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Instruction and Assessment of Mohr's Circle Concepts in Undergraduate Geotechnical Engineering Courses

James Kaklamanos (Associate Professor of Civil Engineering)

Dr. James Kaklamanos is Associate Professor of Civil Engineering at Merrimack College in North Andover, Massachusetts. Jim received his M.S. and Ph.D. in Civil and Environmental Engineering, and a B.S. in Civil Engineering, from Tufts University. He specializes in geotechnical engineering, and his published work has included research on earthquake site response analyses; ground-motion models; uncertainty in earthquake engineering; and engineering education, focusing on undergraduate geotechnical engineering courses. At Merrimack, Jim has taught courses in geotechnical engineering, foundation engineering, earth slopes and retaining structures, earthquake engineering, mechanics of materials, and engineering probability and statistics.

Simon Thomas Ghanat (Associate Professor)

Dr. Simon T. Ghanat, P.E. is an Associate Professor of Civil and Environmental Engineering at The Citadel. He received his Ph.D., M.S., and B.S. degrees in Civil and Environmental Engineering from Arizona State University. His research interests are in seismic site response studies, engineering characteristics of strong ground motions, probabilistic seismic hazard analyses and engineering education. He previously taught at Bucknell University and Arizona State University. He also teaches NHI training courses on seismic design of foundations and geotechnical features of highway systems for the Federal Highway Administration.

Craig M. Shillaber

Craig M. Shillaber is an associate teaching professor in the Department of Civil and Environmental Engineering at Northeastern University. He earned a Ph.D. in civil engineering from Virginia Tech in 2016, an M.S. in civil engineering from Virginia Tech in 2009, and a B.S. in civil engineering from the University of New Hampshire in 2008. His research interests include teaching and learning in geotechnical engineering, sustainability education in civil engineering, geotechnical subsurface characterization, developing and improving methods for assessing life-cycle embodied energy and carbon emissions for the geotechnical profession, and sustainable geotechnics. Dr. Shillaber is a member of the American Society of Civil Engineers (ASCE) and the Deep Foundations Institute (DFI), where he has served on the Sustainability Committee since 2013.

Tanya Kunberger (Professor & Chair)

Dr. Tanya Kunberger is a Professor and Chair of the Department of Environmental Engineering, Civil Engineering and Construction Management in the U.A. Whitaker College of Engineering at Florida Gulf Coast University. Dr. Kunberger's educational research interests are in self-efficacy, persistence, and effective learning approaches in engineering and the development of an interest in STEM topics in K-12 students.

Brock Barry

Dr. Brock E. Barry is the Director of Civil Engineering and Professor of Engineering Education in the Department of Civil & Mechanical Engineering at the United States Military Academy, West Point where he has been part of the faculty since 2009. Dr. Barry holds a Bachelor of Science degree from Rochester Institute of Technology, a Master of Science degree from University of Colorado at Boulder, and a PhD in Engineering Education from Purdue University. Prior to pursuing a career in academics, Dr. Barry spent 10 years as a senior geotechnical engineer and project manager on projects

throughout the United States. He is a licensed professional engineer. Dr. Barry's passion is teaching the Army's future engineers. He was recognized for his remarkable teaching with the American Society for Engineering Education 2020 National Outstanding Teaching Award.

Shawn Griffiths

Shawn Griffiths is an Associate Instructional Professor at the University of Wyoming. His interests include helping students fulfill their dreams to become Engineers and he strives to inspire students to become lifelong learners.

Corrie Walton-macaulay (Assistant Professor)

Suresh Immanuel

Suresh Immanuel is a professor of civil engineering, a licensed professional engineer, and the associate dean for the school of engineering and computer science at the University of Evansville. He holds a PhD degree from Auburn University. Along with his engineering degrees, he has a master of science degree in leadership with a focus in higher education. His research interests are in engineering education, pavement design and analysis, pavement management, and pavement instrumentation. At the University of Evansville, he has been teaching a variety of courses such as transportation engineering, soil mechanics, geotechnical engineering, advanced pavement design and management, construction management and surveying.

David A Saftner (Associate Professor)

Dr. David Saftner is an Associate Professor in the Department of Civil Engineering. He earned a BS from the United States Military Academy and an MS and PhD from the University of Michigan. Prior to pursuing a career in academics, Dr. Saftner spent five years as an engineer officer in the US Army and serving in Missouri, Colorado, Kuwait, and Iraq. His areas of research include beneficial reuse of waste soil material, geotechnical site investigation and characterization, and teaching and learning in engineering education. He currently serves as his Department's Head, an American Society of Civil Engineers (ASCE) Excellence in Civil Engineering Education (ExCEEd) Mentor, and the American Society of Engineering Education (ASEE) Civil Engineering Division Freshman Director.

Chris Swan (Dean of Undergraduate Education and Associate Professor)

Chris Swan is Dean of Undergraduate Education in the School of Engineering and an associate professor in the Civil and Environmental Engineering (CEE) department. He also serves as the School of Engineering’s representative to the Tufts University Cabinet on Diversity, Equity and Inclusion (TUCDEI). This latter appointment connects to his own education research on diversifying the audience for STEM education through service-learning efforts. Additionally, he has a faculty appointment in the Tisch College of Civic Life and serves on its diversity and inclusion committee as the faculty representative. Previously, he has served as the Associate Dean of Tisch College (2016), Associate Dean of Undergraduate Curriculum Development in the School of Engineering (2012-2015) and as CEE department chair (2002-2007). He received a Doctor of Science (ScD) degree in Civil and Environmental Engineering from MIT in 1994 and both Bachelor (BS) and Master (MS) of Science degrees in Civil Engineering from the University of Texas at Austin in 1984 and 1986, respectively.

Instruction and Assessment of Mohr's Circle Concepts in Undergraduate Geotechnical Engineering Courses

Introduction

Mohr's circle, the graphical representation of the plane-stress transformation equations, is a critical engineering concept for quantifying normal and shear stresses on various planes and for determining the strength of materials. Within undergraduate civil engineering curricula at most institutions, Mohr's circle is introduced in the sophomore-level Mechanics of Materials (or Strength of Materials) course. In the subsequent junior- and senior-level civil engineering curriculum, Mohr's circle arguably receives the greatest emphasis in geotechnical engineering courses. Recent studies have shown that students struggle to retain fundamental Mohr's circle concepts between the Mechanics of Materials course and upper-level geotechnical engineering courses [1, 2]. Due to the importance of Mohr's circle in quantifying subsurface stresses and soil shear strength, an analysis of the effectiveness of various instructional styles on this topic could prove beneficial for increasing student learning. Nonetheless, the current pedagogical literature on Mohr's circle instruction tends to be focused on mechanics courses rather than upper-level civil engineering courses [3–6]. With this literature gap in mind, the purpose of this paper is to describe best practices for teaching and evaluating Mohr's circle concepts in undergraduate geotechnical engineering courses.

