122nd ASEE Annual Conference & Exposition
June 14 - 17, 2015
Seattle, WA
Seattle
Making Value for Society
Paper ID #14113

Integration of Mathematics for Sustainable Energy Applications

Dr. Seunghyun Chun, California Baptist University

Dr. Chun is currently an Assistant Professor at the California Baptist University, College of Engineering Electrical and Computer Engineering department. His area of interest is in power electronics, smartgrid and engineering education.

Integration of Mathematics for Sustainable Energy Applications

Abstract

In this paper an example of a method to present a basic numerical analysis method's such as the Secant Method, Bisection method and the Regula Falsi Method is described in the way it is used in sustainable energy application.

A solar panel is examined and students are provided with its P-V characteristic curve. The arbitrary function f(x), that was the target of finding the root for in a numerical analysis textbook, is no longer a function without any significance (Fig. 3). It becomes a derivative of the P-V characteristic curve which has a root that corresponds to the maximum power point for efficient power extraction of the solar panel. This can be applied to wind energy, fuel cells and so on.

Introduction

The need for strong science, technology, engineering, and math (STEM) workforce is essential and critical in advancing the economy and society of the future. But the U.S continues to trail the world in math and science. And also the number of U.S students pursuing a STEM career or educating is decreasing as mentioned in [1] – [3]. A change in the way math is taught and presented in the classroom is urgently needed. Instructors need to be able to engage the students in learning by communicating that the study of mathematics and its objective is not to study math for math sake but to be able to apply it as a tool to solve the world's complex and essential problems.

The topic of sustainable energy is no longer a topic reserved for scientists and engineers. It is the topic of discussion in many different areas and also age groups. From kindergarten to high school, students have been introduce to at least one sustainable energy source, such as solar energy or wind energy and the importance of energy efficiency have been communicated to them. By taking this area of common exposure and integrating it into the way numerical analysis or root finding method is introduced, the students interest in the material and understanding of it increases. This not only provides the student with a better grasp of the course content but it bridges the gap between the students preconception of math as being purely theoretical and real world engineering problems.

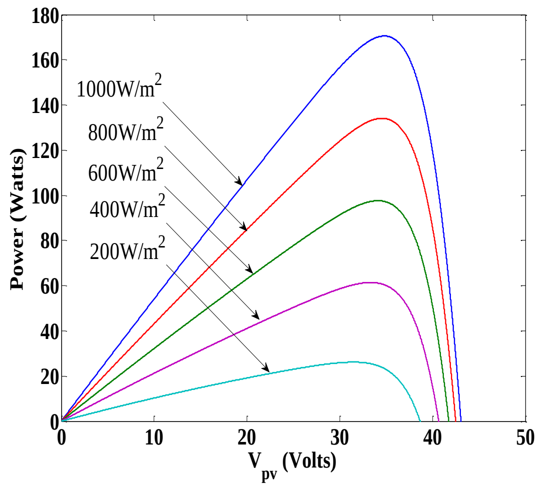

Figure 1: Irradiance effect on P-V

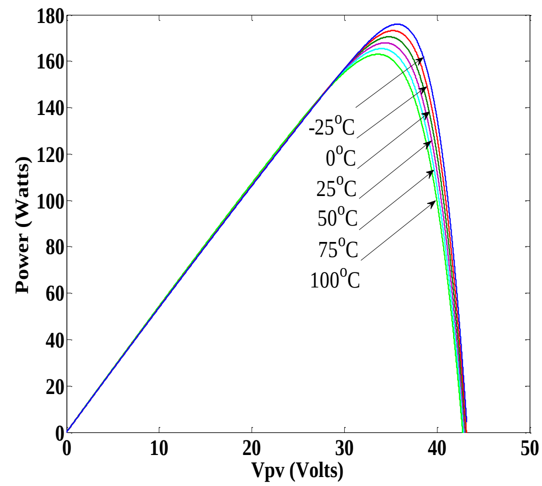

Figure 2: Temperature Effect on P-V Characteristic at constant irradiance (1000W/m^2)

Maximum Power Point Tracking for Sustainable Energy Sources

Most sustainable energy sources used today to generate electricity have an I-V characteristic that is characterized as non-linear. This characteristic is also continuously changing due to the dynamic condition the source is in. In order to extract maximum power being generated at an instance in time an interface between the load and the source is required that can run a smart algorithm that continuously tracks the operating point that produces maximum power. This is called Maximum Power Point Tracking (MPPT) and the algorithm run to do this is called the MPPT algorithm. This can be applied to any energy source that has a non linear characteristic. For this paper our discussion will be focused on a photovoltaic (PV) source, but can be extended to a wind turbine, fuel cells etc.

