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Integral Concept and Decision Making: Do the STEM Majors Know When to Use Numerical Methods for Integral Approximation?

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Miss Hazal Ceyhan

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Making the decision to solve a definite integral exactly or approximately can be a challenge for first time Riemann integral learners. A conceptual cross section between calculus and numerical methods/analysis courses is approximation of definite integrals that cannot be determined exactly by using existing methods. The realization of a need for a numerical method/analysis concept after determining that the definite integral cannot be solved exactly is another conceptual challenge that students can face. In this study, engineering and mathematics students' decision to determine an exact or approximate solution to a definite integral will be analyzed qualitatively and quantitatively. Participating students' responses to the research question are analyzed and the collected information is evaluated by using the schema development idea of Piaget et al. (1989). The research data consists of seventeen senior undergraduate and graduate mathematics and engineering students' responses to a research question who were either enrolled or recently completed (i.e. 1 week after course completion) a Numerical Methods/Analysis course at a large Midwest university during a particular semester in the United States. Missing conceptual knowledge of the participants is observed when they were incapable of determining the solution to the problem.

Key Words: Riemann integral, functions, derivative, triad classification, schema development.

Introduction

Engineering and mathematics undergraduate students' Calculus education during their first two years is one of the most important aspects, or maybe the most important aspect, of a successful undergraduate education. Calculus education of engineering and mathematics students has drawn attention in recent years [Tokgöz (2015), (2016), and (2012); Tokgöz and Gualpa (2015)]. It is not only critical to observe and understand cognitive approaches of students for solving Calculus questions through research but it is also important to redevelop courses according to the lessons learned from the research observations. In calculus, students initially learn practical methods to solve integral questions for a particular set of functions. Determining the definite integral of functions with non-existing exact solutions is taught after teaching practical methods in calculus courses. Deciding whether it is possible to determine the exact or approximate solution to a definite integral is covered in multi-disciplinary courses such as calculus, numerical methods, and numerical analysis by departments in STEM fields. This Institutional Review Board (IRB) approved work is concerned with graduate and senior undergraduate engineering and mathematics students' ability to solve an integral question and determining the Calculus concepts that the participating students are lacking to solve the research question. Decision making process on determining exact or approximate solution to the research question is investigated by collecting written and interview responses. Participants' responses to the research question

Q. Can you calculate the integral $\int_{0.1}^{2} \frac{e^x}{x} dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

are analyzed qualitatively and quantitatively. Throughout this work, written and interview responses of some of the participants will be used for the qualitative analysis. The quantitative work will be consisting of schema classification of the participants. Action-Process-Object-Schema (APOS) introduced by Asiala et al. (1996) originated from Piaget et al. (1977) is initially considered for evaluation of the participant responses to the research question, however Triad classification of the schema development in APOS theory is determined to be a better method for the data evaluation. One of the aims of this paper is to investigate participating students' decision making process on using numerical methods versus traditional integral calculus calculations when a definite integral question is considered to be solved. This investigation can help Professors to redevelop courses who are teaching multidisciplinary topics in Computer Science, Engineering, and Mathematics. Another aim of this article is to understand the circumstances under which students choose to approximate a definite integral.

Participants & Data Collection Procedure

A written questionnaire is given to seventeen participants and all participants are interviewed to explain their written responses to the questionnaire. IRB approved guidelines are followed and participants are compensated for their involvement to the research. The questionnaire questions covered single variable calculus concepts such as functions, limits of functions, function derivatives, Riemann integral, power series of functions, and programming preference of the participants. The questionnaire was given to the participating students outside the course environment under researchers' supervision. The participants of this study are engineering and mathematics undergraduate and graduate students who either were enrolled in or recently completed a numerical methods or analysis course in a particular semester at a large Midwest university in the United States. Pre- and post-interview responses of the participants to an integral question are evaluated in this work. Post-interview results are designed to have a better understanding of the pre-interview (i.e. written) responses of the participants. The answers of the students to the Riemann integral question are evaluated by considering the sub-concepts that take place in the solution of the question.

In this and next sections literature relevant to the theory used for evaluating the research question will be explained. Triad classification will be covered in the next section, which will be used for observing students' ability to answer the research question. The responses of participants are displayed for a better understanding of the qualitative and quantitative analysis of the collected data. Participating students' responses to the research question are evaluated by using the Triad classification of the schema development idea.

The literature on students' ability to determine the Riemann integral of functions by using paper-pencil solution is not extensive. Asiala et al. (1996) observed participants' difficulty to write a code for calculating the integral of functions for which students were asked to write a code to approximate the integral by sampling points. Senior mathematics undergraduate and graduate students' weak rate of change concept knowledge that resulted in weak understanding of the integral concept is observed by Thompson (1994).