Instructional strategies for Mohr's circle as implemented by geotechnical engineering instructors from ten different U.S. institutions are described in this paper, along with methods of evaluating student knowledge in each instructor's course. This paper delineates ready-to-implement techniques that may be useful in geotechnical engineering courses, as well as in Mechanics of Materials and other engineering courses. Student learning of Mohr's circle concepts is assessed at different institutions, highlighting techniques that may lead to retention of concepts.

In this study, students' understanding of Mohr's circle was assessed via a three-question concept inventory at a subset of five of these institutions. This concept inventory, administered on or about the first day and last day of the semester in the introductory geotechnical engineering course, allows for the quantification of students' knowledge gained in Mohr's circle throughout the semester, and an evaluation of the pedagogical techniques employed. The same concept inventory was also employed at two institutions in a subsequent geotechnical engineering course (Foundation Engineering), allowing for an analysis of the retention of Mohr's circle concepts after completion of the introductory geotechnical engineering course. The instruction and assessment strategies in this paper will contribute to a better understanding of the effectiveness of instructional methods on students' understanding and retention of Mohr's circle in geotechnical engineering courses.

Mohr’s Circle in Geotechnical Engineering Education

Mohr’s circle is a graphical depiction of the two-dimensional state of stress at a point [7–9]. This tool is introduced to engineering students in order to make stress-transformation concepts easier to understand and apply. In soil mechanics, Mohr’s circle is used to visualize relationships between normal and shear stresses, and to estimate the maximum stresses at points within the subsurface. If the normal and shear stresses are known on any two orthogonal planes, the construction of Mohr’s circle enables the stresses acting on different planes through a point to be determined. Since many practical situations can be approximated as plane-strain problems, Mohr’s circle receives significant usage in civil engineering. Perhaps the most important application of Mohr’s circle is the determination of strength and failure behavior. In geotechnical engineering, Mohr’s circles are often constructed from the results of triaxial tests, which provide data for determining strength properties and stress-strain relations for soils. The resulting Mohr’s circle and shear strength parameters are critical for solving many geotechnical engineering problems.

Generally, civil engineering undergraduates are first introduced to Mohr’s circle concepts in a Mechanics of Materials (or Strength of Materials) course, which is usually a prerequisite to any courses in geotechnical engineering. The first geotechnical engineering course (i.e., Soil Mechanics or Geotechnical Engineering) usually includes a module covering Mohr’s circle; for most students, this will be the second time they are presented with the concept. The importance of understanding Mohr’s circle in soils is paramount to learning more advanced topics in future geotechnical classes and is a key component of understanding shear failure and shear plane orientation in soils. Mohr’s circle has been identified as a “threshold” concept [3], a concept best learned from multiple methods or thought processes. Threshold concepts are not unique to engineering and are often difficult for students to understand [10].

The concept of Mohr’s circle can be challenging for students to master, as evidenced by the number of different studies conducted concerning the education and teaching of Mohr’s circle [3–6, 11]. Studies have shown that Mohr’s circle is among the most difficult topics for students to comprehend and apply [4, 5, 12]. Much of the available literature concerning Mohr’s circle provides useful insights and can inform educators of common pitfalls and better approaches for Mohr’s circle instruction. However, most of the literature is focused on either mechanics or structural engineering courses, with little or no references concerning the education of Mohr’s circle in geotechnical engineering courses. While the concept of Mohr’s circle is not fundamentally different in geotechnical applications when compared to structural or mechanics applications, it still presents challenges for many learners throughout their undergraduate education. These challenges include the differences in sign conventions for geotechnical applications, the continued complexity of the topic, and that the students have not mastered the concept from the previous course(s).

A phenomenographic analysis of 25 students revealed qualitative conceptions about Mohr’s circle that affect understanding [13]. Student conceptions were grouped into four categories: topic, procedure, tool and visualization. This study yielded student perceptions that can be used to re-design or guide instruction in various categories. It was also found that approaching the instruction using multiple perspectives resulted in the enhancement of student learning. For example, by only focusing on the procedure of how to compute principal stresses, students may not understand that it can be used to visualize stresses or as a tool to determine stresses for other orientations. Using multiple perspectives identified from the phenomenographic analyses is similar to diversifying learning activities and/or bringing real world experiences into the classroom, both of which have been shown to increase student understanding of threshold concepts [3, 14, 15].

One unique study encouraged students in a mechanics course to compute stresses using data from strain gauges affixed to a stadium column [3]. This exercise required students to convert strains to stresses, and then apply Mohr's circle to determine if the maximum stresses exceeded allowable values. This comprehensive approach likely deepened student understanding [3] and allowed students to struggle with the difficult concepts of stress and strain, giving them greater appreciation for these topics. This assignment also required students to program a solution for Mohr's circle using specific software. This activity is much more involved than the more common "configuring approach," which gives students a software program, requires them to change stresses and orientations from pre-determined inputs, and allows them to visualize Mohr's circle [3]. This kind of "configuring approach" type of software is useful for student visualization, but at least one study confirmed that they offer little benefit to develop necessary vocabulary and induce deeper understanding [10]. These studies, however, rarely focus on geotechnical engineering courses in particular.

Instructional Strategies for Mohr's Circle in Geotechnical Engineering

With the goal of identifying common practices for Mohr's circle instruction and assessment in undergraduate geotechnical engineering courses, strategies at ten educational institutions are profiled in this section. The ten institutions participating in this study have a range of different characteristics, from enrollment to curriculum structure, to the amount of learning time dedicated to Mohr's circle, as detailed in Table 1. The overall undergraduate enrollment at the institutions ranges from approximately 1,300 to 15,000, and enrollment in the undergraduate civil engineering programs at these institutions ranges from 40 to 335 students.

