Integrating digital technology for the innovation of Calculus curriculum

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Abstract

The aim of this submission is to show a sequence of activities integrating technology in a new approach to calculus taking place at the first mathematics course for engineer students at Tecnologico de Monterrey, Mexico. Different specialized software is used to focus the learning process on a global image including derivative and antiderivative. The first scenario of motion over a straight line offers the platform to deal with these notions related to the meaning of position (function/antiderivative) and velocity (derivative). From there, the transference to other real context scenarios is provided. The sequence favors the development of this approach since it deals simultaneously with both calculus notions. In classroom, technology is a mediator to identify, at an early stage of the course, important calculus' results (theorems) about the relationship between function and derivative. The sequence guides a symbolization process that rests in signs that students show through body gestures and visual images. The learning goal is to interpret the graph of a function in terms of its derivative graph; this way, process of visualizing the antiderivative is becoming an important fact at the first contact with calculus, where the Fundamental Theorem of calculus takes a special place as background knowledge throughout the course.

Introduction

The development of new digital technologies must have a positive impact in the learning process of Mathematics, but the speed that is characteristic of this development limits the time needed to understand the importance of these resources and their inclusion in the courses. On the other hand, a traditional curriculum, the standard in many classrooms, actively resists questioning and creates difficulties in the establishment of defined criteria that can guide us into making allies out of technologies currently available.

When relating technology to the Calculus learning process, the result undoubtedly steers toward graphing software, be it for computers or calculators. Nevertheless, the integration of these well known technological resources should not be taken as a guarantee of a better learning process. In our judgement, innovation can only come when the software is implemented correctly and thoughtfully in the curriculum, and when it brings a significant, notable improvement in the teaching and learning process. In this work, we provide some issues to consider when pondering the impact that digital technologies can have when introduced in a visual learning process for Calculus. For this, we have two important factors to consider: the characteristics of the technological environment and the design of didactic activities centered on harnessing these characteristics.

The aim of this work is to describe a didactic experience that uses a kind of software created specifically for the improvement of the Mathematics learning process. We want to share the design of a sequence of activities carried out in the Introduction to College Mathematics course with the help of said software. The conditions of plenty of time at classroom that this course offers made us perform the first experience there. Nevertheless it its worth to say that the

sequence offers a new way to deal with graphs and the evaluation of this constitutes a main goal of an ongoing PhD dissertation in Mathematics Education. It is part of a broad work of educational research that we have been practicing in our institution in order to give elements of discussion about the opportunity to transform curriculum by means of taking advantage of the new technologies that offer great expectation about the way Mathematics could be learned.

At present, some of these changes are available in a Calculus learning process that has been practiced in the current Mathematics for Engineering courses at Tecnologico de Monterrey. The textbook collection edited by Cengage Learning[11], provides an approach to Calculus where the desired interaction promotes the development of visualization, modeling and flexibility between the different mathematic representations. In order to do this, the integration of different specialized mathematical technologies, like graphing software and spreadsheets, has become a key element. The didactic design that textbooks promote, provides a new structure of mathematical knowledge that presents an integration of the concepts of derivative and integral from the very beginning of the Calculus learning process. It uses technology to favor the interaction a student can have with a main problem: the prediction of values of a magnitude that is changing. In order to address this problem, both are key concepts to intervene, empowering Calculus as the study of change and variation in turn [1, 9, 10].

We will now establish some elements to support the academic validity of this work's content, as well as providing reasons to consider the development of educational innovation through the experience shown. After this, we will describe the didactic sequence, illustrating its use in the classroom and considering the establishment of generalizations that can be identified as theoretical results of Calculus. Finally, some general reflections outline the curriculum innovation of the Calculus at College level that has been developed in our educational institution.

Background and rationale

SimCalc MathWorlds® (http://www.kaputcenter.umassd.edu/products/software/), the educational software hereon SimCalc, offers the advantage of direct and dynamic manipulation of Cartesian graphs that are linked to the simulation of motion in a straight line. From its conception, James Kaput (1942-2005) was interested in providing a semiotic perspective for mathematical symbols in education. This distinguished mathematician and educational researcher established a frame for understanding the elements that emerge when analyzing the way Mathematics are taught and learnt in the classroom.