Schema Development

Clark et al. (1997) used Triad classification for investigating how first-year calculus students construct the chain rule concept. Authors initially attempted to use the APOS theory and ended up using the schema development idea introduced by Piaget et al. (1989). Students are placed in the intra stage by Clark et al. (1997) if they knew some of the derivative rules and were able to apply the chain rule but did not know the relationship between these concepts, and participants took place in the inter stage if they had the ability to recognize all different cases and figure that they are related. Participants were assumed to be in the Trans stage if they were able to construct and apply the chain rule.

Students' responses in this work are evaluated by using the following Triad classification:

- Participants are classified to be in the Intra stage if they doubt about the need of an approximation method to solve the research question and considered integral solution methods to solve the research problem.
- Participants are classified to be in the Inter stage if they recognized the need of an approximation method to solve the research question but did not use a numerical method.
- Participants are classified to be in the Trans stage if they are placed in the inter stage and specified the approximation method that they would use to calculate the integral.

Research Question

All seventeen participants' written and interview responses to the following research question are evaluated in this work:

Q. Can you calculate the integral $\int_{0.1}^{2} \frac{e^x}{x} dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

One of the researchers interviewed the participants and interviews are video recorded and transcribed by the main author of this article. The questionnaire (i.e. written) data collected prior to the interviews are displayed in the next section.

Written Responses

The written questionnaire responses of the students are collected prior to the completion of the numerical methods/analysis course that participants were enrolled. Six of the participants tried to use a technique to determine the solution to the research question by using either a numerical or an integration technique that they know. These students' used either Integration by Parts or series expansion of the integrand to determine the numerical value of the given integral.