At all but one institution, the first geotechnical course is offered during the third (junior) year of study, and in-class instructional time during class dedicated exclusively to Mohr's circle in this course ranges from 0.75 to 2.5 hours. The vast majority of instructors teach Mohr's circle at the beginning of the unit on shear strength, typically in the latter part of semester. At two institutions (Merrimack College and the University of Wyoming), however, Mohr's circle is taught earlier in the semester as part of the module on subsurface stresses; at a third institution (the University of Evansville), Mohr's circle is briefly introduced in the stress module, and is later expanded upon in the shear strength module. Two institutions (The Citadel and the University of Evansville) require a second course in geotechnical engineering, while the others offer more advanced geotechnical courses as electives (which many students may take in order to satisfy degree requirements for upper-level design electives). At all institutions, more advanced geotechnical courses only briefly address Mohr's circle, with 0 to 1 hours of learning time, generally in the form of a review at the beginning of the semester (Table 1).

Table 1. Summary of characteristics of participating institutions.

Institution	Total undergrad. enrollment	Civil Engin. undergrad. enrollment	First geotechnical course			Second geotechnical course	
			Semester typically taken	Mohr's circle instructional time (class hours)	Module in semester when Mohr's circle taught	Required?	Mohr's circle instructional time (class hours)
The Citadel	2,858	161	Senior Fall	0.75	Shear strength	Yes	0.25
Florida Gulf Coast Univ.	13,871	310	Junior Spring	2.5	Shear strength	No	0
Merrimack College	4,202	100	Junior Fall	2.0	Stress	No	0.5
Northeastern University	15,156	335	Middler / Junior Spring or Sumr.	1.25	Shear strength	No	0.5
Saint Martin's University	1,394	93	Junior Fall	1.5	Shear strength	No	0
Tufts University	6,114	67	Junior Spring	1.5	Shear strength	No	0
U.S. Military Academy	4,536	125	Junior Spring	1.5	Shear strength	No	1
University of Evansville	2,041	40	Junior Spring	1.0	Stress, Shear strength (split)	Yes	0.5
Univ. of Minnesota Duluth	9,301	280	Jr. Fall or Spr.	1.5	Shear strength	No	0
University of Wyoming	9,342	215	Junior Spring	2.0	Stress	No	0

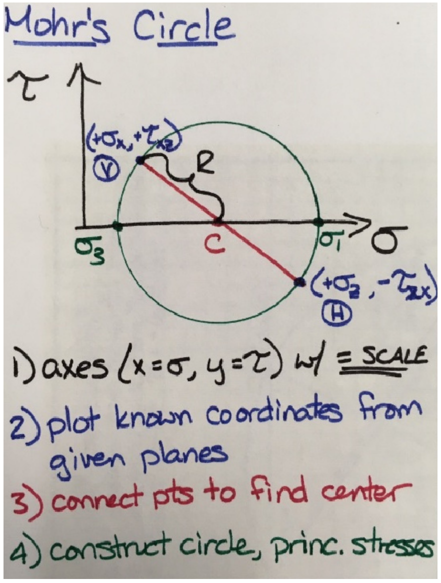
Several strategies for Mohr's circle instruction are employed in geotechnical courses at the institutions involved in this study, including both in-class and outside-of-class learning activities. As illustrated in Table 2, for in-class activities, all instructors make use of lecture methods in tandem with other learning activities. Many instructors augment traditional lecture with handouts or skeleton notes (i.e., a handout with notes that is partially completed, with blank spaces filled in by students as they take notes during class). Instructors also make use of worked example problems, physical demonstrations, and collaborative in-class problem solving. The collaborative problem solving is sometimes used as a formative assessment to adapt following classroom content; however, one instructor takes a full formative approach and adapts the lecture based on a pre-class exercise and in-class activities.

Some instructors noted that they developed their classes using the American Society of Civil Engineers (ASCE) Excellence in Civil Engineering Education (ExCEEd) teaching model [16], incorporating learning objectives, structured organization, connection with prior knowledge, active learning techniques, and board notes. One instructor's board notes for the initial construction of Mohr's circle are provided in Figure 1. Consistent with the ExCEEd teaching model, learning objectives are presented to the students at the start of each class, with the goal of facilitating the structured organization of the course. Some example learning objectives for Mohr's circle used by the instructors in this study include:

- **Explain** the purpose of stress-transformation equations and Mohr's circle.
- **Describe** the sign conventions for normal and shear stresses for plotting Mohr's circle.
- **Construct** Mohr's circle for a given state of stress.
- **Locate** the origin of planes (pole) for a given state of stress.
- Given a Mohr's circle, **determine** the principal stresses and the maximum in-plane shear stress; and show these stresses in an appropriate sketch.
- Given a Mohr's circle, **determine** the normal and shear stresses at a transformation angle; and show these stresses in an appropriate sketch.

Table 2. Summary of instruction methods for Mohr's circle in geotechnical engineering courses at participating institutions.

Institution			The Citadel	Florida Gulf Coast Univ.	Merri-mack College	North-eastern Univ.	Saint Martin's Univ.	Tufts Univ.	U.S. Military Academy	Univ. of Evans-ville	Univ. of Minn. Duluth	Univ. of Wyo-ming
Instruction / Learning Activities	**In-Class**	**Lecture (board notes and/or PowerPoint)**	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
		Handouts / study guides / skeleton notes	Yes	Yes	Yes	Yes	Yes	Yes			Yes	Yes
		Physical demonstrations	Yes		Yes						Yes	Yes
		Example problems	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
		Collaborative in-class problem-solving	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	
		Instructional time on Mohr's circle (class hours)	0.75	2.5	2.0	1.25	1.5	1.5	1.5	1.0	1.5	2.0
	Outside Class	**Pre-class reading**	Yes						Yes		Yes	
		Pre-class quiz / exercise	Yes						Yes (review HW)			
		Homework	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Assessment		**Formative**	Yes (1 min. paper)			Yes (prob. Solving)						
		Homework	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
		Exams / quizzes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes



1) axes $(x=\sigma, y=\tau)$ w/ = SCALE

2) plot known coordinates from given planes

3) connect pts to find center

4) construct circle, princ. stresses

Figure 1. Example board notes for the initial construction of Mohr's circle from Florida Gulf Coast University.

