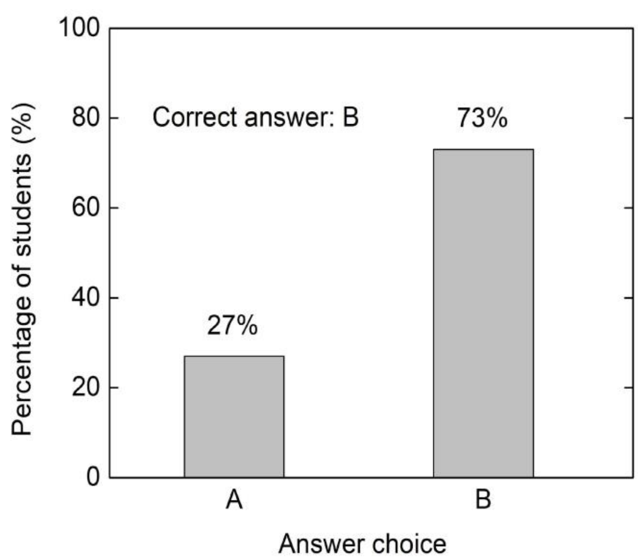

Figure 7. Students' responses to clicker question #2

Because there were still 27% of students choosing the wrong answer, the instructor presented the *third* problem shown in Fig. 8, where two slider blocks are connected by a rod of length 2 meters. The displacement of block A $S_A = 2\cos\theta$. Velocity of block A, V_A, can be calculated by taking the time derivative of S_A. We can get $V_A = -2\sin\theta \cdot \omega$, where ω is angular speed of bar AB. The chain rule should also be applied because angle θ is a function of time.

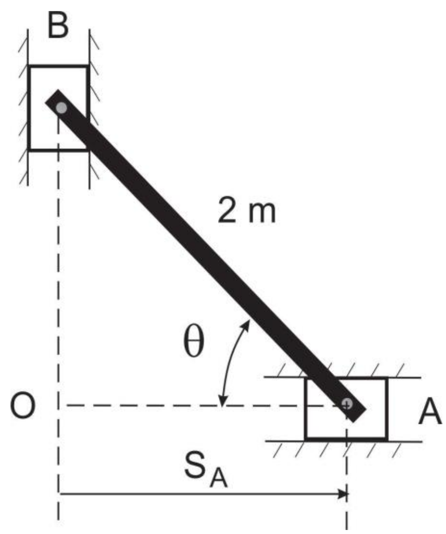

Figure 8. Problem #3 (reproduced from a figure in [2])

The instructor posed clicker question #3 as follows:

Is $V_A = -2\sin\theta$ correct?

A) Yes, it is correct
B) No, it is wrong

Students' responses were shown in Fig. 9. As seen, the percentage of students who chose the wrong answer dropped from 27% to 14%.

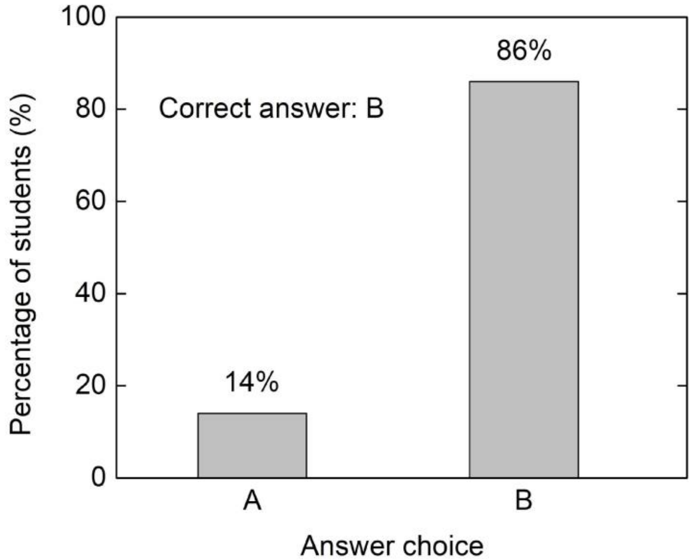

Figure 9. Students' responses to clicker question #3

An end-of-the-chapter exam involving problem solving with calculus was conducted. The exam performance of the 10 students (14%) who chose the wrong answer to clicker question #3 was compared to the class average. The results show that the exam scores of 7 students (out of 10) were below the class average. The average exam score of these 10 students was 12% lower than the class-average.

Discussions

It should be noted that the purpose of the present study is not to prove clicker technology works. The effectiveness of clickers has been well documented in the literature [12] [13] [14] [15]. Therefore, no efforts were made in the present study to involve two groups of students (with one group using clickers and another group not using clickers) and then compare student learning outcomes with and without clickers.

Instead, the present study develops an unconventional method of implementing clickers in the classroom. Is this method, clickers are employed in class for *multiple times* to detect and assess student understanding of *the same learning topic until student misunderstanding is corrected*. Although only 70 students were involved in the present study, the method described in this paper can be scaled up to larger classes because the signal receiver used in clicker technology can accept real-time clicker signals sent by hundreds of students in the class.

The results described in the previous section demonstrate that in-class real-time assessments via clicker technology helped the instructor diagnose students' skills in vectors and calculus. It turns out that some engineering students did not have sufficient vector and calculus skills needed for effective problem solving in engineering dynamics. In other words, even if those students had taken a series of prerequisite courses including calculus, they were neither vector-ready nor calculus-ready for taking engineering dynamics. As a consequence, the instructor in engineering dynamics had to spend a significant amount of engineering class time to do remedial mathematics lessons.

The author of this paper further interviewed engineering students about their perceptions of mathematics courses. Some indicated that when they took mathematics courses (e.g. calculus), they could not see real-world applications and felt that many concepts in mathematics are very abstract and irrelevant to students' real-world life experiences. Therefore, some students lacked extrinsic motivation to learn mathematics. As a result, their performance in mathematics did not meet expectations and also caused a chain reaction affecting their subsequent performance in engineering.

One potential approach to improving engineering students' mathematical skills is integrating engineering into mathematics classrooms via collaboration between mathematics and engineering instructors. Thus, students not only see real-world applications and usefulness of mathematics, but also see the relevance and meaningfulness of abstract concepts in mathematics [16] [17] [18]. Mechanics instructors can also use a small portion of class time to reinforce student understanding of some fundamental math concepts. In addition, mechanics instructors or their teaching assistants can hold recitation sessions after class to help students overcome their math challenges and motivate students learn engineering and math as well.

In the present study, the instructor used three engineering examples to re-teach chain rule, a fundamental concept in calculus I. The results were encouraging: The percentage of students who chose the wrong answer to three clicker questions dropped from initially 43% to 27%, and finally to 14%. This same teaching method used in the engineering classroom can also be adopted in the mathematics classroom. A detailed discussion of opportunities and challenges for integrating engineering into mathematics classrooms is out of the scope of this paper and will be dealt with in a future separate paper.

Concluding remarks

This paper has described in-class real-time assessments of students' fundamental skills of applying vectors and calculus to solve problems in engineering dynamics. This paper has also shown how the instructor made just-in-time adjustments to the lecture to maximize student learning outcomes.

The assessment data collected from the present study reveals that many engineering students were neither vector-ready nor calculus-ready prior to taking engineering dynamics. It is recommended that mathematics and engineering instructors collaborate to integrate engineering into mathematics classrooms.

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