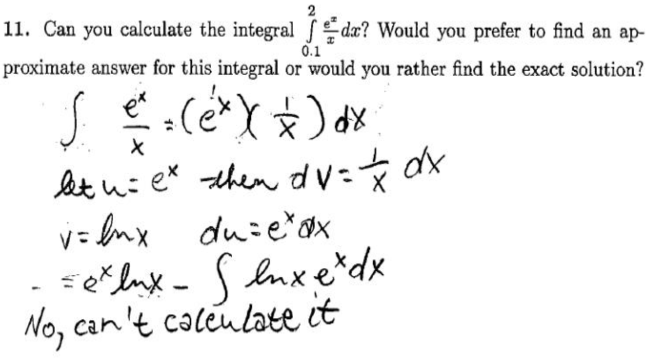


Figure 1: Written response of RP 3

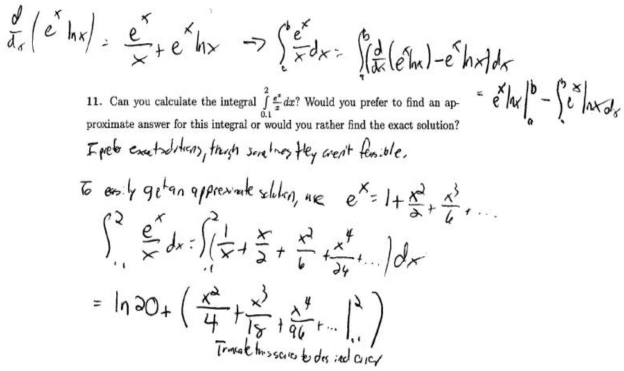


Figure 2: Written response of RP 13

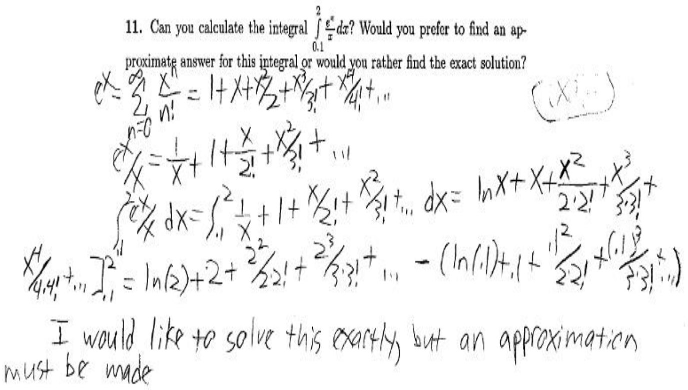


Figure 3: Written response of RP 17

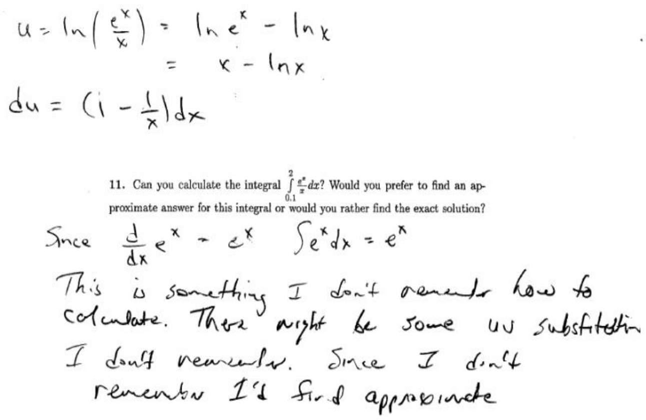


Figure 4: Written response of RP 12

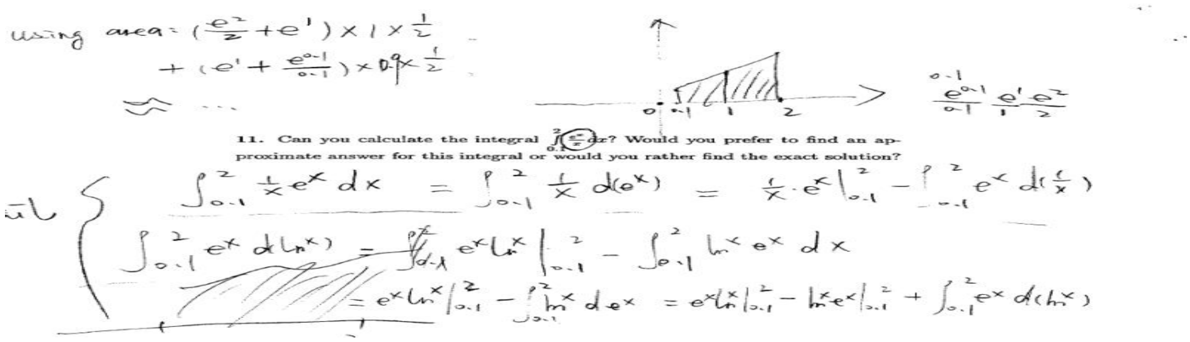
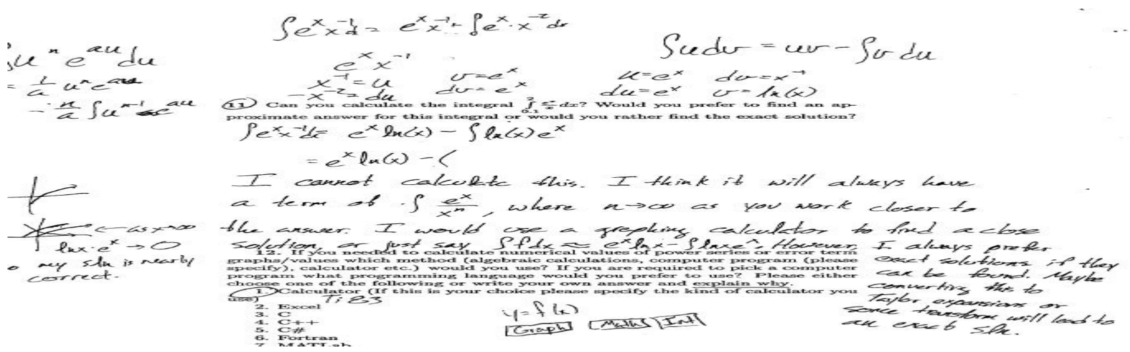


Figure 5: Written response of RP 5



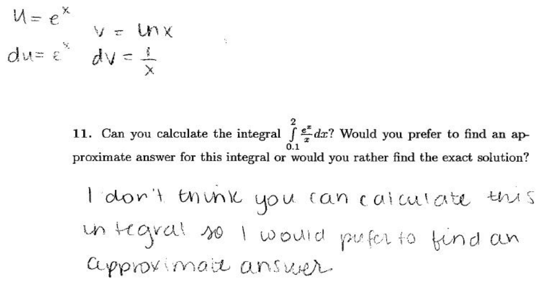
(11) Can you calculate the integral $\int_{0.1}^{2}\frac{e^x}{x}dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

I cannot calculate this. I think it will always have a term of $\int \frac{e^x}{x^n}$, where $n \to \infty$ as you work closer to the answer. I would use a graphing calculator to find a close solution or just say ... However, I always prefer exact solutions if they can be found. Maybe converting this to Taylor expansions or some transform will lead to an exact sln.

12. If you needed to calculate numerical values of power series or error term graphs/values which method (algebraic calculations, computer program (please specify), calculator etc.) would you use? If you are required to pick a computer program what programming language would you prefer to use? Please either choose one of the following or write your own answer and explain why.

Figure 6: Written response of RP 6

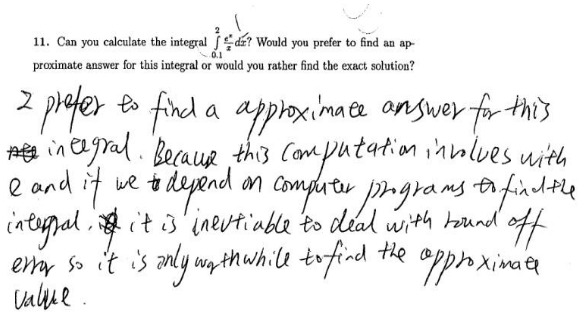
The rest of the participants did not try to calculate the integral and only commented on whether it makes sense to approximate or determine an exact solution to the integral.



11. Can you calculate the integral $\int_{0.1}^{2}\frac{e^x}{x}dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

I don't think you can calculate this integral so I would prefer to find an approximate answer.