A. Characteristic of a Photovoltaic Source

When a photovoltaic panel is exposed to sunlight due to photovoltaic effects electrical energy is generated from the panel. The amount of electricity is dependent on the dynamic conditions of the environment and the two major factors are irradiance and temperature. As shown in Figure 1 and 2, irradiance and temperature changes the P-V characteristic of the PV panel but at every instance a maximum power point (MPP) exists. In order to extract maximum power from the PV panel at any moment in time the solar power system needs to always operate at the MPP in order to be efficient.

Once the student can identify that the characteristic curve of a PV panel is essentially a mathematical function and that the MPP or the optimum operating point is the root of the derivative of the characteristic curve, students studying root finding algorithms can apply the concepts learned in the classroom to a simple setup that is widely used in industry. The study of root finding algorithms will no longer become a dry experience of calculating the approximated root iteratively. Each new approximated value of the root will change the operating point of the energy system and will directly tie in with the efficiency of the overall system. This creates an ultimate goal for the students to want to achieve and makes learning more interesting and fun.

Many different algorithms to do MPPT have been developed but in this paper a brief introduction of root finding algorithm as MPPT algorithms have been presented and how they can be presented in a root finding algorithm or numerical analysis class.

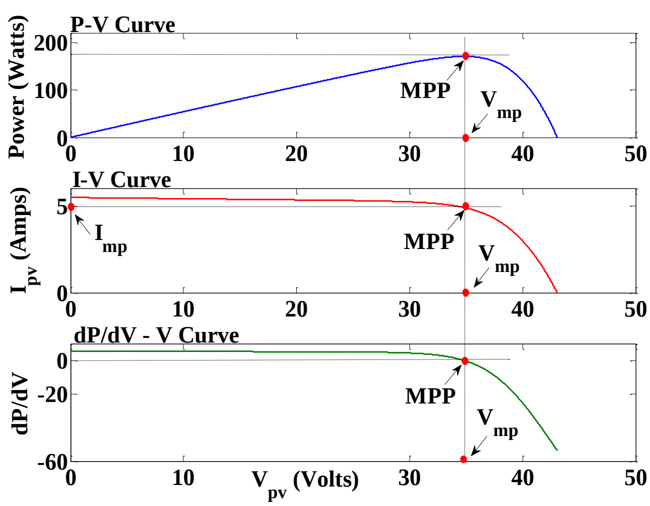

Figure 3: Maximum Power Point for different curves of a PV module.

B. Maximum Power Point Tracking Algorithms

There are many different kinds of MPPT algorithms ranging from very simple ones such as the Perturb and Observe (P&O) to very complicated algorithms. But the majority of these algorithms rely on the fact that at the MPP the derivative value of the power respective to the voltage equals zero ($\frac{dP}{dV} = 0$) . The operating voltage of the solar system is altered so that it is at the point where $\frac{dP}{dV} = 0$ and this is continuously monitored by a microcontroller with a feedback circuitry.

Another way of looking at this algorithm is to consider the $\frac{dP}{dV} = 0$ as a function f(x) = 0 where x is the root, corresponding to the operating voltage that will be at the MPP. This enables the application of root finding algorithms to MPPT algorithms. This is illustrated in Figure 3 on the final plot. The dP/dV – V curve will be the curve for which the root finding algorithm will be applied.

There are many different root finding algorithms, for this paper the secant method, bisection method and the regula falsi method will be applied as MPPT algorithms.

