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Learning about Blood through a Property Data Base Project

Introduction
An understanding of the physical nature of blood is critical in a biomedical engineering program. For programs with a mechanical engineering orientation, knowing and understanding the thermophysical properties of blood is essential in modeling the operations of such biomedical devices as artificial heart valves, blood pumps, left ventricular assist devices, and artificial hearts. To build this knowledge base with mechanical engineering students, a project was developed and implemented in a senior level elective course in thermal design. The project involved the students building an Excel spreadsheet calculator for blood properties. With the user specifying blood hematocrit, temperature, and pressure, the spreadsheet will calculate values for 13 different thermophysical properties, which include thermodynamic properties such as enthalpy and entropy and transport properties such as thermal conductivity and kinematic viscosity.

Since little data exists in the form needed for the project, a portion of this paper will focus on the constitutive equations that were developed by the authors for the project. The models used in the development of functional forms of the properties dependent on hematocrit, temperature, and pressure are explored. The paper provides the details of the project assignments, including lecture material. Student feedback on this project was collected and will be shared in the paper.

Project Description
The blood project serves as the first of 5 projects in a senior level thermal design course. When a biomedical theme is used in the course, subsequent projects use the blood property data base developed for the modeling of cardiopulmonary bypass systems and hemodialysis machines. The course in which this project is used has an extensive set of course learning objectives. Those that are pertinent to the blood project are provided below.

Students are able to determine thermodynamic properties using mathematical models
Students are able to represent design data in terms of curve fits
Students are able to develop a computerized property data base
Students are be able to program in Excel
Students are able to graph in Excel

For the blood project the student is asked to develop an Excel spreadsheet that will allow the calculation of a number of human blood properties at specified hematocrit (over a range from 0 to 0.60), temperature (over a range from 280 K to 330 K), and pressure (over a range from 80 kPa to 150 kPa). The set of thermodynamic properties to be calculated include yield stress, specific volume, density, specific heat, internal energy, enthalpy, entropy, and the thermal expansion coefficient. For the thermodynamic properties constitutive equations are provided to the students (shown in Figure 1). The students are directed to use the symbolic manipulator associated with MATLAB to evaluate thermodynamic properties that require integration or differentiation. The
following transport properties are to be included in the spreadsheet: thermal conductivity, thermal diffusivity, dynamic viscosity; kinematic viscosity, and Prandtl number. Tabular data are provided for the transport properties (see Table 1). The students are directed to utilize a curve fit for one transport property and a table look-up for a second transport property. The other three transport properties can then be evaluated through the three relationships among the transport properties shown below.

\[ \nu = \frac{\mu}{\rho} \quad \alpha = \frac{k}{\rho \cdot C_p} \quad Pr = \frac{\nu}{\alpha} \quad (1) \]

Though this is an extensive set of properties, engineers designing cardiopulmonary bypass systems, hemodialysis machines, and blood preservation systems will need values for these properties.
Figure 1 Constitutive Equations for Human Blood

Yield Stress

$$\tau_Y = (0.0008)(\phi - 0.05)^3 \text{ in N/m}^2$$

Hematocrit ($\phi$) in decimal form

Density

$$\rho = \left[1.06\phi + 1.03(1 - \phi)\right] \left[\frac{1 + d\delta^{1/3} + e\delta}{3.1975 + a\delta^{1/3} + b\delta + c\delta^4}\right] \text{ in g/cm}^3$$

where

$$\delta = 647.27 - T$$
$$a = -0.3151548, b = -1.2003374 \times 10^{-3}, c = 7.48908 \times 10^{-13}$$
$$d = 0.1342463, e = -3.946263 \times 10^{-3}$$

Temperature ($T$) in Kelvins

Specific Volume

$$v = \frac{1}{\rho} \text{ in m}^3/\text{kg}$$

Specific Heat

$$c_p = 0.0571 + (3.7234)\phi + (1-1.02\phi)(0.93)(a+bT+cT^2+dT^3+eT^4) \text{ in kJ/(kg \cdot K)}$$

where

$$a = 32.256, b = -0.29368, c = 1.1372 \times 10^{-3}$$
$$d = -1.93494 \times 10^{-6}, e = 1.2369 \times 10^{-9}$$