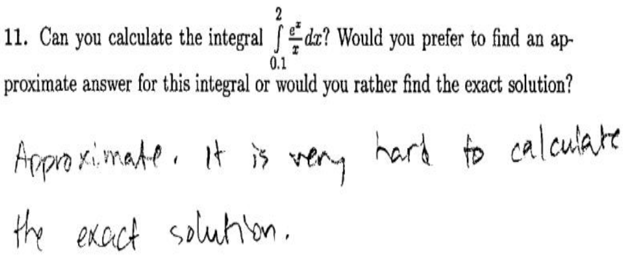
Figure 7: Written response of RP 1



11. Can you calculate the integral $\int_{0.1}^{2}\frac{e^x}{x}dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

I prefer to find a approximate answer for this integral. Because this computation involves with e and if we depend on computer programs to find the integral, it is inevitable to deal with round off error so it is only worthwhile to find the approximate value.

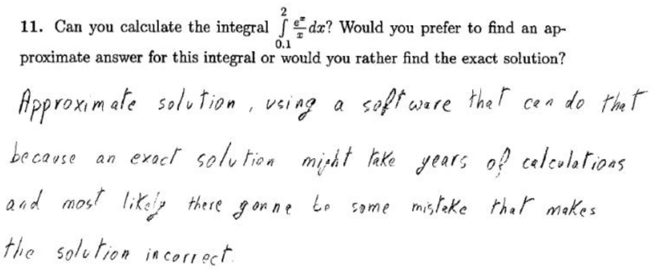
Figure 8: Written response of RP 2



11. Can you calculate the integral $\int_{0.1}^{2}\frac{e^x}{x}dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

Approximate. It is very hard to calculate the exact solution.

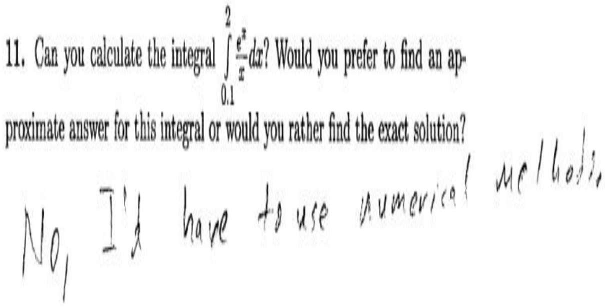
Figure 9: Written response of RP 9



11. Can you calculate the integral $\int_{0.1}^{2}\frac{e^x}{x}dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

Approximate solution, using a software that can do that because an exact solution might take years of calculations and most likely there gonna be some mistake that makes the solution incorrect

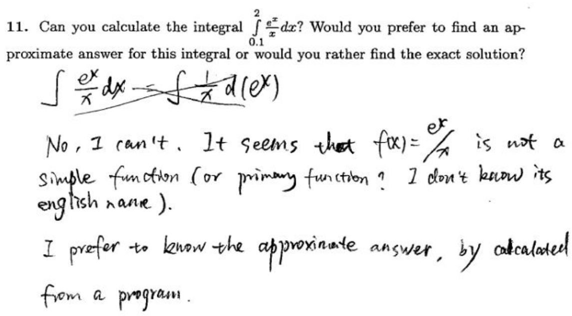
Figure 10: Written response of RP 10



11. Can you calculate the integral $\int_{0.1}^{2}\frac{e^x}{x}dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

No, I'd have to use numerical methods

Figure 11: Written response of RP 11



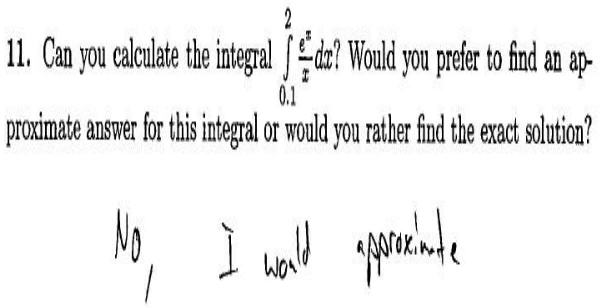
11. Can you calculate the integral $\int_{0.1}^{2}\frac{e^x}{x}dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

$\int \frac{e^x}{x}dx = \int \frac{1}{x}d(e^x)$

No, I can't. It seems that $f(x)=e^x/x$ is not a simple function (or primary function? I don't know its english name).

I prefer to know the approximate answer, by calculated from a program.