At most institutions in this study, the instructors attempt to link the study of Mohr's circle in geotechnical engineering to the students' prior experience in mechanics of materials. In fact, multiple geotechnical engineering instructors also teach the prerequisite Mechanics of Materials course, at least on a rotating basis. When reviewing stress and strain, sign conventions, and states of stress, they can highlight (to the same students they previously taught) the similarities and differences between what students learned in Mechanics of Materials and how the concepts are applied within Geotechnical Engineering (in particular, differences in the sign convention, with compression being positive instead of negative). Multiple instructors require students to bring a geometry set to class (e.g. graph paper, straightedges, compasses, and protractors) to facilitate the accurate construction of Mohr's circle. In their geotechnical engineering courses, some instructors focus primarily on graphical methods of constructing and analyzing Mohr's circle (including the origin of planes, or pole), while others also incorporate the stress transformation equations to solve problems in an analytical or numerical manner.

A few instructors employ pre-class exercises that build on students' prior experiences with Mohr's circle before discussing it in the geotechnical engineering course. One instructor from The Citadel uses web-based pre-class reading responses to motivate students to prepare for class. Prior to the lesson, students are required to respond to two qualitative questions, such as "What is Mohr's circle?" and "What is the significance of Mohr's circle?" Several hours prior to the lesson, the instructor examines students' responses, and develops in-class activities to meet their needs. As students enter the classroom, this instructor plays a song with the word 'circle' in its lyrics, such as "Circle in the Sand" by Belinda Carlisle; "Circle of Life" by Elton John; or "Draw Me a Circle" by Barbara Streisand to stimulate learning and build students' enthusiasm about Mohr's circle. As a refresher of concepts learned (and ideally retained) in the Mechanics of Materials course, another instructor at the U.S. Military Academy assigns a review homework set related to Mohr's circle. No in-class review is provided prior to issuing the homework. The homework presents a stress element with both normal and shear stresses, and students are required to analyze the states of stress using both the stress-transformation equations and Mohr's circle. This pre-class assignment then leads into a class about Mohr's circle in geotechnical engineering, including the introduction to the origin of planes (pole) method, which is largely unique to geotechnical engineering.

A variety of in-class example problems and collaborative activities have been employed by the instructors in this study. Students often struggle with the linkage between the analytical stress-transformation equations and Mohr's circle. After deriving the stress-transformation equations (in an abbreviated manner, drawing upon students' prior experience in Mechanics of Materials), one instructor from Merrimack College attempts to bridge this gap using an in-class exercise that illustrates how Mohr's circle arises from the stress-transformation equations. Students are provided with the following state of stress: normal stress in the horizontal direction: $\sigma_x = 72$ kPa, normal stress in the vertical direction: $\sigma_z = 115$ kPa, and shear stress on horizontal and vertical planes: $\tau_{xz} = 80$ kPa. Pairs of students are then assigned different angles of orientation (θ) ranging from 0° to 180° (in various increments, depending on the size of the class), and they work in pairs to calculate the normal and shear stresses on their assigned plane using the stress-transformation equations. Upon completion, students bring their answers to the front of the room, where the instructor inputs their computed values into a spreadsheet with a live graph. Gradually, as groups complete their calculations and points are added to the graph, students see that the relationship between normal and shear stresses at different orientations begins to form a circle (Figure 2). Throughout the remainder of the class session, when reviewing principal stresses and maximum in-plane shear stresses, this numerical example is continually referred to.

All instructors incorporate one or more example problems involving Mohr's circle into their classes, but one instructor at the University of Minnesota Duluth has a particularly unique framework for developing the in-class example problems. Two example problems are performed in class: the first problem is defined by the instructor, and the second problem is defined by the students in the course. Student athletes' numbers are used as the given quantities in the problem: the normal and shear stresses, and the inclination angle of the stress block. This activity not only keeps students engaged, but helps build positive rapport and connections with students, consistent with the ExCEEd model of effective teaching [16].

Physical demonstrations are employed by a few of the instructors profiled in this study. Two instructors (The Citadel and Merrimack College) use samples of failed soil specimens to illustrate the importance of failure in geotechnical engineering, and how Mohr's circle can be used to identify failure conditions. The instructor from The Citadel takes this demonstration a bit further, and actually conducts an unconfined compression test on a sample of clay and a triaxial consolidated drained shear test on a sample of sand (Figure 3). In groups of three, students are required to use the data from the conducted tests to compute (1) the undrained shear strength of the clay, and (2) the friction angle and shear stress at failure. Students are then asked to draw a Mohr's circle, along with the Mohr-Coulomb failure envelope for sand.

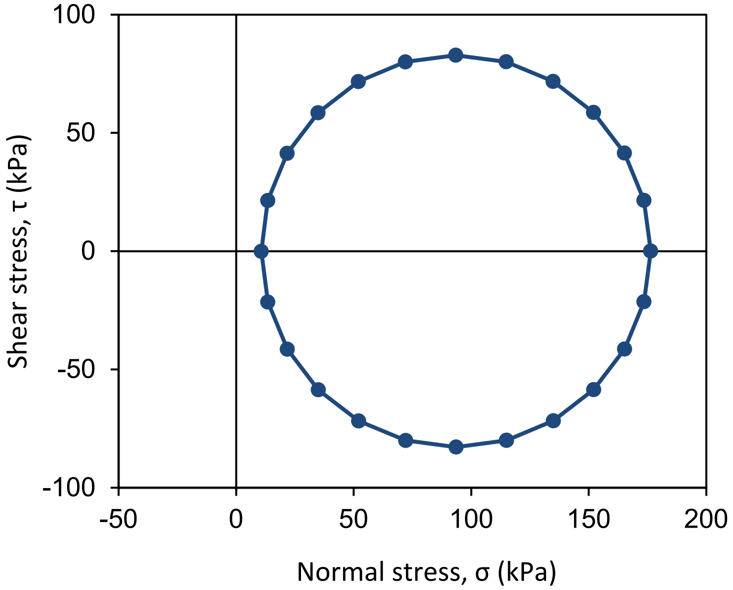


Figure 2. Interactive in-class activity at Merrimack College used to establish the connection between the stress-transformation equations and Mohr's circle, for the following state of stress: $\sigma_x = 72$ kPa, $\sigma_z = 115$ kPa, and $\tau_{xz} = 80$ kPa.