Internal Energy

$$\hat{u} - \hat{u}_0 = \int_{T_o}^{T} c_p(\phi, T')dT' \text{ in kJ/kg}$$

Base State Temperature and Internal Energy ($T_o$ and $\hat{u}_0$)

Enthalpy

$$\hat{h} - \hat{h}_0 = \int_{T_o}^{T} c_p(\phi, T')dT' + \int_{P_o}^{P} v(\phi, T)dP' \text{ in kJ/kg}$$

Entropy

$$\hat{s} - \hat{s}_0 = \int_{T_o}^{T} \frac{c_p(\phi, T')}{T'}dT' \text{ in kJ/(kg \cdot K)}$$

Thermal Expansion Coefficient

$$\beta = -\frac{1}{\rho(\phi, T)} \left(\frac{\partial \rho(\phi, T)}{\partial T}\right)_P \text{ in K}^{-1}$$
### Table 1 Transport Properties for Blood at 310 K and 101 kPa

<table>
<thead>
<tr>
<th>Φ</th>
<th>μ (Nt s/m²)</th>
<th>ν (m²/s)</th>
<th>k (W/m K)</th>
<th>α (m²/s)</th>
<th>Pr</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>8.3040E-04</td>
<td>8.3603E-07</td>
<td>0.2907</td>
<td>7.2500E-08</td>
<td>11.53</td>
</tr>
<tr>
<td>0.06</td>
<td>9.7694E-04</td>
<td>9.8180E-07</td>
<td>0.3420</td>
<td>8.5568E-08</td>
<td>11.47</td>
</tr>
<tr>
<td>0.12</td>
<td>1.0742E-03</td>
<td>1.0776E-06</td>
<td>0.3760</td>
<td>9.4391E-08</td>
<td>11.42</td>
</tr>
<tr>
<td>0.18</td>
<td>1.1836E-03</td>
<td>1.1852E-06</td>
<td>0.4143</td>
<td>1.0435E-07</td>
<td>11.36</td>
</tr>
<tr>
<td>0.24</td>
<td>1.2942E-03</td>
<td>1.2936E-06</td>
<td>0.4531</td>
<td>1.1447E-07</td>
<td>11.30</td>
</tr>
<tr>
<td>0.30</td>
<td>1.4158E-03</td>
<td>1.4126E-06</td>
<td>0.4956</td>
<td>1.2564E-07</td>
<td>11.24</td>
</tr>
<tr>
<td>0.36</td>
<td>1.5615E-03</td>
<td>1.5553E-06</td>
<td>0.5466</td>
<td>1.3904E-07</td>
<td>11.19</td>
</tr>
<tr>
<td>0.44</td>
<td>1.8299E-03</td>
<td>1.8183E-06</td>
<td>0.6406</td>
<td>1.6368E-07</td>
<td>11.11</td>
</tr>
<tr>
<td>0.48</td>
<td>2.0210E-03</td>
<td>2.0059E-06</td>
<td>0.7075</td>
<td>1.8119E-07</td>
<td>11.07</td>
</tr>
<tr>
<td>0.54</td>
<td>2.4451E-03</td>
<td>2.4224E-06</td>
<td>0.8560</td>
<td>2.1996E-07</td>
<td>11.01</td>
</tr>
<tr>
<td>0.60</td>
<td>3.2184E-03</td>
<td>3.1830E-06</td>
<td>1.1267</td>
<td>2.9053E-07</td>
<td>10.96</td>
</tr>
</tbody>
</table>

The source of these equations and data will be addressed in a later section.

To assist the students and ease the burden of grading, a template is provided, which is shown in Figure 2.