The projects developed with SimCalc (http://www.kaputcenter.umassd.edu/projects/simcalc/) share the common goal of seeking to provide students with the opportunity to access to important ideas concerning change and variation. SimCalc facilitates the design of activities that can be modified to match the researcher's intention. In the present work we seek to share a sequence of activities designed to provide a classroom learning environment that develops the visual skills necessary to graph a function through its derivative's behavior. To this end, it is necessary to work with this software, because it has the Fundamental Theorem of Calculus incorporated into its infrastructure, such that any alteration done in the velocity graph will provide direct feedback in the position graph and vice versa, representing the consequences of the theorem's results. This

executability characteristic makes of SimCalc a *continuous dynamic* software, which provides a benefit in the learning process worthy of being explored[5].

It is easy to recognize the students' cognitive difficulties when transitioning from a graphic representation to an algebraic one, ability widely recognized as key in the understanding of Mathematics. Duval[2] clarifies the notion of representations when contemplating underlying cognitive aspects that can be a source of difficulty in the learning of Mathematics. The representations are also complexly associated signs, produced through the fulfillment of rules as part of a system. Like language, they are tools useful for providing knowledge as a result of operations, and mind cognitive structure organization.

Duval[3] provides a central idea to analyze the cognitive processes involved in mathematical thinking: there are different representation systems that must be coordinated during the mathematical activity. He established two types of possible transformations, identified as *treatment* and *conversion*. The first referring to changes contained in a single type of representation, and the second referring to the ability to change representation type, including the transformation of sentences from natural to numeric, algebraic or graphic language.

We cannot question the importance that a fluid and simultaneous symbol and graphic coordination has for Calculus students, including an understanding of graphic and numeric behavioral patterns. The ability to identify a single representation pattern in different contexts is also desirable. However, educational research has given us a deep understanding of this problem and it is currently accepted that these goals are still far from achievable. We must then understand what activities can be offered to incite cognitive actions effectively, and not just consider that "the most obvious action is to show various registries of possible representations at the same time" [4].

Software is currently a valuable tool because it permits an "instantaneous" display of any solicited representation, making it seem like the problems in the learning process are nonexistent, however, this is the precise time at which we must insist that innovation should be associated with a full reinvention of our perspective, and evaluate the software's utility in Calculus classes through this. When teaching Mathematics, it is important to differentiate between the various representations available, between what is mathematically different and what is mathematically relevant. "This condition is particularly strong when cognitive representations are linguistic or visual, and not only symbolic"[4].

Moreno-Armella, Hegedus and Kaput[5] establish that *generalization* and *symbolization* are two key elements in mathematical reasoning, associated very closely with one another. To generalize, that is, to establish a statement that can be applied to multiple instances, we can try to create a unifying expression, treating all situations as one and the same. But this expression requires a symbolic structure, a way to unify multiplicity; in this sense, symbolism is a supporting element in generalization.

Using technology to trigger the emergence of generalizations leads to consider the possibility of a transformation in mathematical knowledge; Noss and Hoyles[8] have embraced this in their search of new theoretical and methodological tools in order to better understand the available

learning processes linked with technology. They use the term *situated abstraction* to define the act by which a community of students can develop a common discourse and agreement with their professor on the fact that they are all dealing with the same mathematical abstractions. This act gives a certain mathematic legitimacy to the students' expressions, "even if they differ substantially from the traditional mathematical discourse"[8].

Moreno-Armella and Sriraman[7] discuss the necessity to conceive new ways of thinking about the meanings that students develop. They are so familiarized with computational tools that exploration through these resources can eventually lead to a reorganization of strategies to solve problems. Students can first develop observations situated inside the technological environment through exploration; here, the word "situated" expresses the role taken by the environment that includes the computational tool and the activity being carried out. The observations can refer to a certain property or result being expressed through the technological tool, where the environment facilitates its detection. This defines a *situated proof*, the result of a systematic exploration carried out purposefully inside a technological environment with the intention of "proving" mathematical relations.