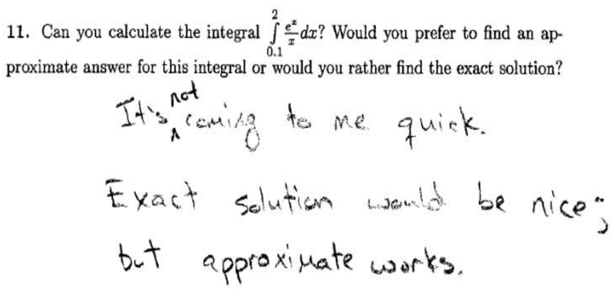
Figure 12: Written response of RP 14



11. Can you calculate the integral $\int_{0.1}^{2} \frac{e^x}{x} dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

No, I would approximate

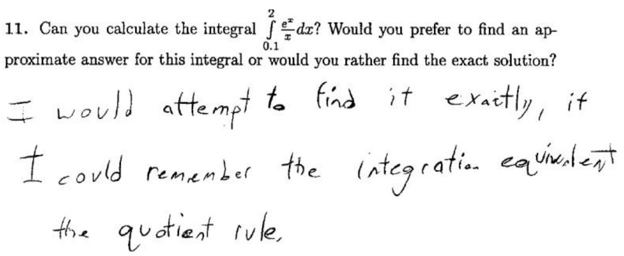
Figure 13: Written response of RP 15



11. Can you calculate the integral $\int_{0.1}^{2} \frac{e^x}{x} dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

It's not coming to me quick.
Exact solution would be nice; but approximate works.

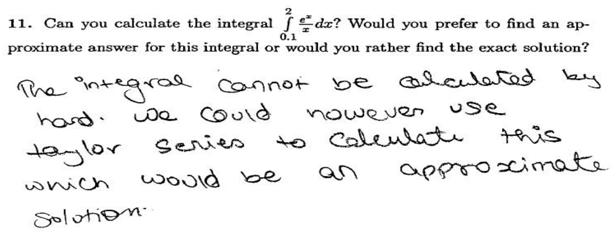
Figure 14: Written response of RP 16



11. Can you calculate the integral $\int_{0.1}^{2} \frac{e^x}{x} dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

I would attempt to find it exactly, if I could remember the integration equivalent the quotient rule.

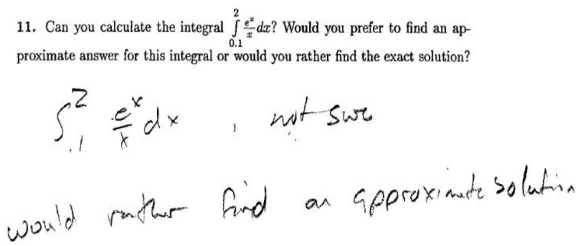
Figure 17: Written response of RP 7



11. Can you calculate the integral $\int_{0.1}^{2} \frac{e^x}{x} dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

The integral cannot be calculated by hand. We could however use taylor series to calculate this which would be an approximate solution.

Figure 8: Written response of RP 8



11. Can you calculate the integral $\int_{0.1}^{2} \frac{e^x}{x} dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

$\int_{.1}^{2} \frac{e^x}{x} dx$, not sure
would rather find an approximate solution

Figure 4: Written response of RP 4

Triad classification of the written responses are not implemented in this work because of insufficient details of the participant responses. Participants explained their written questionnaire responses during the video recorded interviews and the next section is devoted to these interview responses with the evaluation of the corresponding data.

Interview Responses

The majority of the transcribed interview responses will be displayed in this section. The interviews questions are structured in a way to have a good understanding of the participants' written responses. The interview questions posed by the researcher varied depending on the written responses of the participating students.

Interviewer: …You are saying I don't think you calculate this integral so I would prefer finding an approximate answer. So how would you find an approximate answer? Do you remember any method, any method you recall?

RP 1: I know that you can probably use Taylor series to approximate this (pointing $\frac{e^x}{x}$.)... I'm sure there is some a method you can use for integration but I don't remember on top of my head.

Interviewer: Where did you learn that method, do you recall?

RP 1: The Taylor series method?

Interviewer: No, just like to approximate that (pointing the integral.)

RP 1: …in Numerical Analysis course.

…

Interviewer: So here we have $\int \frac{e^x}{x}dx$ and you say "I prefer to find an approximate answer for this integral because this computation involves e and if we depend on computer programs to find the integral, it is inevitable to deal with round off error so it is only worth to find the approximate value." So is there any method that you would use to be able to calculate this or would you just type it in Matlab and the program will give you the answer?

RP 2: Yeah, I think you calculate this one (pointing $\int \frac{e^x}{x}dx$) by hand. But if you have an e in your final, generalized solution, something like this (writes $e^x + c$) you need to calculate e.

Interviewer: Is there any method you know to be able to calculate this (integral $\int \frac{e^x}{x}dx$) by hand?

RP 2: Oh, okay maybe...

Interviewer: Do you recall any method? Or do you think it is possible to do it by hand?

RP 2: Maybe you can change this to. I don't remember just exact way to solve this by hand but I think there is some methods in the book.

…

Interviewer: There is another integral question here. $\int \frac{e^x}{x}$ dx and you said "Not sure, I would rather find an approximate solution." You are not sure about...?

RP 3: Whether or not this can be integrated. If this can be integrated then I can find exact solution. If it's not able to then we can use Taylor methods, I'm taking Numerical Methods course so I would use some sort of numerical way to find a close solution.