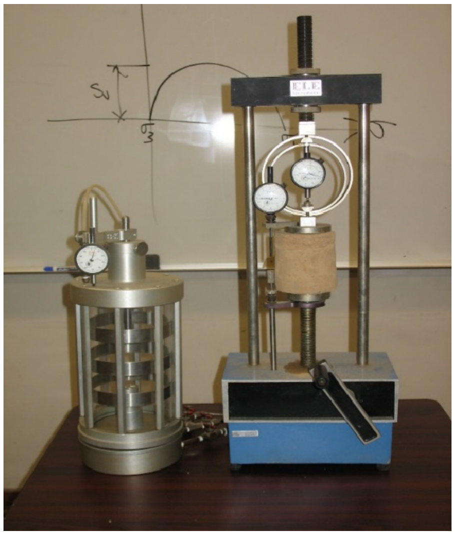


Figure 3. In-class demonstration of triaxial test on sand (left) and unconfined compression test on clay (right) at The Citadel.

Another instructor at the University of Wyoming performs a unique skit to help students visualize different planes through an element. The professor's upper body (shoulders to waist) is considered to be an element of soil. Two student volunteers are brought to the front of the room; standing a distance from the professor, one student points their arm to the professor's midsection, and the other student points to the professor's head. The students are instructed to keep their arms perpendicular to the professor, as he rotates his body left and right. The professor then discusses how different stresses will arise on different planes through the soil element (the professor's midsection), but that the soil element stays the same, no matter which direction in which it is rotated. That is, the soil element is the professor, and although the stresses change on different planes, the soil element remains the professor.

As noted in Table 2, outside of class, the majority of instructors make use of traditional homework assignments given after the material is covered in-class, in order to provide students with additional practice with Mohr's circle. However, three instructors require students to complete a pre-class reading and/or a web assignment, one of whom offers a review homework assignment based on content from a prior Mechanics of Materials course before addressing Mohr's circle in the Geotechnical Engineering course. Most instructors assess student learning with regards to Mohr's circle in a primarily summative manner, via a graded homework assignment and examination problems given after the content is covered in class. One instructor employs a formative approach by evaluating student responses on a pre-class assignment, during class, and an end-of-class one-minute paper. Some instructors incorporate additional assessments of Mohr's circle concepts in tandem with shear strength laboratory experiments later in the course.

Assessment of Mohr’s Circle Concepts in a First Geotechnical Engineering Course

At a subset of five institutions (The Citadel, Merrimack College, Northeastern University, Saint Martin’s University, and the University of Minnesota Duluth), a three-question concept inventory was administered to assess students’ understanding of Mohr’s circle concepts. This concept inventory, administered on or about the first and last class of the semester in the introductory geotechnical engineering course, allows for the quantification of students’ knowledge on Mohr’s circle prior to starting the semester (e.g. after the Mechanics of Materials course), as well as the knowledge gained throughout the semester. The list of questions and solutions for the concept inventory is provided in Table 3. The concept inventory assesses students’ learning at various levels of Bloom’s Taxonomy, with Question 2 focusing on more fundamental concepts, and Questions 1 and 3 requiring students to apply calculations. The questions on Mohr’s circle were deliberately intended to be brief, because they were a component of a broader concept inventory on geotechnical engineering with nine other questions [1, 2, 17].

A total of 349 students completed the concept inventory: 115 from The Citadel, 65 from Merrimack College, 132 from Northeastern University, 15 from Saint Martin’s University, and 22 from the University of Minnesota Duluth. Data were collected from the Fall 2019 semester through the Fall 2021 semester. Each question was graded on a scale of 0 to 1, with 0 points (no credit) awarded for an incorrect, off-base answer, or no answer at all; 0.5 points (partial credit) awarded for an answer with some issues (but for which students’ work illustrates some conceptual understanding); and 1 point (full credit) awarded for a fully correct answer. Partial credit was not awarded on Question 2. For uniformity across the institutions, no instructors factored the concept inventories into students’ course grades. To ensure that students were not ‘guided’ to the correct answers to these particular questions throughout the term, the instructors did not use the exact numerical examples in Questions 1 and 3 throughout the course, although students were exposed to similar examples and problems. Question 2, on the other hand, represents a fundamental concept about the principal planes that was emphasized in class, reading, and homework.

Table 3. Mohr’s circle concept inventory and solutions.

No.	Question
Q1	The center and radius of a Mohr’s circle have been computed to be C = 800 psf and R = 500 psf, respectively. What is the smallest normal stress that will be developed on any plane? Solution: Minimum normal stress = $\sigma_3 = C - R = 800 - 500 =$ **300 psf**.
Q2	For a given state of stress, what level of shear stress acts on the principal planes? Solution: **Zero shear stress** acts on the principal planes.
Q3	The major and minor principal stresses at a certain point in the ground are 450 and 200 kPa, respectively. Determine the maximum shear stress at this point. Solution: Maximum shear stress = $(\sigma_1 - \sigma_3)/2 = (450 - 200) / 2 =$ **125 kPa**

Figure 4 depicts students' performance on each question on the pre-test (at the start of the semester) and post-test (at the end of the semester) for offerings of the first geotechnical engineering course between Fall 2019 and Fall 2021 at participating institutions. The student performance (< 30%) on all three questions of the pre-test is considered poor performance, indicating minimal retention of Mohr's circle concepts from the Mechanics of Materials course. The strongest scores on the pre-test were for Question 1 (27.6%), and the weakest scores on the pre-test were for Questions 2 and 3 (just 9–10% for each question). However, students performed poorly on both the pre-test and post-test on Question 2 (over 50% of the students missed this question on both the pre-test and the post-test), suggesting that students are failing to comprehend that zero shear stress acts on the principal planes. Alternatively, the low scores on Question 2 could perhaps indicate confusion with the wording of Question 2, and/or reflect the lack of partial credit available on this question. Although the post-test scores on Question 2 were lower than anticipated, students' scores did increase on all three questions from the pre-test to the post-test; the mean pre-test score for all participants was 15% and the mean post-test score for all participants was 64%, as shown in Figure 4. Post-test scores for Questions 1 and 3, which were more calculation-oriented, rose to 65–78%. These results show that the majority of students in this study entered and exited the course with a poor understanding of some of the conceptual aspects of Mohr's circle, but that their ability to perform calculations involving Mohr's circle increased substantially throughout the semester.