It is worth mentioning the use that Moreno-Armella and Hegedus[6] give to the term *coaction*, which describes the way in which a user of a dynamic environment guides the actions occurring in it, and is simultaneously guided by the environment in a fluid activity. This perspective can lead to understanding the way in which digital technologies can be effectively included in current educational trends as well as the way in which communication is being transformed in the mathematics classroom.

Didactic sequence design

The didactic experience described hereafter was implemented in a course of Introduction to Mathematics during the Spring semester of 2011. The participating group consisted of 32 students belonging to different majors. They are enrolled in the course to review the mathematical knowledge that is useful to them during their first Mathematics course at College level. A low score in a Math Placement Test leads to enrollment in this course. It is worth mentioning that although the sequence was implemented during that semester, the design of the didactic sequence using the software has been worked on during several previous semesters, and this in order to support the development of the approach to Calculus that was emerging in our educational institution.

In September 2009, our institution formalized a Collaboration Agreement with the Kaput Center for Research and Innovation in STEM Education (http://www.kaputcenter.umassd.edu/news/index.php?path=localglobal), where the software discussed here has been developed; and the contact between the authors of this work and the researchers of the mentioned Center has facilitated work on the design of a special, college level scenario for SimCalc. The resulting sequence's design was worked on during 2010, such that when the experience here described was carried out, there already existed enough information and an established idea about the way in which the software could be beneficially implemented in the course[12].

The classroom that served as a base for this didactic experience is technologically equipped with a computer, projection screen and the software with its special connectivity features, which is preinstalled by students on their own laptop computers. Printed material is also provided in these sessions in the form of worksheets for the students, where they can express their comments and conclusions on the results of the activity and establishment of generalizations. Each student works in his or her own laptop and the connectivity features are used over the classroom's wireless network.

Regarding the design of the sequence, a decision was made to prioritize the graphic representation as a didactic guide towards the development of the *conversion* processes aimed to interpret graphic information and apply it in various real life contexts. Through the activity developed on a motion context, students can establish the relationship between position and velocity graphs, the latter establishing the former's behavior. The interpretation of each of these graphs becomes a useful tool in order to describe in words the effects in the character motion simulation that they represent.

In what follows we describe a path of four didactic activities developed in the classroom, which allow the establishment of *generalizations* on the *qualitative relationships* of the position and velocity graphs; but through an environment that allows a different way of reaching these relations. It must be said that the classroom sessions allowed for plenty of motion oriented scenarios, but in this work we only mention some of them for the purpose of describing the way to reach some generalizations to get established.

Scene One: Motion at a constant velocity.
Making use of the first activity designed in the SimCalc environment, students identified a graph of constant velocity with a horizontal line; the *dragging* feature included in the software, when used on the velocity graph, caused a corresponding response in the position graph (a straight line), a change in its slope according to the students' input. The animated character was used to provide a meaning for the axes involved in the scene, specifically the horizontal one, which becomes "highlighted" by the continuous motion of a faint vertical line in both graphs, signifying the change in time during the simulation. Fig. 1 shows two SimCalc screens representing motion at a constant velocity, the first being positive and the second, negative.
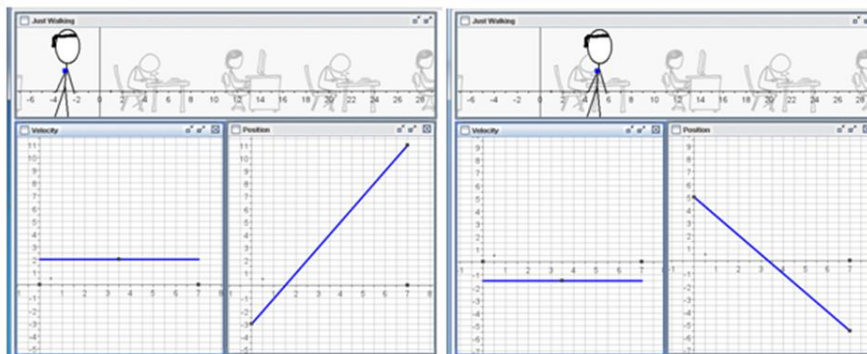

Fig. 1. SimCalc environment showing graphs for constant velocity.

