AC 1999-273: Learning Modules for Finite Element Method on the World--Wide Web

Eron Flory,

Javed Alam, Youngstown State University

Joseph Rencis, University of Arkansas

Peter Kwok,

Learning Modules for Finite Element Method on the World Wide Web

Joseph J. Rencis[1], Eron Flory[2], Peter Kwok[3]/ Javed Alam[4]
Worcester Polytechnic Institute/Youngstown State University

Abstract

Web based learning modules have been developed for linear algebra, single degree of freedom spring, bar element, beam element and industrial applications of FEM. These modules can be accessed by students and engineers twenty-four hours a day since they reside on a World-Wide Web server. This paper will review the format of the bar and beam element learning modules and the experiences of the first author in integrating all five modules into the introductory undergraduate finite element course at WPI. The issue of student feedback is also addressed.

1. Introduction

The Internet/World-Wide Web (WWW) is emerging as a new medium for transmitting information globally, created in multimedia form. Engineering educators have been using the Web the past few academic years in courses, for posting course guidelines, homework, etc., and to develop courseware (instructional modules) [1-4]. Two Web pages that include some Finite Element Method (FEM) learning material can be found at [5, 6].

To exploit the latest developments in the enabling technology of networked multimedia for instructional purposes a website entitled the “Finite Element Method universal resource” (*FEMur*) is being developed by a consortium of universities. The *FEMur* homepage is located at the URL address http://femur.wpi.edu/ and is shown in Figure 1. The *FEMur* homepage includes: 1. Browse Finite Element Resources on WWW; 2. Learning Modules for the Finite Element Method. 'Resources for FEM on WWW' is a collection of organized links to FEM on the World-Wide Web. Some resources include commercial FEM codes, textbooks, other FEM homepages, consulting services, newsgroups, university, government and personal homepages, and professional societies. ‘Learning Modules for the Finite Element Method’ is a modular learning resource that includes theoretical aspects and analysis applications of typical elements. This paper will review the format of the learning modules and the experiences of the first author in integrating them into the undergraduate FEM course at WPI.

[1] Associate Professor of Mechanical Engineering <jjrencis@wpi.edu, http://jjrencis.wpi.edu/>
[2] Manufacturing Engineer, FMI Corporation, Novato, CA <eron@earthlink.net, http://femur.wpi.edu/Developers/Students/Eron/Eron.html>
[3] M.S. Student in Mechanical Engineering at MIT <http://femur.wpi.edu/Developers/Students/Kwok/main.htm>
[4] Professor of Civil and Environmental Engineering <jalam@cc.ysu.edu, http://www.eng.ysu.edu/civil/javed.html>