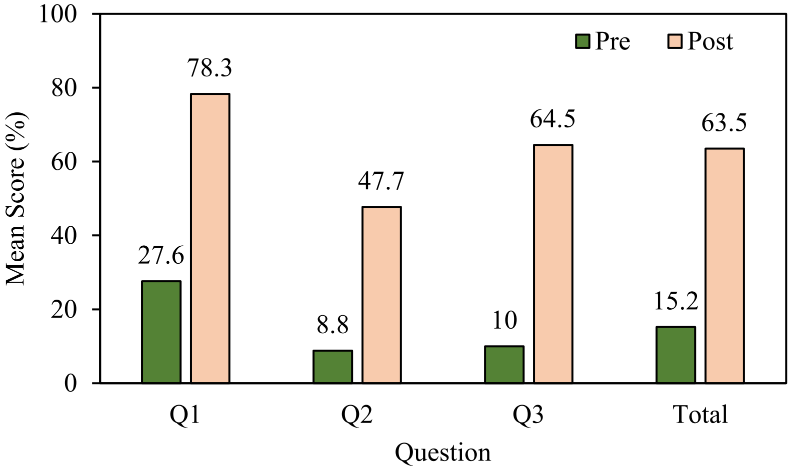


Figure 4. The pre- and post-test means for Questions 1-3 from 2019 to 2021 across five institutions (349 students), administered in the first geotechnical engineering course. Differences between the post- and pre-test means are 50.7% for Question 1, 38.9% for Question 2, 54.5% for Question 3, and 48.3% overall.

A two-sample t-test at the 5% level of significance was conducted for each question to identify any statistically significant differences between the pre- and post-test scores, as illustrated in Table 4. The difference between the means was statistically significant for each question and overall scores, showing substantial improvement from pre-test to post-test at the 5% level of significance. There was an increase from an average score of 15.2% on the pre-test to an average score of 63.5% on the post-test ($t = 22.2$, P-value < 0.001). All three questions showed a statistically significant difference between the pre- and post-tests, although the difference was the smallest for Question 2.

Table 4. Pre-test and post-test means and standard deviations for Questions 1-3 from 2019 to 2021, administered at five institutions in the first geotechnical engineering course.

Question	Sample size, *n*	Pre-Test		Post-Test		Test statistic, *t*	P-value
		Mean (%)	Std. Dev. (%)	Mean (%)	Std. Dev. (%)		
Q1	349	27.6	44.7	78.3	40.8	15.6	< 0.001
Q2	349	8.8	28.3	47.7	50.0	12.1	< 0.001
Q3	349	10.0	29.6	64.5	46.8	18.3	< 0.001
Overall	349	15.2	40.7	63.5	56.7	22.2	< 0.001

Figure 5 separates the results by institution, illustrating the students' total scores on the pre-test and post-test at each of the five institutions. The mean pre-test scores range from 9 to 32% (a noticeable variation), and the post-test scores range from 57% to 73% (slightly less variation). The mean increase from pre-test to post-test is approximately 30% at Saint Martin's University and the University of Minnesota Duluth, approximately 40% at Merrimack College and Northeastern University, and a larger increase of 64% at The Citadel. Overall, the patterns are generally similar for Merrimack, Northeastern, Saint Martin's, and the University of Minnesota Duluth, but The Citadel noticeably has the smallest pre-test score and the largest post-test score. One possible explanation is that the time gap between students' completion of their mechanics of materials course and the start of their first geotechnical engineering course is larger at The Citadel than at other institutions, meaning that the pre-test is capturing a longer time period of Mohr's circle retention. Students at The Citadel complete their first course in geotechnical engineering in the first semester of their senior year, rather than during the junior year for the other institutions in this study. On a similar token, the higher post-test scores at The Citadel may suggest a greater degree of academic development by the end of the semester, because these students are seniors rather than juniors.

Another distinguishing characteristic of Mohr's circle instruction used at The Citadel that may be contributing to the improvement in scores from the pre-test to post-test is the use of a web-based pre-class assignment and a formative assessment approach. Both the web-based pre-class assignment and the formative assessment had positive effects on student learning of Mohr's circle concepts. The web-based pre-class assignment provided opportunities for students to actively construct new knowledge from prior knowledge, as well as offering prompt feedback. The formative assessment helped the instructor to identify students' strengths and remedy weaknesses to achieve a higher level of student learning. Northeastern University is the only other institution in this study to employ formative methods through instructional adaptation based on student performance on an in-class problem solving exercise. Northeastern University observed the second largest increase in scores from the pre-test to the post-test. While there is currently insufficient data to draw strong conclusions, the data suggests implementation of formative methods and adapting instruction to student needs may improve student learning with Mohr's circle.

Merrimack College experienced the third-largest increase in scores from pre-test to post-test, with differences slightly less than Northeastern's. The slightly longer class instructional time on Mohr's circle (an entire 2-hour session) and extensive in-class activities and physical demonstrations may support the large increase observed in scores throughout the semester. However, when considering the entire set of institutions, the amount of class instructional time for Mohr's circle does not appear to be positively correlated with higher post-test scores, or with larger gains in scores from the pre-test to the post-test. Thus, dedicating more class time to Mohr's circle instruction does not necessarily lead to better learning outcomes. In fact, the Citadel employs the least Mohr's circle classroom instruction time and saw the highest post-test scores and gain in scores from pre-test to post-test. These findings further support the notion that a formative approach leads to better learning with respect to Mohr's circle.