Dragging the horizontal segment representing the velocity graph up and down allows to allocate different constant velocity values. As a result, through guided exploration inside the software environment, we can promote *situated abstraction*, establishing that the character's motion towards the right corresponds with positive velocity and, logically, a graph showing an increasing position. In the same way, motion toward the left is established to correspond with negative velocity and a graph showing a decreasing position.

This type of guided exploration used with the software facilitates and invites the students to move the character to different positions and observe the changes produced in the graphs. It is in this way that the infrastructure and *executability* of the software allows a "proof" of the relationship between different position graphs and a single constant velocity graph, with a change in the initial motion position as the only difference. It is also important to point out that by making the velocity graph take subsequent increasing positive values, students were able to recognize the corresponding changes in the slopes of the position graphs and were then able to interpret that the character's motion to the right becomes successively faster. Finally they should work in the motion to the left successively faster.

Scene Two: Motion at a constant velocity by intervals.
Once motion at a constant velocity, its corresponding position graph and the interpretation of the associated motion became familiar to students, SimCalc allows for the introduction of a motion at constant velocity (initially positive) for different intervals, like a step function. To do this, various horizontal segments are constructed in contiguous intervals for the velocity function. When the position graph was analyzed, it was easy to observe that its increasing behavior corresponded to a positive velocity, which also showed its increasing behavior linked to the upward concavity in the position graph. With the observation of this global display of position and velocity graphs, it was possible to interpret the motion shown: character is moving to the right progressively faster. The next objective was to make students interpret the motion shown through a single graph, be it the one representing velocity or the one representing position.

The activity then goes on with a mixture of different situations to be interpreted by the class: motion to the right progressively slower, motion to the left progressively faster and motion to the left progressively slower. Once the students finished their activity, they made a collective analysis of their work by sending their result images to the shared projection screen through the software's connectivity features. Even when the scenarios provided featured constant velocity step functions, the professor's comments offered the opportunity to understand the necessity of applying this knowledge to processes dealing with infinity.

As a final step in this session, the professor asked the students to think of a linear velocity graph as the limit of step functions, where the time intervals used were every time smaller. Through this, students understood that a step velocity graph can be thought as an almost literal straight line when shortening the size of its steps. As a result, the position graph was then perceived as a curve instead of a polygonal connected by linear segments with different slope values. Fig. 2, shown below, helps to better explain this situation.
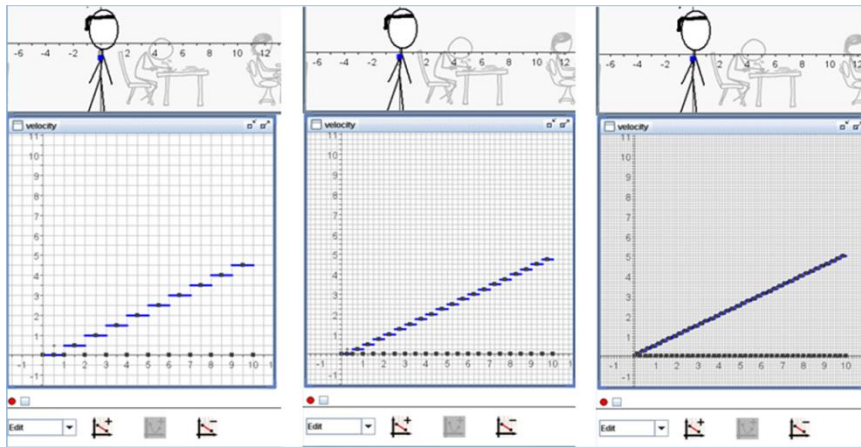
Fig. 2. Sequence showing the SimCalc environment allowing the understanding of the velocity graph as a straight line by shortening the intervals of constant velocity.

Scene Three: Generating four cases with linear velocity.