**Figure 2. Excel Template for Blood Property Project**

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Student Values</th>
<th>Course Values</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield Stress</td>
<td>N/m²</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
</tr>
<tr>
<td>Density</td>
<td>kg/m³</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
</tr>
<tr>
<td>Specific Volume</td>
<td>m³/kg</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
</tr>
<tr>
<td>Specific Heat</td>
<td>kJ/(kg·K)</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
</tr>
<tr>
<td>Internal Energy</td>
<td>kJ/kg</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
</tr>
<tr>
<td>Enthalpy</td>
<td>kJ/kg</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
</tr>
<tr>
<td>Entropy</td>
<td>kJ/(kg·K)</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
</tr>
<tr>
<td>Thermal Expansion</td>
<td>1/K</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>W/(m·K)</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
</tr>
<tr>
<td>Thermal Diffusivity</td>
<td>m²/s</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
</tr>
<tr>
<td>Dynamic Viscosity</td>
<td>N·s/m²</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
</tr>
<tr>
<td>Kinematic Viscosity</td>
<td>m²/s</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
</tr>
<tr>
<td>Prandtl Number</td>
<td>NONE</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
<td>#DIV/0!</td>
</tr>
</tbody>
</table>

Dark shaded cells will be user input. Values for light shaded cells will be inputted during testing by the course instructor.
Once the spreadsheet is completed the student is required to assess its correctness through three comparison studies.

1. Quantitative comparison with transport property data provided in project statement.
2. Quantitative comparison with blood properties found in the literature.
3. Quantitative comparison of thermodynamic properties with those of water.

A technical memo is required to be submitted that documents the student’s work. Additionally, the student must submit his/her spreadsheet that is tested against the instructor’s spreadsheet. Grading is done following the rubric shown in Figure 3.

**Figure 3 Grading Rubric for the Project**

**Project Grade Evaluation**
**Project #1 Computerized Data Base for Blood Properties**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Assigned Score</th>
<th>Maximum Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>User’s Manual</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Equation Fit of Transport Properties</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>Base State of Thermodynamic Properties</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>MATLAB Integration</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Comparison with Source Data</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Table Look-up</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Test</td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>Quality</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

**Modeling of Blood Properties**
During the development of the project, it became clear that there was limited data and equations associated with the properties of human blood. In fact, it surprised the authors that with the limited data available such devices as cardiopulmonary bypass machines and hemodialysis machines could be effectively designed. Since the general format of the project was set to be consistent with similar projects used, it was necessary for the authors to develop their own models for blood properties. As will be shown, this often required some creative judgments to be made and, in no way, should this model development be viewed as a scientific endeavor, but rather an engineering one. Note that
the use of these models will be first educational and second in the engineering design of biomedical devices. As such, approximate values and correct trends are the goals, rather than exact values.

The first property to be considered is the yield stress, which defines the stress at which blood will flow. For hematocrit greater than 0.8%, the yield stress is found to be given by [1]

$$\tau_0 = A(\phi - \phi_m)^3 \quad (2)$$

where

$$A = 0.0008 \text{ Nt/m}^2$$
$$\phi = \text{blood hematocrit}$$
$$\phi_m = \text{hematocrit below which there is no yield stress} (\approx 0.05)$$

Next the density is considered. Assuming that blood is composed of plasma and erythrocytes, a standard mixture rule can be used to write

$$\rho_b = \rho_H \phi + \rho_p (1-\phi) \quad (3)$$

The following subscript notation is being used.

- **b**: blood
- **H**: hematocrit
- **p**: plasma
- **w**: water

To determine the density of the erythrocytes and plasma, the specific gravity, \( \gamma \), is used. Then we can write

$$\rho_p = \gamma_p \rho_w \quad (4)$$
$$\rho_H = \gamma_H \rho_w \quad (5)$$

From [1] we find \( \gamma_p = 1.03 \) and \( \gamma_H = 1.06 \). Thus, it is now possible to return to Eqn. (3) and write

$$\rho_b = 1.06 \phi \rho_w + (1-\phi)(1.03)\rho_w \quad (6)$$

The density of water can be determined from the equation for the specific volume of saturated liquid water from Keenan and Keyes [2], so that

$$\rho_w = 1/v_{sat} \quad (7)$$
where

\[ v_{\text{sat}} = \frac{v_{\text{crit}} + a \theta^{1/3} + b \theta + c \theta^4}{1 + d \theta^{1/3} + e \theta} \text{ in } \text{cm}^3 / \text{g} \]  