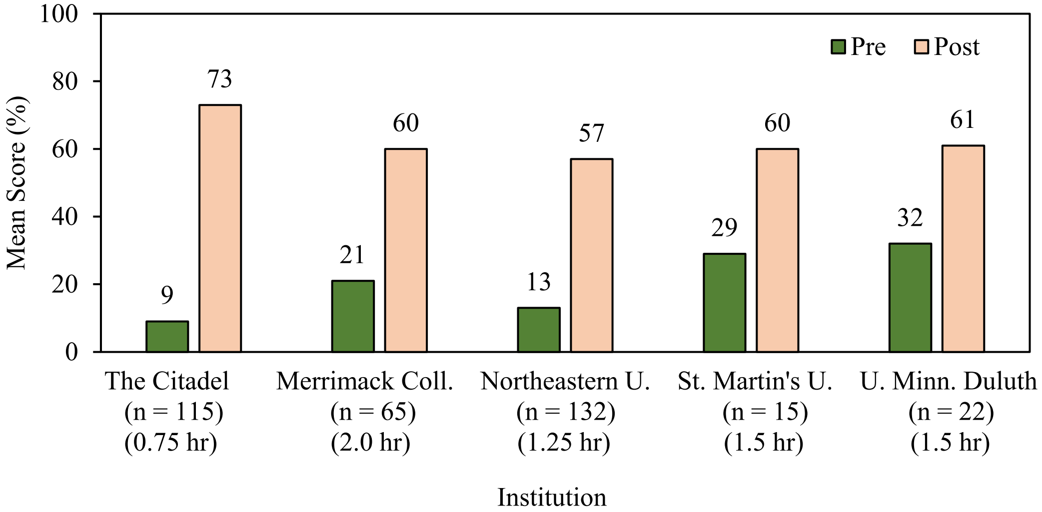


Figure 5. The total pre- and post-test means for concept inventories administered in the first geotechnical course, separated by institution. Sample sizes and amounts of class instructional time for Mohr's Circle are also provided. Differences between the post- and pre-test means are as follows: The Citadel, 64%; Northeastern, 44%; Merrimack, 39%; St. Martin's, 31%; and Univ. Minn. Duluth, 29%.

Question 3 (the calculation of the maximum shear stress given the principal stresses) was previously administered on a concept inventory with additional questions related to geotechnical engineering [1, 2, 17]. A total of $n = 922$ students at eight institutions completed this instrument over the course of eight years, from 2014 to 2021 (note that this question appeared as Question 10 on that broader instrument [1, 2, 17]). Figure 6 illustrates the means of the Question 3 scores on the pre and post-test within the broader dataset. Across all years, the trends observed in Figure 4 are confirmed: extremely low scores on the pre-test (illustrating poor retention after the Mechanics of Materials course), and significantly higher scores on the post-test (illustrating a stronger, although not perfect, ability to perform Mohr's circle calculations at the end of the semester). Observing the patterns in eight years of data, some minor increases and decreases are observed, but there are no significant trends versus time.

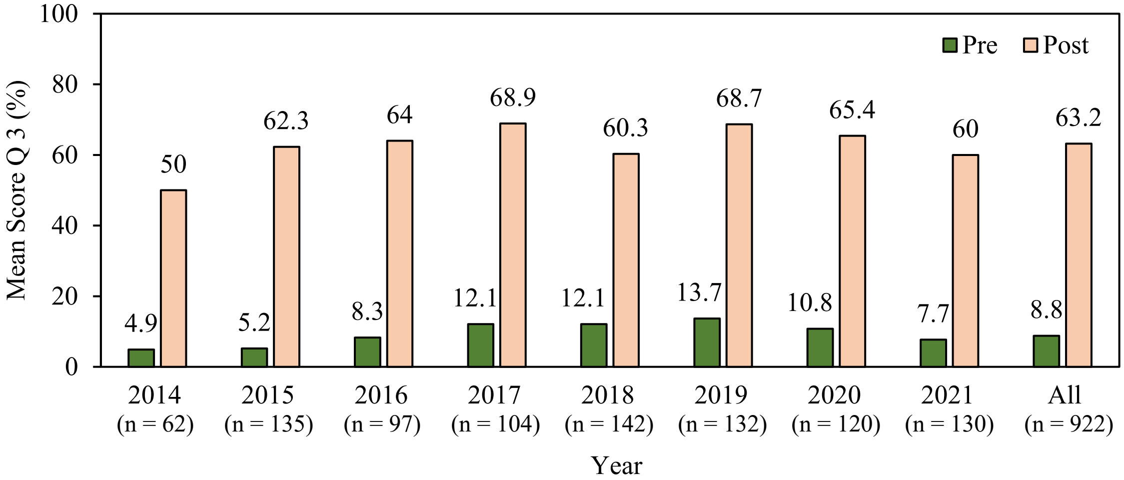


Figure 6. The pre- and post-test means for Question 3 for years 2014-2021 across a broader dataset of eight institutions (922 students).

As documented in Table 5, a statistical analysis was conducted on all pre-test and post-test data for Question 3 to detect changes in students' understanding of the Mohr's circle concepts over the course of the semester. Comparison of the pre- and post-test mean scores was completed using the two-sample t-test at the five percent level of significance, and the results are shown in Table 5. Significant differences are observed between the means of the pre-test and post-test for each year and all years combined. In the aggregate, there was an increase from an average score of 8.8% on the pre-test to an average score of 63.2% on the post-test (mean difference = 54.4%; t = 25.8, P-value < 0.001). During the study period, the pre-test means and standard deviations ranged from 5% to 14%, and 20% to 35%, respectively. The post-test means and standard deviations both increased relative to the pre-test values, ranging from 50% to 69%, and 42% to 50%, respectively. The relative difference between the pre- and post-test means ranges from 45% to 57%. There is also noticeably low variation among the different years' post-test standard deviations, although there is slightly greater variation in the pre-test standard deviations.

Table 5. Pre-test and post-test means and standard deviations, and differences for Question 3, years 2014 to 2021.

Year	Sample size, *n*	Pre-Test		Post-Test		Test statistic, *t*	P-value
		Mean (%)	Std. Dev. (%)	Mean (%)	Std. Dev. (%)		
2014	62	4.9	21.8	50.0	50.0	6.64	< 0.001
2015	135	5.2	20.6	62.3	47.6	13.2	< 0.001
2016	97	8.3	27.7	64.0	45.7	9.98	< 0.001
2017	104	12.1	31.6	68.9	42.1	12.4	< 0.001
2018	142	12.1	31.6	60.3	48.4	10.49	< 0.001
2019	132	13.7	34.8	68.7	46.5	11.57	< 0.001
2020	120	10.8	30.0	65.4	46.6	10.58	< 0.001
2021	130	7.7	26.0	60.0	47.9	10.73	< 0.001
Overall	922	8.8	11.4	63.2	22.1	25.8	< 0.001

Assessment of Mohr's Circle Concepts in a Second Geotechnical Engineering Course

The same three-question Mohr's circle concept inventory was also employed at two institutions in a subsequent geotechnical engineering course (Foundation Engineering), allowing for an analysis of the retention of Mohr's circle concepts after completion of the introductory geotechnical engineering course. During 2020 and 2021, the concept inventory was completed by 28 students at Merrimack College and 20 students at Northeastern University at the start and end of their Foundation Engineering course. Most of the students enrolled in the second course were civil engineering seniors, and therefore had successfully completed both the mechanics of materials course and the first course in geotechnical engineering, with exposure to Mohr's circle in at least these two courses.