Interviewer: … do you remember any method to be able to calculate this integral? Just the name of the method.

RP 3: I thought about using integration by parts.

…

Interviewer: Okay, now there is an integral question. And $\int \frac{e^x}{x}$ dx. Can you calculate this integral? Would you prefer to find an approximate solution or precise solution? …which method are you using here. Can you briefly tell me?

RP 4: I think I tried to integrate it by parts.

Interviewer: And did it work?

RP 4: …no.

Interviewer: ... Is that the only method you know or could there be any other method you can use. Can you actually determine that you cannot calculate this integral by any method?

RP 4: I cannot remember.
Interviewer: And you said "No can't calculate it" Would you calculate it by using your calculator?
RP 4: I'd try.
Interviewer: So you would rather use the calculator I guess at this point…
RP 4: Yeah.
Interviewer: And would you prefer to approximate it by any method you know or you learned in the numerical methods course if you can't calculate it precisely?
RP 4: Yeah but I can't remember which method on top of my head? Yeah, I can't remember right now.
Interviewer: Do you know which method your calculator uses?
RP 4: I have no idea.
Interviewer: Alright, let's move on to the next question.
RP 4: Probably a numerical method, I don't know...
…
Interviewer: And here you are calculating this function's integral, $\frac{e^x}{x}$ integral from point one to two's integral. Can you briefly explain how you calculated or were you able to calculate it?
RP 5: Yeah, I first used this method.
Interviewer: ...What is that method?
RP 5: ...I don't know the name. I don't know the name of it in English so.
Interviewer: Integration by parts…
RP 5: I just failed to get the result.
Interviewer: Okay. And do you think you will be able to calculate this integral $\frac{e^x}{x}$ by hand; does it look possible?
RP 5: Yeah, this I know we can calculate the result.
Interviewer: If you wanted to use the computer program, or like a method, what would you use to be able to approximate this integral?
RP 5: I would use Matlab.
Interviewer: Using Matlab. Do you remember any methods to be able to calculate this integral from Numerical Methods course?
RP 5: I think I can plot this function (pointing $\frac{e^x}{x}$) based on this interval (draws xy-coordinates and draws a graph from 0.1 to 2) I can calculate the area (sketches the area of the region between the curve and the x-axis)
Interviewer: Okay, so will you be able to calculate the exact area or will you be able to calculate something else? Would it be exact or approximate?
RP 5: Maybe it should not be exact value. Yeah, I think so.
…
Interviewer: And here it says can you calculate the integral $\int \frac{e^x}{x}dx$? Would you prefer to find an approximate answer for this integral or … can you find the exact solution…?
RP 6: I'm not sure exact solution exists…At least I couldn't figure out. I tried to find it using this (pointing $\int udv = uv - \int vdu$), whatever method it is called and I always got an irreducible term that I couldn't integrate…I'm not sure if there is an exact solution.

Interviewer: Okay. Would you use a calculator or would you use, I guess you would use a calculator based on what you wrote there but would you consider writing a program in this case? No.
RP 6: No. Because with a calculator like the one I use, TI-83, you can easily, you can easily sat it there. It is easy to do, it is just a few key strokes there.
Interviewer: Okay. And how would you do that? What type of method would you use if you would do it, if you would calculate it?
RP 6: Oh, you can just define it in the window, I mean, as a y equals, define your function. And then I guess graph, and then math and then I think it is integral or something.
Interviewer: Okay.
RP 6: It gives you two bounds and you type in two bounds and it pops out automatically.
Interviewer: Okay. So you do remember how to calculate it.
RP 6: Yeah.
…
Interviewer: Do you recall any way to determine whether you can find this definite integral? …is it possible to calculate this… You are saying that you can use approximation here … to be able to calculate this when you don't remember how to calculate it algebraically?
RP 7: It will be better.
Interviewer: …which method would you use? ...Can you suggest something?
RP 7: I'm trying to think what we went over but my memory is really bad whenever I try to recall one specific thing.
Interviewer: So would you just look it up in the book and just calculate it.
RP 7: Yeah... I know it is continuous so it continuous up to zero so. Trapezoid rule. There you go, trapezoid method. Not the lower sum or upper sum. It is really annoying honestly. I can find another way to do it automatically do the lower or upper. I guess max, or supremum, infimum. I guess those were the terms.
Interviewer: So you would use trapezoid rule?
RP 7: Yes.
Interviewer: ...Do you remember any details about the rule?...
RP 7: Just helps to write (starts writing 1/2). Doing the area so take the first point (continues writing

$$\frac{1}{2}\left(\frac{e^x}{x}+\frac{e^{x+\Delta x}}{x+\Delta x}\right)$$

I know that there is a reduced form but I can't quite remember that... Sum (continues writing

$$\sum_{i=0.1}^{1.9}\frac{1}{2}\left(\frac{e^x}{x}+\frac{e^{x+\Delta x}}{x+\Delta x}\right)\Delta x$$

If I was doing, I guess, twenty things I would do it 1.9. One interval short so that this would be two.
Interviewer: What is Δx? Δx is two?
RP 7: The upper bound of the interval.
Interviewer: What is Δx in this setting? Δx you have there.