1) Secant Method(SM)

The SM is an iterative algorithm, in which the fundamental equation is given by,

$$x_{n+1} = x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \quad n = 0,1,2\cdots$$

For the SM, x_n+1 is the root for a secant line $h_n(x)$ to the function $f(x)$ at the point x_n such that $h_n(x_n+1) = 0$ with

$$h_n(x) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} \cdot (x - x_n) + f(x_n) \quad n = 0,1,2\cdots$$

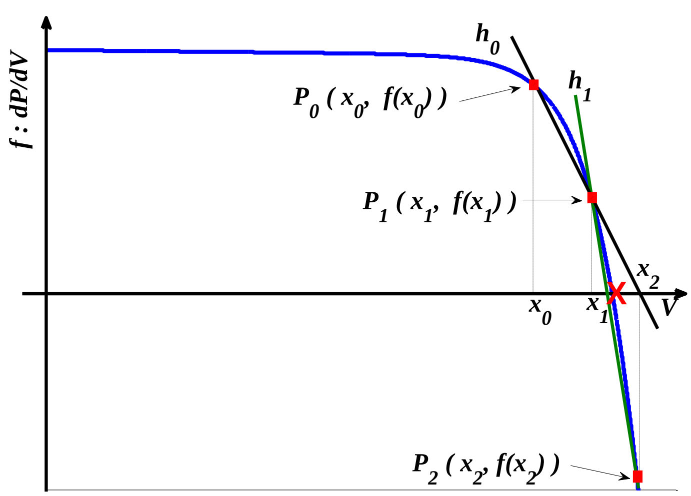

Figure 4: Secant Method (SM)

Figure 4 illustrates the SM applied as an MPPT algorithm on a solar panel.

2) Bisection Method (BSM)

The BSM algorithm, represented in Figure 5, can be summarized in the following steps:

(i) Given a well-defined function f(x), choose a lower value x_l and an upper value x_u. These two points define an interval $[x_l, x_u]$ that must include the root x* of f(x). That is, f(x) has opposing signs in x_l and x_u, e.g. $f(x_l)f(x_u) < 0$.

(ii) Approximate the root to the midpoint x_m of the interval $[x_l, x_u]$. That is

$$x_m = \frac{x_u + x_l}{2}$$

(iii) If $f(x_l)f(x_m) < 0$ then set $x_u = x_m$ and repeat the previous step. If $f(x_l)f(x_m) > 0$ then set $x_l = x_m$ and repeat the previous step. If $| f(x_m) | \leq \varepsilon$ (where ε is the tolerance) then take x_m as the root or approximation.

The BSM convergence rate is slower than the SM. Yet, with the BSM root convergence is guaranteed.

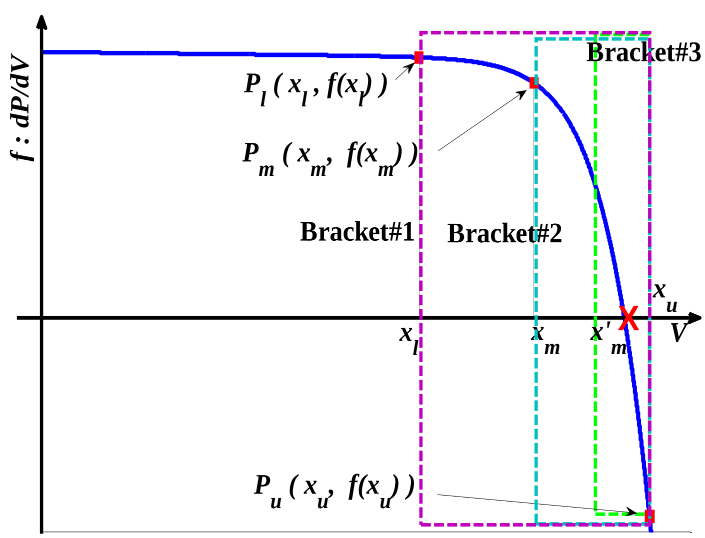

Figure 5 : Bisection Method(BSM)

Figure 5 illustrates the BSM applied as an MPPT algorithm on a solar panel.