Once the visual perception of "softness" in the position graph was discussed, the new activity consisted of providing the students with a new SimCalc document, where the character moves with a velocity that varies according to a linear model. A worksheet was handed to the students where they were asked to draw the different scenarios produced inside the SimCalc environment, this allowed an exploration of the SimCalc scenario, where we detected students noticing certain characteristics that they were expressing in natural language and gestures. Now the students had the option to change the slope of the velocity graph through *dragging*, this made the *coaction* between user and software very evident. After this, the software's connectivity feature was again used to gather images produced by the students showing their fulfillment of the objective. Students were then able to understand the different valid solutions available to describe motion, particularly through their observation of the character's motion, after knowing the characteristics that it had to follow. Fig. 3 shows two of these cases, where the "likeness" of the velocity graphs became a source of difficulty when interpreting the differences in motion.
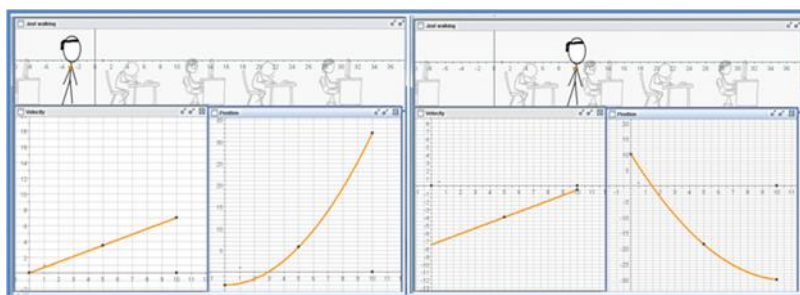

Fig. 3. Graphs representing increasing linear velocity, describing a motion progressively faster to the right and slower to the left.

The new result was the establishment of the relationships between positive or negative values in the velocity graph, corresponding respectively to an increasing or decreasing behavior in position. A second result was the establishment of conditions linked to the increasing or decreasing behavior of the velocity graph in correspondence to the upward or downward concavity for the position graph, respectively. With the support that the SimCalc environment

provides, students interpreted the motion made toward the right or left as progressively faster or slower. After analyzing the four types of motion, it became possible to establish the two conditions of velocity that allow the simple recognition of the types of motion according to the case provided: progressively faster to the right, progressively slower to the right, progressively faster to the left, progressively slower to the left.

Scene Four: Combining the generalizations to allow backtracking. In this scene, students were asked to make the character return to its initial position, which forced students to combine two of the four previous scenarios addressed. To achieve this, they were first required to produce a motion where the character was moving toward the right, but in a way progressively slower until stopping, and then begins moving to the left progressively faster. They also produced the other possible scenario. Through their interaction with SimCalc, they were able to observe the simulation of motion in the character, and understood that when the velocity graph intersects the horizontal axis, it is situated at the exact instant where velocity is zero, corresponding with the time at which the character changes direction in its motion, from towards right to towards left, or vice-versa. This way, the notions of maximum and minimum relative values of a function were established.

Making this connection with the help provided by the simulation allows the establishment of a result: the maximum point in the position graph occurs when it stops increasing and begins decreasing, this relates to the graph of velocity, which shows an intersection with the horizontal axis changing from positive to negative values. Through SimCalc, *dragging* the character horizontally in the motion display generated a vertical shift only in the position graph, fulfilling the requirement of a different initial position for the character; the velocity graph was not modified by the software during this process. The *coaction* inside the environment supported the feedback that any change performed in velocity, without affecting its change of sign from positive to negative, brings a different graph of position always showing a maximum value attained at that same time value where velocity changed sign. Up to this point, the general requirements that must be met in the velocity graph to guarantee the existence of maximums and minimums in the position function were already identified. Fig. 4 helps us explain this.

Fig. 4. Situations that show backtracking made by the character when its velocity is zero.

The didactic experience described in the four previous scenes is shown in the real context of straight line motion offered in the software; however, in our approach for the learning of Calculus, this sequence is then transferred to other real life contexts through a focus of attention

in the visual image of both graphs that have been established already to be tightly linked. The character's position is assigned as the value of certain magnitude that varies with time and, correspondingly, his velocity is identified with the rate of change of such magnitude according to time.