RP 7: …Δx...For this it would be 0.1. 0.1-0.2, 0.2-0.3. Oh wait, I'm missing something. (Adds Δx term in the expression above and makes it

$$\sum \frac{1}{2}\left(\frac{e^x}{x}+\frac{e^{x+\Delta x}}{x+\Delta x}\right)\Delta x$$

Interviewer: So you just need to calculate this and that gives the area right? That's what you are saying.
RP 7: That will give an approximation.
Interviewer: Okay, alright.
RP 7: Because it is going to be always positive. It should check for that but.
…
Interviewer: And here we have integral $\int \frac{e^x}{x}$ dx and you are saying "The integral cannot be calculated by hand. We could however use Taylor series to calculate this which would be an approximate solution." So is there anything you would want to add to this?
RP 8: Probably Riemann sum.
Interviewer: Okay, and is there any other method you recall from numerical methods?
RP 8: Trapezoid rule.
…
Interviewer: And here we have can you calculate this integral (pointing to the integral given in the question), given integral and would you prefer to find an approximate answer to this question and you are saying "Approximate. It is very hard to calculate the exact solution."
RP 9: Yes.
Interviewer: What would you use to approximate it? Do you recall any technique to be able to do so?
RP 9: I don't know, I might use a series or something like that. Yeah, I'd use like a Taylor series to calculate that, to approximate that.
Interviewer: And you are saying it is very hard to calculate the exact solution. Why?
RP 9: Because we learned that in the class...We learned that it is very hard to calculate $\frac{e^x}{x}$ and I remember that
Interviewer: What about calculus? …if you use calculus, would you come up with the solution?
RP 9: You mean, like actually if I try to do it?
Interviewer: ... Yes
RP 9: Oh, yeah.
Interviewer: What would you use? Do you recall any method?
RP 9: For approximating it? Yeah, like Taylor series...Finding the exact solution?
Interviewer: …Exact solution? Can you do that? Do you recall any method?
RP 9: No I don't.
…
Interviewer: And here you are saying "approximate solution" to the given integral "using a software that can do that because an exact solution might take years of calculations and most likely there gonna be some mistake that makes the solution incorrect." So what do you mean by this?
RP 10: If you try to calculate it by hand it's gonna take forever and you might make calculation mistakes.
Interviewer: With the program or when you solve by hand…
RP 10: By hand.
Interviewer: …okay.
RP 10: With the program the calculations will be fast and exact.

Interviewer: Do you recall any method to be able to calculate this?
RP 10: We took a lot of methods this semester.
Interviewer: Do you recall anything?
RP 10: The names?
Interviewer: Like…
RP 10: Taylor series, Newton's method, bisection method.
…
Interviewer: And here you are saying "No, I'd have to use numerical methods." And I guess here you needed to make the decision to be able to decide you know what you should do and which method would you use from numerical methods?
RP 11: …probably trapezoid rule or I'm assuming I can probably use some previous things that we were just working on.
Interviewer: And can you calculate this by hand? Do you think it is easy to do that or is it possible?
RP 11: I believe that is impossible. Well, I don't know, maybe with what we just did, I don't know, but.
…
Interviewer: And here you are given integral of $\frac{e^x}{x}$ from 0.1 to 2. The question is saying would you prefer to find an approximate answer for this integral or would you rather find the exact solution? And here I see some of the calculations you have, you are looking at the derivative of e^x times ln(x) and you are trying to find a solution. Were you able to find a solution based on the technique you know or techniques you know?
RP 13: …Let me see if I messed it up since it being recorded by the camera. I don't see any mistakes right off the bet. So it appears what I have down is an answer in terms of an infinite series. So you can truncate that to get the area. Series is probably conversing, something simple to see, something I observed right off the bet.
…
Interviewer: …here you are saying if you wanted to use this integral you would approximate it.
RP 15: Yes.
Interviewer: How do you make that decision? How do you decide for such things?
RP 15: At first I just wasn't sure if you could, if it is possible to that integration so then you would have to approximate it. And yeah…I wasn't sure, I could use a program or a calculator. And I didn't know how to do it so approximating is the only way to be able to get this.
Interviewer: Okay, and when there is a given question like this do you directly go for your calculator first and see if it is really possible to solve it or do you use any calculus technique or any other program actually to solve this problem, type of problem?
RP 15: When I was in calculus, when I was more familiar with it, I would probably be able to tell you if it was integrable or not but now I'll probably just plug it in really quickly and make sure.
Interviewer: And do you remember any technique to be able to calculate this definite integral?
RP 15: …I'll probably use, like, Riemann sums.
Interviewer: Okay. And here there is integral given…from 0.1 to 2, $\frac{e^x}{x}$ and "It is not coming to me quickly. Exact solution would be nice but approximate works." So what type of method would you use approximate this integral?
RP 16: …yeah, I don't know. Maybe some of the stuff we learned last semester but.