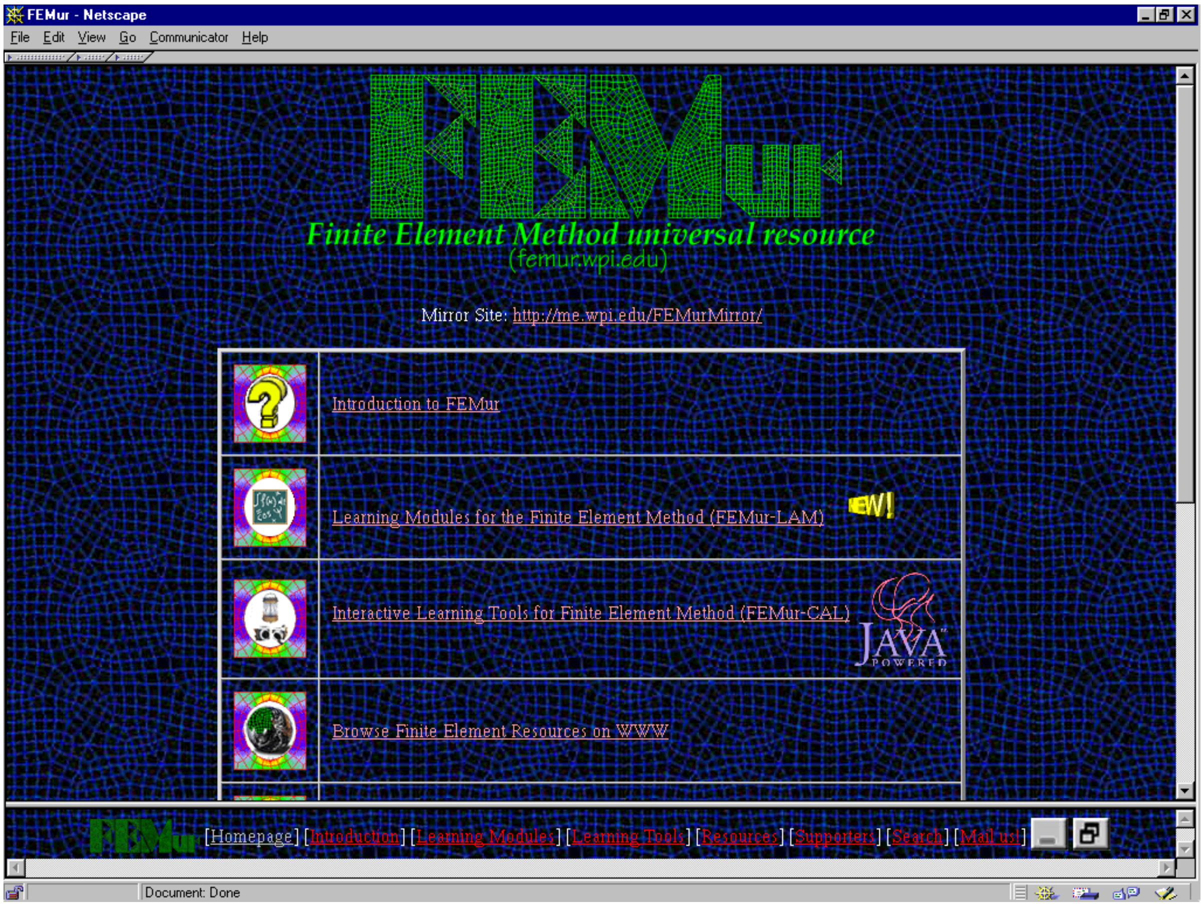


Figure 1. A snapshot of the FEMur Homepage.

2. Format of Bar and Beam Element Learning Modules

The learning modules are called *FEMur-LAM* (Finite Element Method universal resource - Learning Assistance Modules) and are located at http://femur.wpi.edu/Learning-Modules/. *FEMur-LAM* currently includes the following: linear algebra, industrial applications of FEM, single degree of freedom spring, bar element and beam element [7]. These modules were developed by WPI undergraduate students under the primary supervision of the first author at WPI. The direct stiffness approach is used to derive the finite element equations. The authors feel that the direct approach makes it easier for undergraduates to understand basic FEM fundamentals, e.g., linear algebra, degrees of freedom, element stiffness, assembly, imposing boundary conditions, etc., using simple mathematical concepts. With these fundamentals a student will have an easier time understanding the more general and mathematically complex variational (total potential energy) and weighted residual approaches. These learning modules can be used as part of an introductory finite element course, matrix structural analysis course or any other course in which the instructor would like to introduce basic FEM concepts, e.g., mechanics of materials.

In the 'Learning Modules for the Finite Element Method' the content of the bar and beam element modules will only be reviewed. The bar and beam element modules are located at location http://femur.wpi.edu/Learning-Modules/Stress-Analysis/ and were developed by the second and third authors (undergraduate students) as part of their degree requirements for a B.S. in Mechanical Engineering at WPI. The Bar and Beam element learning modules of *FEMur* are divided up into the same sub-modules and can be viewed separately or in succession. The Bar and Beam element modules each have an index that lists the sub-modules that can be accessed within each module as shown in Figure 2.