3) Regula Falsi Method (RFM)

The RFM is a linearly convergent root finding algorithm for continuous functions with one independent variable. It is a hybrid of the bisection search theorem (BST) and the secant method. A value c_i, is derived from

$$c_i = \frac{x_l \cdot f(x_u) - x_u \cdot f(x_l)}{f(x_u) - f(x_l)}, \quad i = 0,1,\cdots \qquad (1)$$

This found value c_i is then used to substitute the mid-point of each interval $[x_l, x_u]$ as the root approximation used in the BST method. This process is described in both the next algorithm and through Figure 6.

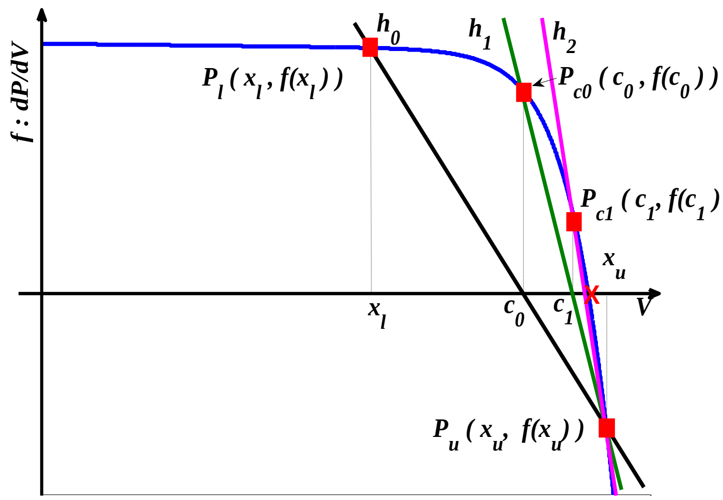

Figure 6: Regula Falsi (RFM)

The RFM algorithm, represented in Figure 6, can be summarized in the following steps:

(i) Given a continuous function f(x) find initial points x_l and x_u, such that $x_l \neq x_u$ and $f(x_l)f(x_u) < 0$. Hence, according to the intermediate value theorem the root of f(x) is located inside the interval $[x_l, x_u]$.

(ii) Calculate the approximate value for the root c_i with (1)

(iii) If $|f(c_i)| \leq \varepsilon$ (where ε is the tolerance) then it is considered that the root have been reached and that c_i is the root. Else, if $f(c_i) \cdot f(x_u) < 0$ then let $x_l = c_i$, else if $f(c_i) \cdot f(x_l) < 0$ then let $x_u = c_i$. These changes yield a smaller interval.

(iv) Iterate steps (ii) and (iii) until the root is reached.

Figure 6 illustrates the RFM applied as an MPPT algorithm on a solar panel.

C. Simulation Results

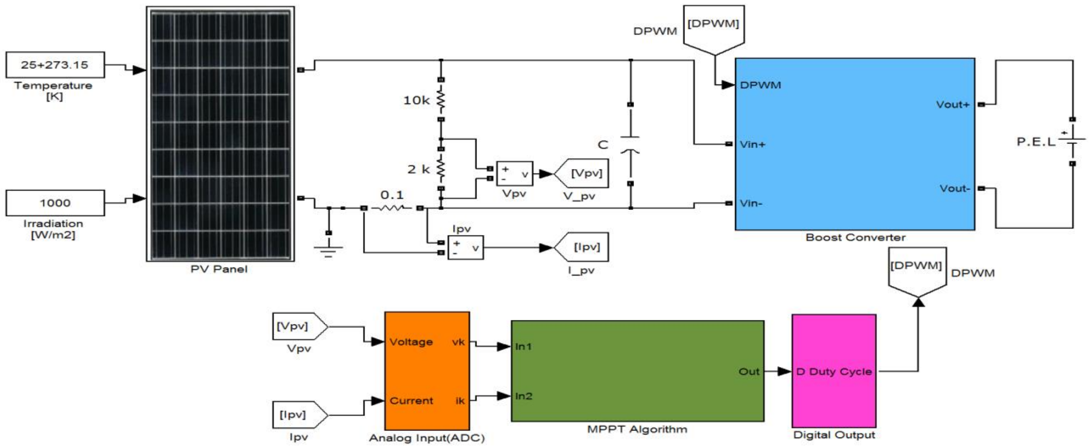

Figure 7: MPPT MATLAB Simulation Setup

Figure 7 shows the MATLAB setup used to implement root finding algorithms as a MPPT algorithms. Students can implement different root finding algorithm as an M-file and paste it into the MPPT Algorithm section in Figure 7. Without any configuration of the simulation setup, just by switching out the algorithms, students can explore the performance difference of multiple root finding algorithms, even ones not explored in this paper and come up with more better and creative algorithms.