Transference to other contexts

Once the graphic representation of magnitude is associated with the graph of a quadratic function, and the graphic representation of the rate of change of magnitude is associated with the graph of a linear function, it is possible to establish generalizations that signify the presence of relative maximum or minimum values in an element of the function's domain. These are drawn in terms of a zero value in the derivative, and a change of sign in its values, calculated before and after the element in question. Fig. 5 serves to illustrate this situation, which is now part of the approach for the teaching and learning of Calculus that has been discussed[11], where derivative and function (antiderivative), included in a single coordinate system, provide a display used by the students to remember the conditions that represent the *maximum* and *minimum* of the function.
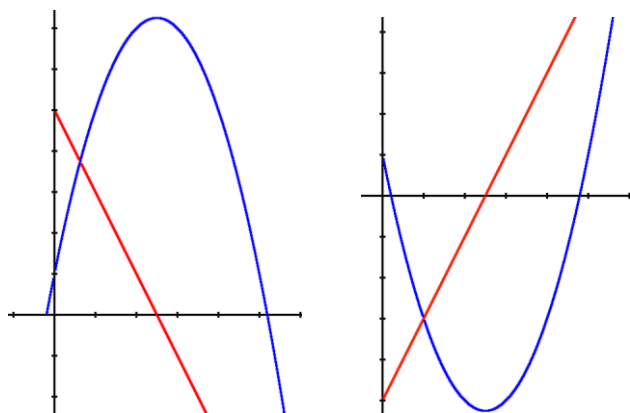
Fig. 5. Representations of the relative maximum and minimum value of the (blue) function, where the derivative (red) is zero and crosses the horizontal axis from positive to negative values or negative to positive values, respectively.

It should be noticed that, although the image correspond to the case of a quadratic function, it can just as well be generalized to a case of *differentiable* functions with a *continuous derivative* function, where the existence of a relative maximum or minimum value is related to the existence of a zero in the derivative function with the corresponding change of sign in the derivative values before and after that zero.

In the newly developed approach to Calculus, graphing vertical parabolas has become an immediate consequence of the interpretation of the image shown in previous Fig. 5. The vertex of the parabola is obtained by equating to zero the derivate of the function and solving the linear equation obtained, this gives the first coordinate, and finally, evaluating the quadratic function in such value the second coordinate is obtained. The coordinates of the vertex and the point of intersection of the parabola with the vertical axis are the only data required to obtain a clear idea of the location of the parabola in the coordinate plane.

This handling of the quadratic function allows us to establish a next step in the generalization of results concerning Calculus; we do this by proposing the following situation, one that now is able to be solved without the software. The goal is to fill a water tank, where the water level is modeled through the cubic function $h(t)=15+2t-2t^2+t^3$. The request is to interpret the behavior of the water level. Does it increase always or sometimes? Does it decrease? Is the increasing or decreasing behavior done progressively faster or progressively slower?

The graph of the function is not included in the initial statement; certainly it can be easily plotted by the students using different software; however, it is noticeable that there is a lack of clarity in the behavior of the water level with the first input done with software. Fig. 6 offers an image that has been previously manipulated by graphing software in order to display an adequate representation to visualize its behavior; the inclusion of the graph of the derivative allows for the correct interpretation of the water level's behavior.

Fig. 6. The water level in a tank is modeled with a cubic function (blue graph) and its derivative is a quadratic function, a parabola (red graph).

It should be mentioned that the presence of a negative sign in the algebraic representation of the cubic function $h(t)=15+2t-2t^2+t^3$ leads some students to conclude that the water level must decrease and also increase, this without having the visual representation of the graph available. However, analyzing the behavior of the derivative $h'(t)=2-4t+3t^2$ we can verify that it maintains a positive sign, which confirms the always increasing behavior of the water level.

On the other hand, the way the water level grows is different; initially it increases progressively slower, and without stopping, at time $t = \frac{2}{3}$ continues its increasing behavior, but now progressively faster, until the tank fills up. That value for time is obtained by getting the derivative of the water level function, and equating it to zero; this in order to find the vertex of the parabola.

Solving this problem leads to the establishment of the requirement for the existence of an *inflection point* in the graph of a function, now representing the water level in the tank. Inflection point reveals the existence of a minimum value for the graph of the derivative function in this case. With this scenario, the class can ponder the different possibilities for an inflection point, where the graph of the function shows a change of concavity. And linking the behavior of the function graph with the behavior of the derivative graph (at least in differentiable functions with a continuous derivative) we can relate the inflection point in the function with the relative maximum or minimum value of its derivative function.