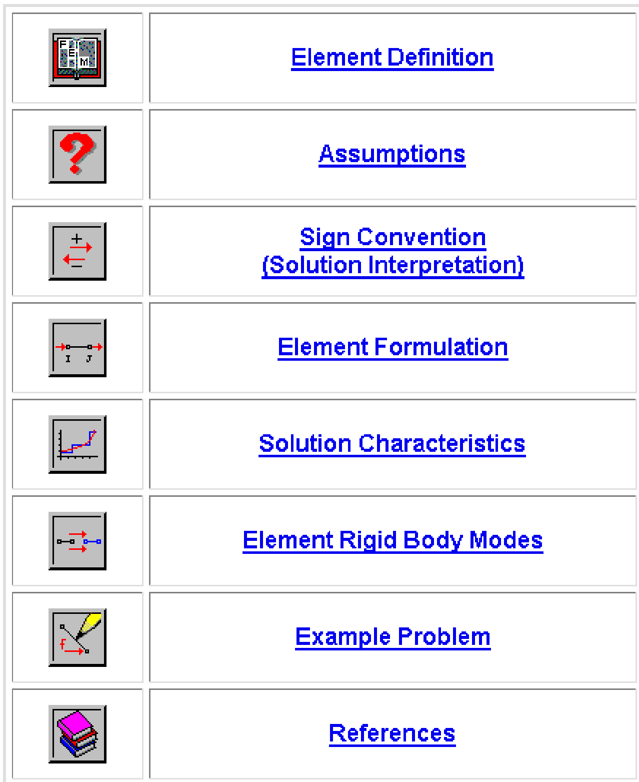
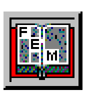


Figure 2. Element index page.

The sub-modules of the bar element and beam element in sequential order according to Figure 2 include the following:



Element Definition - The element type is defined and a table is shown, listing various characteristics of the element along with a graphical interpretation of it's characteristic. The characteristics include the following: physical discipline, dimensionality, geometric shape, material law, types of nodal degrees of freedom, and method of formulation. Along with each characteristic is a hyperlink that provides a more in-depth discussion of that characteristic. Figure 3 shows the portion of the table that links to the discussion of the types of degrees of freedom that an element can have and an explanation of the degrees of freedom associated with the bar element.

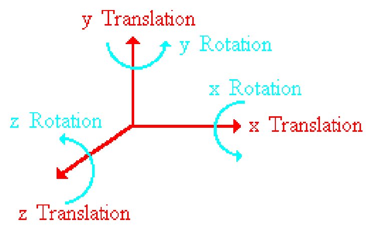
Types of Degrees of Freedom per Node		**Translation in the x-direction**

Figure 3. Element Definition Index Page from the Bar Element Module.

Assumptions - The assumptions associated with a particular element are presented in a table form. Each assumption has a graphical representation and a hyper-link to a more in-depth discussion. The in-depth discussion addresses why the particular assumption is made when applicable, and how to handle a case where the assumption does not apply. Items included for the bar element include: prismatic member, weightless member, nodal forces, axially loaded, constant load, buckling effects not considered, isotropic member, homogenous material, Poisson's effects are not considered, and cross section remains plane. Figure 3.6 shows the portion of the Assumptions Index page that links to a discussion of the assumption that the beam element is a prismatic member.

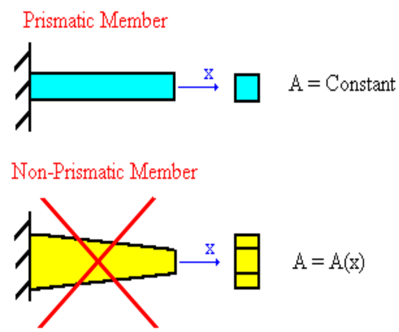
Prismatic Member	

Figure 4. Assumptions Index Page from the Bar Element Module.

Sign Convention - The sign conventions used to interpret nodal and element solution quantities of an element and mesh assembly are presented in tabular form, along with a graphical interpretation. Sign convention for the bar element includes the following: nodal displacements, nodal forces, element stress, and element strain. A segment of the sign convention table, which is shown in Figure 5, is linked to a discussion of the sign convention used for the axial displacement of a bar element.

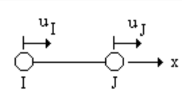
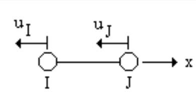
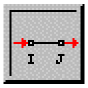
Solution Quantity	Common Units	Nodal or Element Quantity	Positive Value(s)	Negative Value(s)
Axial (Horizontal) Displacement (u)	inches or millimeters	Nodal	 Positive to the right.	 Negative to the left.

Figure 5. Sign Convention Index Page from the Bar Element Module.

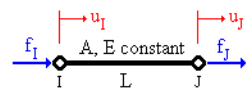


Element Formulation - Mechanics of materials (physical) principles are used to develop the element stiffness matrix. Equilibrium, compatibility and material law are emphasized throughout the formulation. Hyperlinks are used to link to other components, e.g., element definition, assumptions, sign convention, etc., as needed. The element formulation for the bar includes the following: element stiffness matrix, element strain relationship, and element stress relationship. Figure 6 shows the beginning of the formulation for the bar element stiffness matrix.