(8)

with

\[ v_{\text{crit}} = 3.1975 \text{ cm}^3 / \text{g} \]

\[ \theta = 647.27 - T \]

T: temperature in Kelvins

\[ a = -0.3151548 \quad b = -1.203374 \times 10^{-3} \quad c = 7.48908 \times 10^{-13} \]

\[ d = 0.1342463 \quad e = -3.946263 \times 10^{-3} \]

The combination of Eqns. (6), (7), and (8) provide us with a mathematical model for the density of blood as a function of hematocrit and temperature.

The key remaining thermodynamic property is the specific heat, \( c_p \). Using a mixture rule the specific heat may be defined as follows

\[ c_{p,b} = Z c_{p,H} + (1 - Z)c_{p,p} \]

(9)

where Z is the mass percent of erythrocytes in the blood. The mass percent of erythrocytes can be related to the hematocrit by

\[ Z = (\rho_H/\rho_b) \phi \]

(10)

Once again using standard specific gravities, we can write

\[ Z = (1.06/1.04) \phi = 1.02 \phi \]

(11)

To determine the specific heat of plasma, we begin by considering the composition of plasma as shown in Table 2.

**Table 2 Plasma Composition [1]**

<table>
<thead>
<tr>
<th>Component</th>
<th>Weight Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.91</td>
</tr>
<tr>
<td>Proteins</td>
<td>0.07</td>
</tr>
<tr>
<td>Inorganic Solutes</td>
<td>0.01</td>
</tr>
<tr>
<td>Other Organic Substances</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Thus, using the above table in conjunction with a mixture rule an equation for \( c_{p,\text{plasma}} \) can be written

\[ c_{p,p} = (0.93)c_{p,w} + (0.07)c_{p,\text{protein}} \]

(12)
The specific heat of water is given by [3]

\[ c_{P,w} = 32.256 - 0.29368T + (1.1372 \times 10^{-3})T^2 - (1.9394 \times 10^{-6})T^3 + (1.2369 \times 10^{-9})T^4 \]  

(13)

with units of kJ/(kg·K) for temperature (T) in Kelvins.

The specific heat of plasma is not readily available, which forces us to be somewhat creative. Consider a substance, chicken, that is composed of 74% water and 26% protein. Then the mixture rule can be used to write

\[ c_{p,chicken} = (0.74)c_{p,w} + (0.26)c_{p,protein} \]  

(14)

At 310 K, the specific heat of chicken is given as 3320 kJ/(kg·K). Using Eqn. (13), the specific heat of water is found to be 4279.35 J/kg K. Substituting into Eqn. (14) and solving gives

\[ 3320 = (0.74)(4200) + (0.26)c_{p,protein} \]  

(15.1)

\[ c_{p,protein} = 815 \text{ J/(kg K)} \]  

(15.2)

Neglecting the temperature dependence of the specific heat of protein, a final expression for the \( c_{p,p} \) is shown below

\[ c_{p,p} = (0.93)c_{p,w} + (0.07)(815) \]  

(16)

The specific heat of blood can now be written

\[ c_{p,b} = 1.02 \phi c_{p,H} + 0.02 \phi \{0.93c_{p,w} + 57.06\} \]  

(17)

To determine a value for the specific heat of erythrocytes, we follow a similar procedure as for the specific heat of plasma. From [6] we find a value for \( c_{p,blood} \), of 3889 J/(kg K) at a temperature of 310 K and hematocrit of 0.44. Solving Eqn. (17) gives

\[ c_{p,erythrocytes} = 3707.4 \text{ J/(kg K)} \]  