Results and Discussion

The present work is not intended to summarize partial results of the broad ongoing research for which the didactic sequence presented takes part. The aim of the work has been to share the way technology could be embedded into Mathematics allowing a visual display that could support students' thoughts and reasoning dealing with graphs. In this sense, the didactic experience may be considered useful in various ways. Among them it is worth stressing the accessibility that SimCalc software allows in the presentation, identification and general establishment of the relationships between a graph of a magnitude (function) and the graph of its rate of change (derivative). The didactic design of the SimCalc document enables emerge what can be identified as *situated proofs*[7] within the real context of straight line motion. The motion scenario sequence prepared in SimCalc supports the emergence of these generalizations and their symbolization throughout the students' interaction and experience with the software and the professor's didactic intentions.

The didactic sequence we presented in this work offers a visual way through which important elements of Calculus are readily understandable. Being able to establish these relationships between the derivative's sign and the increase/decrease behavior of the function, and between the increase/decrease behavior of the derivative and the upward/downward concavity of the function at an early stage of the course, allows us to work with a graphic representation of functions and use it as a visual environment for the introduction to Calculus. Therefore, the graphic representation of functions was established as the stage in which students would interpret the global behavior of a changing magnitude. The simultaneous visualization of the graph of the magnitude and of its rate of change bring those relationship elements between them that become an important part of the analysis.

At the same time, with the support of the motion simulation, it was possible to confront some common cognitive conflicts associated with the graphic representation of a function, particularly, the lack of a dynamic view of the tracing process in curves. It also offers an adequate environment to discuss the differences between natural and scientific language (as is the case of

velocity and speed) and to give a proper meaning to the presence of negative numbers in this motion context.

The inclusion of the public screen has the intention of providing a place where students may share their contributions and discuss and ponder the different solution alternatives and the important differences that may or may not be valid to satisfy the activity tasks. In the same way, this resource becomes important in the undertaking of true mathematical discussions with everyone's participation; the motion simulation then allows the validation or refutation of the proposals given in this discussion.

Finally, this kind of experience using software in class has led to the opportunity to gauge the students' abilities when working with electronic documents, which brings an additional element of design and creativity in the assignments they carry out. This, in Mathematics courses, represents an achievement that is worth being explored in the service of innovation.

Final reflections

Visual learners require visual activities that attract their interest and with which they can interact. The SimCalc software offers the opportunity of fulfilling both requests and additionally helps to foster visualization in the Calculus learning process. This type of digital environment puts content in the hands of students, with which they can interact using Calculus concepts and develop tools to use in the study of change.

By using SimCalc in the design of the activities discussed in this work, and through the experience gained by carrying them out with students in the classroom, we can develop content organized in such a way that important ideas behind the construction of the derivative and integral may be worked with intuition, giving the students confidence in their knowledge when sharing results with the class. This context is accompanied by meaning, and within it, we might establish Calculus theorems that in a rethought curriculum could be identified as the subject: *qualitative relations between a function and its derivative*.

This new way to deal with graphs, as a common tool in mathematical reasoning, establishes the classroom as an ideal stage in which students can produce thought processes where conjectures emerge, hypotheses are confronted and conclusions are supported. A visual learning process becomes possible through the use of technology with didactic goals. It is worth mentioning that in conventional teaching, the study of algebraic strategies has been the conventional way to access the graphing of functions at the end of a Calculus course; instead, through the didactic sequence described and operated in SimCalc, it is possible to establish a function's graph as a visual answer to represent the behavior of a magnitude that is changing.

The experience gained when putting together this sequence allows us to truly observe the limits of conventional Calculus, undoubtedly, the inclusion of digital resources must lead to an understanding of the effects they have on the content being taught and learned. Our point of view is not to see the same content in courses with the added help of technology, but to open the possibility of allowing the thoughtful use of technology to add innovation into the curriculum. In this sense, the approach to Calculus described in this work attempts to offer innovation, putting a

new perspective of mathematical knowledge in the hands of students, where digital software allows dynamic interaction and promotes numeric, algebraic and graphical visualizations at a time.

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