$\underline{K}_E$ Formulation:

The stiffness matrix $\underline{K}_E$ of the one-dimensional truss element will now be developed. Since the one-dimensional bar element has two translational degrees of freedom, the element force-displacement relationship relates two nodal forces (f_I, f_J) to two nodal displacements (u_I, u_J) through a 2 x 2 stiffness matrix. In symbolic matrix form the *linear* force-displacement relationship is:

$$\underset{2\times1}{\underline{f}_E} = \underset{2\times2}{\underline{K}_E}\ \underset{2\times1}{\underline{u}_E}$$



In expanded matrix form:

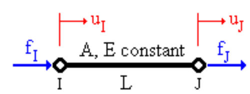
$$\underset{2\times1}{\underline{f}_E}\ \begin{Bmatrix} f_I \\ f_J \end{Bmatrix} = \underset{2\times2}{\underline{K}_E}\ \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \underset{2\times1}{\underline{u}_E}\ \begin{Bmatrix} u_I \\ u_J \end{Bmatrix}$$

where K_{IJ} is the force at DOF I due to a unit displacement at DOF J. In the stiffness matrix $\underline{K}_E$, the rows are related to forces and the columns are related to displacements, e.g., K_{12} relates f_I to u_J. In our discussions, capital and small letters denote square matrices and column vectors, respectively.

The direct method will be used to determine the elements K_{IJ} of $\underline{K}_E$. Recalling the definition of stiffness, the element stiffness matrix $\underline{K}_E$ is formulated by determining the *nodal* forces (f_I, f_J) that must be applied in order to produce *nodal* displacements (u_I, u_J).

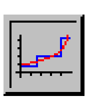
Assuming tension positive, the internal normal force at node J is

$$f_J = A\,\sigma$$



where σ and A are the normal stress and cross-sectional area at node J, respectively. Introducing the *third condition (stress-strain law)* and assuming a linear elastic material in accordance to Hooke's law for a uniaxial state of stress:

Figure 6. Element Formulation Page from the Bar Element Module.



Solution Characteristics - The solution characteristics that are associated with the element type are presented. These characteristics are presented as they apply to an individual element as well as how they vary over a mesh. Graphics are used to illustrate how each solution characteristic, e.g., displacement, internal force, etc., varies over the element and mesh. Each solution characteristic has three hyperlinks that provides an in-depth explanation of the following items: the solution variable, the equation that describes the particular function and the continuity of the function over the element and mesh. The bar element includes the following on the element and mesh levels: displacement, strain, stress, and internal force. Figure 7 shows the solution characteristic for the axial displacement variation throughout a bar element mesh.