(18)

Then a final constitutive equation for the specific heat of blood can be written

\[ c_{p,b} = 3781.2 \phi + 0.02 \phi \{0.93c_{p,w} + 57.06\} \]  

(19)

To determine the transport properties as a function of temperature and hematocrit, we first recognize that with the use of Eqn. (1), only two properties need to be expressed as independent equations. In developing an equation for the dynamics viscosity, \( \mu \), it was assumed that a Newtonian fluid model would be appropriate. This assumption can be shown to be valid for vessels whose diameters exceed 100 µm [4]. Using Einstein’s equation for spheres in suspension, we can write
\[ \mu_b = \mu_p/(1-\alpha\phi) \] (20)

where \( \alpha \) is the shape factor and can be determined from

\[ \alpha = 0.076 \exp[2.24\phi + (1107/T)e^{-1.69\phi}] \] (21)

In order to define the viscosity of plasma is approximated to be

\[ \mu_p = 1.2\ \mu_{\text{water}} \] (22)

Then Eqns. (20), (21), (22), and a temperature table for the dynamic viscosity can be used to generate values of the dynamic viscosity of blood as a function of temperature and hematocrit.

Thermal conductivity was chosen as the other transport property to be represented by an independent equation. Using the analogy between momentum transfer and heat transfer we can write

\[ k_b = k_p/(1-\alpha\phi) \] (23)

To determine the thermal conductivity of plasma, the literature value of 0.642 W/(m K) at a hematocrit of 0.44 is used in Eqn. (23) to determine \( k_{\text{plasma}} = 0.2907 \) W/(m K). So that

\[ k_b = 0.2907/(1-\alpha\phi) \] (24)

**Student Performance and Feedback**

Student performance on the project is shown in Figure 4. The average score of 86.75 and the median score of 90.5 show good student performance on the project.
Student feedback for the blood project was obtained through two questions on the course survey, administered on the last day of class. These questions are given below.

4. The blood property project gave me a strong understanding of blood as a substance.

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Strongly Agree</td>
</tr>
<tr>
<td>3</td>
<td>Agree</td>
</tr>
<tr>
<td>2</td>
<td>Neutral</td>
</tr>
<tr>
<td>1</td>
<td>Disagree</td>
</tr>
<tr>
<td>0</td>
<td>Strongly Disagree</td>
</tr>
</tbody>
</table>

5. Briefly describe the physical nature of blood.

The results of question 4 are shown in Figure 5. Students self report that they have a good understanding of blood. Question 5 was graded as passed (answer includes most of the key elements), marginal pass (answer has some of the key elements), and fail (answer has few or none of the key elements). These results are shown in Table 3.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass</td>
<td>7</td>
</tr>
<tr>
<td>Marginal Pass</td>
<td>6</td>
</tr>
<tr>
<td>Fail</td>
<td>11</td>
</tr>
<tr>
<td>No Response</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3 Results of Question 5
Here are some typical responses:

**Typical Pass Answer:** “Blood is made up of particles, proteins, enzymes, red cells, white cells, etc. suspended in a water-like substance called plasma. The particles are of different sizes and perform various functions.”

**Typical Marginal Pass Answer:** “It’s a homogeneous mix of solids in the blood to the plasma and its concentration of solids to liquids is hematocrit.

**Typical Fail Answer:** “Blood is dependent on several thermodynamic properties, which are required to transport blood through the body.”

Though the response to Question 5 is somewhat disappointing, I must be noted that this was an “ungraded” quiz that was conducted some 10 weeks after the blood project was completed.

![Figure 5 Student Responses to Question 4](image)

**Conclusions and Recommendations**
A project involving the development of a spreadsheet property calculator for human blood has been developed and implemented for a senior level mechanical engineering course. To fit the project into the model used for the course, several property models had to be developed. Comparison of the values predicted by these models with the limited data available for blood, shows good agreement. The students did well on this project and they believed that they developed a good understanding of the nature of blood.
References