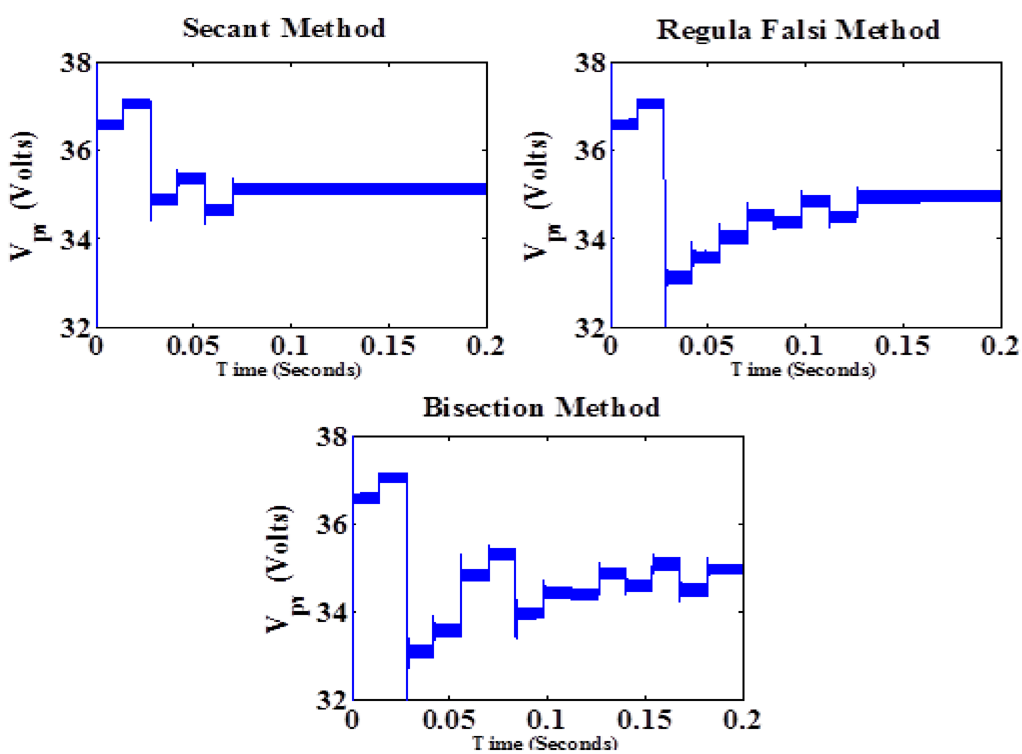

Figure 8: Simulation results

Figure 8 shows the simulation results of the three root finding algorithms applied as MPPT algorithms.

Conclusion

By introducing mathematic in a well-known real world application helps the future STEM students understand math as a tool for innovation and supports the depth of leaning new mathematical concept such as root finding algorithms explored in this paper. Also by having an already configured setup where students can change out different algorithms will help students to focus on the mathematical concept being explored and will enhance their learning experience. In this paper a simulation approach was given but an actual experimental setup can be built to demonstrate the concept. This will enhance the student's interest even more but need to be done under supervision due to high currents and voltages.

Reference

[1] Sterba, J., "Fewer Kids Pursue Science, Engineering," *Arizona Daily Star*, **163**, (355), 1-11, (2004)

[2] American Association of Engineering Societies, *Engineering Societies Diversity Summit Report*, www.aaes.org, (2003).

[3] American Society of Engineering Educators, "Bachelor's Breakdown," *Prism,* 14(1), 20, (2004)

[4] Chun, Seunghyun, 2011**,** *Analysis of Classical Root-Finding Methods Applied to Digital Maximum Power Point Tracking for Photovoltaic Energy Generation*, Ph.D dissertation, The University of Texas, Austin