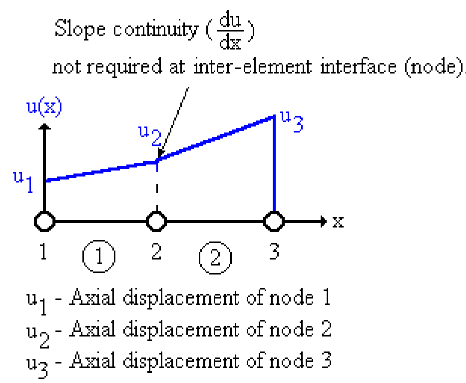
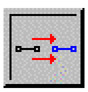
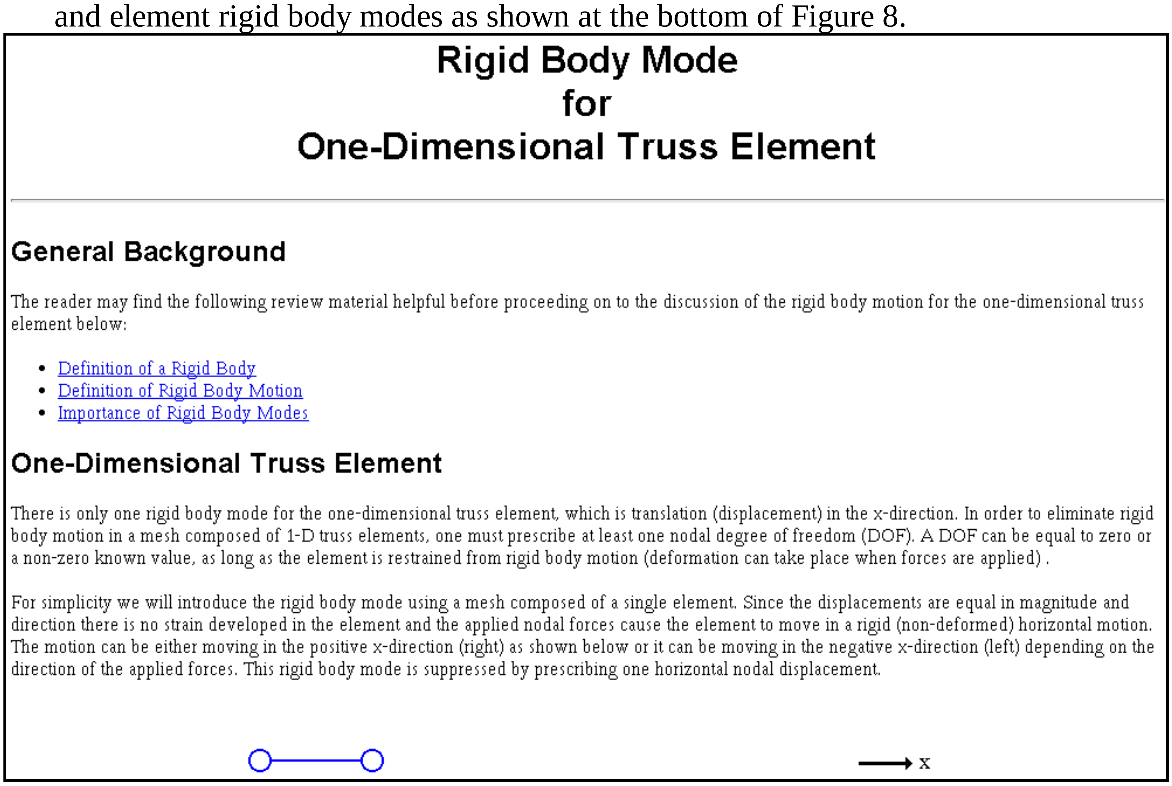
Solution Variable	Variation over Assemblage	Inter-Element Continuity
Axial Displacement (u)	Piecewise Linear Polynomial	Continuous
		

Figure 7. Solution Characteristics Page from the Bar Element Module.



Element Rigid Body Modes – The problem of an inadequately restrained element/mesh is presented, along with the necessary requirements to prevent rigid body motion and why it is necessary to prevent it for a given element type. Figure 8 shows the rigid body mode page for the bar element. The page includes the following: definition of rigid body, definition of rigid body modes and the importance of rigid body modes. Animations are used to enhance the understanding of a rigid body, rigid body motion and element rigid body modes as shown at the bottom of Figure 8.



Rigid Body Mode
for
One-Dimensional Truss Element

General Background

The reader may find the following review material helpful before proceeding on to the discussion of the rigid body motion for the one-dimensional truss element below:

- Definition of a Rigid Body
- Definition of Rigid Body Motion
- Importance of Rigid Body Modes

One-Dimensional Truss Element

There is only one rigid body mode for the one-dimensional truss element, which is translation (displacement) in the x-direction. In order to eliminate rigid body motion in a mesh composed of 1-D truss elements, one must prescribe at least one nodal degree of freedom (DOF). A DOF can be equal to zero or a non-zero known value, as long as the element is restrained from rigid body motion (deformation can take place when forces are applied).

For simplicity we will introduce the rigid body mode using a mesh composed of a single element. Since the displacements are equal in magnitude and direction there is no strain developed in the element and the applied nodal forces cause the element to move in a rigid (non-deformed) horizontal motion. The motion can be either moving in the positive x-direction (right) as shown below or it can be moving in the negative x-direction (left) depending on the direction of the applied forces. This rigid body mode is suppressed by prescribing one horizontal nodal displacement.

Figure 8. Rigid Body Mode Page from the Bar Element Module.