Interviewer: And does this function look like something that you would calculate by hand or does it look simple or would you just?
RP 16: No it is not something that it would be simple.
Interviewer: Okay.
RP 16: Maybe they did it but I don't, it doesn't look good.
…
Interviewer: …can you calculate the integral, given in this question and as I see you wrote e^x as a Taylor series.
RP 17: Right.
Interviewer: And then you wrote $\frac{e^x}{x}$ as a power series. And then you are integrating $\frac{e^x}{x}$ from point one to two by integrating each term I guess, right.
RP 17: Right.
Interviewer: And you are saying "I would like to solve this exactly but an approximation must be made." Why "must be made"?
RP 17: So because you never going to be able to find every value starting here and going all the way to the infinity. If you can't even do that, you can't find all the values here and certainly you can't add them together. At some point you are going to have to cut it off and make an approximation.
Interviewer: …can you calculate this with any method that you know theoretically instead of approximating it?
RP 17: Not with any method that I can think of…There could be a method that I don't realize when I looked at the problem but I can't think of any method.

The interview results indicated 11 out of 17 (64.71%) participants to be placed in the inter stage. The major role of this classification was the "doubt" factor that the participants faced. Four out of 17 (23.53%) participants are classified to be in the intra stage. Two of the 17 participants (11.76%) placed in the Trans stage due their ability to implement series expansion solution in their responses.

Qualitative and Quantitative Data Analysis

Making the decision to solve a definite integral exactly or approximately can be a challenge for first time Riemann integral learners. A conceptual cross section between calculus and numerical methods/analysis courses is approximation of definite integrals that cannot be determined exactly by using the existing integral calculation methods. The realization of the need for a numerical method/analysis concept after determining that the definite integral cannot be solved exactly is another conceptual challenge that students can face. In this work, seventeen engineering and mathematics undergraduate and graduate students' responses to the research question

Q. Can you calculate the integral $\int_{0.1}^{2} \frac{e^x}{x}\,dx$? Would you prefer to find an approximate answer for this integral or would you rather find the exact solution?

are evaluated by using a triad classification. The participating students were either enrolled to or recently completed a numerical methods or analysis course in a particular semester at a large Midwest university in the United States. The purpose of the research question was to understand how STEM majors decide to solve a definite integral question. The participants completed a series of pre-requisite calculus courses in which the questionnaire concepts are covered. Pre- and post-interview responses of the participants to the research question are evaluated and displayed in this work. Post-interview results are designed to have a better understanding of the pre-interview (i.e. written) responses of the participants. The interview results indicated 64.71% of the participants to be classified at the inter stage; 23.73% of the participants classified to be in the intra stage, and 11.76% to be classified in the Trans stage. The following are observed by evaluating the interview and written data responses:

- Most of the participants who preferred to determine an exact solution to the research question attempted to find the solution by using an integration technique called Integration by Parts method that is well known in Calculus. Students concluded that integral cannot be calculated precisely after implementing several steps of Integral by Parts integration technique.
- Some of the participants made mistakes while trying to determine a numerical solution to the question by using an integration technique.
- Several of the participants concluded that the solution cannot be determined to the research question without a justification.
- Majority of the participants who supported to determine an approximate solution to the question supported the use of power series expansion of the integrand after writing the Taylor series expansion of the exponential function. These participants rewrote the integrand by using Taylor series of e^x and calculated the integral of the integrand term-by-term in its power series form.
- Some of the participants supported the use of technology; these participants preferred to determine a solution to the problem by using Matlab programming language.
- Majority of the participants who supported to determine the approximate solution of the integral also supported the use of a numerical method to approximate the solution but some of them did not recall the names of the methods or how to apply them.

The analysis of the collected data indicates the need of a better numerical method knowledge development and an in-depth understanding of the error term calculations as a part of the power series concept through developed courses. Engineering and mathematics majors' decision making for solving an integral problem needs to be improved and the use of technology appears to be needed for solving problems by the educators. The interview process indicated usefulness of a set of follow-up questions for the solution of the research question when students were not capable of solving the question. This is due to the fact that students put more thought into solving the research question when follow-up questions were asked to solve the research question. An interactive online platform can be developed for students to solve problems in several steps; a series of interactive questions guiding towards the solution of the questions can be implemented in an online setting since such questions may not be possible to ask during a written exam. This could be also done in the classroom setting with students working in groups and/or using technology in an instructor-led setting. Students' awareness of numerical methods can be increased by using technology and graphing errors for numerical calculations. Facilitation of the correct method to approach an integral can be customized by ruling out the implementation of the known integral rules for calculating definite integrals. Online exercises can be very affective in learning numerical methods as long as the materials and the instructional methodology are structured right.

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