Figure 7 provides an overview of the means of the students' question-by-question and overall scores on the pre-test and post-test in the second geotechnical engineering course. The means and standard deviations of the pre- and post-test scores are tabulated in Table 6, along with the results of a two-sample t-test to compare the pre- and post-test mean scores. There are minor increases in the scores from the pre- to post-test for Questions 1 and 2, but no change in the Question 3 scores; however, the P-values in Table 6 illustrate that the changes are not statistically significant. Students appear to be entering and leaving the second geotechnical engineering course with roughly the same level of knowledge related to Mohr's circle, with a slight increase from the pre-test overall mean of 43.1% to post-test mean of 51.0%. The within-semester increase in students' understanding of Mohr's circle is much smaller for the second geotechnical engineering course than the first geotechnical engineering course, perhaps due to the fact that Mohr's circle is not used as heavily in the second course on foundation engineering and receives much less instructional time (about 0.5 hours at these institutions). Another possible explanation is that the emphasis on Mohr's circle in the first geotechnical engineering course is often at the end of the semester (directly before students complete the post-test), whereas the emphasis on Mohr's circle in the second geotechnical engineering course is early in the semester, and therefore a greater time period passes before the post-test is completed.

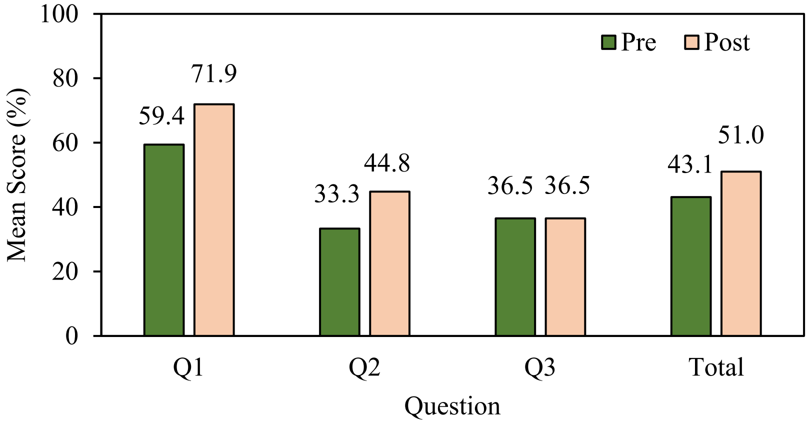


Figure 7. The pre- and post-test means for Questions 1-3 administered in a second geotechnical engineering course in 2020 and 2021 at two institutions: Merrimack College and Northeastern University (48 students total).

Table 6. Pre-test and post-test means and standard deviations for Questions 1-3 administered in a second geotechnical engineering course at two institutions in 2020 and 2021

Question	Sample size, *n*	Pre-Test		Post-Test		Test statistic, *t*	P-value
		Mean (%)	Std. Dev. (%)	Mean (%)	Std. Dev. (%)		
Q1	48	59.4	49.1	71.9	44.8	1.30	0.196
Q2	48	33.3	47.6	44.8	49.7	1.15	0.252
Q3	48	36.5	48.1	36.5	44.6	0.00	1.000
Overall	48	43.1	37.6	51.0	37.6	1.04	0.301

Comparing the results of Figure 7 (question-by-question scores for the second geotechnical engineering course) to the results of Figure 4 (question-by-question scores for the first geotechnical engineering course), students do appear to retain most of their knowledge between the first and second courses for Questions 1 and 2. For these questions, the pre-test scores in the second course are slightly less than the post-test scores in the first course; by the end of the semester in the second course, students' scores approach those of the post-test in the first course. However, the results of Question 3 (on the maximum shear stress) seem to deviate from this trend; at both ends of the second course, there is a noticeable decrease from the post-test scores in the first course, and no improvement is observed during the second course. Students' peak understanding of Mohr's circle appears to be at the end of the first course in geotechnical engineering. However, comparisons of the pre-test scores in Figures 4 and 7 suggest that the knowledge decay between the first and second courses in geotechnical engineering is much less than that between the mechanics of materials course and the first course in geotechnical engineering. Improvements in the post-test scores in the second geotechnical engineering course could perhaps be improved by a more intentional emphasis on fundamental soil mechanics (such as Mohr's circle) throughout the semester in foundation engineering.

Conclusions

Mohr's circle, a critical concept for the evaluation of subsurface stresses and strengths, remains a challenging concept for students to master. Using the results of concept inventories, this study has shown that students fail to retain fundamental Mohr's circle concepts between the Mechanics of Materials course and the introductory geotechnical engineering course. At the end of the geotechnical engineering course, the majority of students exhibited a significant improvement in their ability to perform calculations involving Mohr's circle. However, most students still exit the course with a poor understanding of some of the conceptual aspects of Mohr's circle (such as realizing that the principal planes are associated with zero shear stresses). For a subset of institutions where this concept inventory was administered in a follow-up undergraduate geotechnical engineering course (Foundation Engineering), students' levels of understanding appear to stabilize at a level slightly less than their peak at the end of the first course in geotechnical engineering. One would expect to see improvement during the semester and in subsequent courses, just as one would expect to see knowledge decay between the end of one course and the start of another. One possible explanation for the loss of retention between courses is that many students view their civil engineering program as a series of individual disconnected courses, rather than a series of linked themes. Improvements in students' retention of Mohr's circle concepts (and other concepts that span multiple courses) may perhaps be enhanced by instructors' intentional efforts to highlight the future applications of Mohr's circle to students when they first learn the topic.

Due to the importance of Mohr's circle in geotechnical engineering, students would benefit from improved pedagogical techniques that highlight its significance in future courses and in engineering practice. The instructional strategies for Mohr's circle described in this paper, as implemented by geotechnical engineering instructors from ten different U.S. institutions, provide some interesting techniques that may work towards increasing students' understanding of Mohr's circle concepts. The data from this study provide some indication that web-based pre-class assignments and formative approaches lead to greater gains in student knowledge of Mohr's circle, but additional study is needed to draw stronger conclusions. Future directions of this research may extend the analyses to a wider group of students and approaches, and evaluate correlations between student performance and instructional strategies. As there remains room for improvement in students' conceptual understanding of Mohr's circle as it applies to geotechnical engineering, an increase in the sharing of best practices from instructors at different institutions may help work towards this goal.

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