Example Problem – A real life application is modeled with FEM and solved in this submodule. The problem is worked through in a step-by-step manner where text and graphics are used to explain each step in-depth. The five steps to solve the bar element include the following: nodal displacements, element strains, element stresses, nodal reaction forces, and element nodal forces. A graphical interpretation is then given with an explanation, so that the student can better understand the significance of each step. Animations are used to visualize the deflected shape of the structure and to demonstrate how to construct plots for displacement, internal force, strain, and stress along the mesh. A two-story building (Figure 9) and traffic light pole are used to demonstrate applications of the bar and beam elements, respectively.

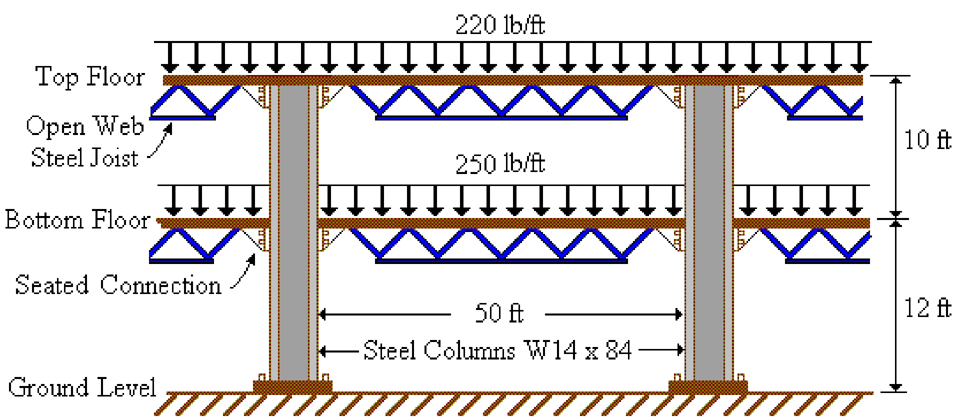


Figure 9a. Example Problem from the Bar Element Module.

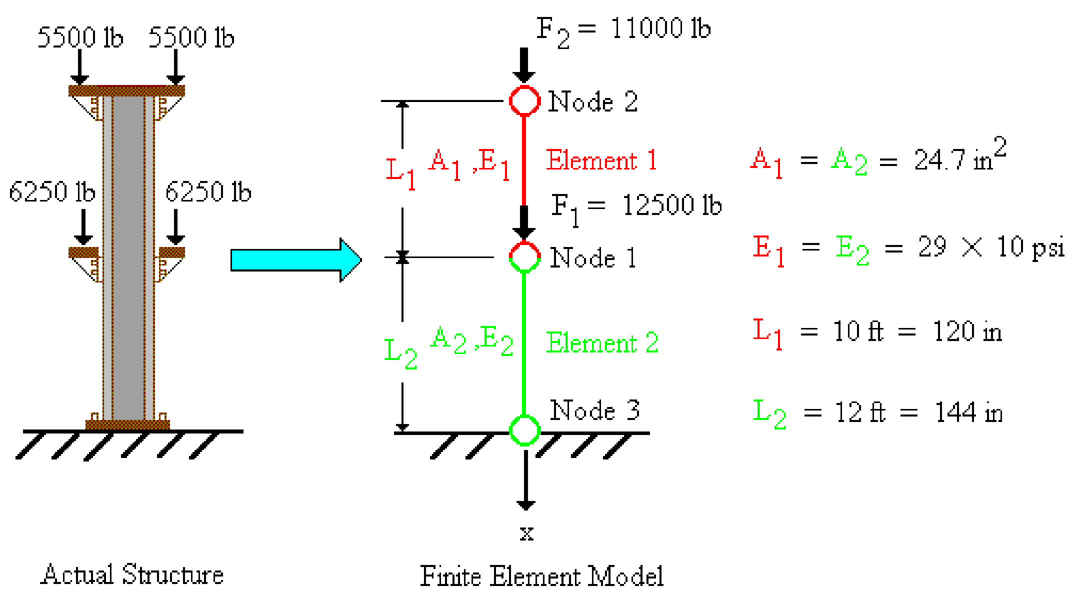
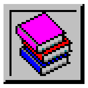


Figure 9b. Finite Element Mesh.



References - This section contains textbook references that can be used to find more information on FEM and Matrix Structural Analysis. The information is presented in a table form as shown in Figure 10. Hyperlinks provide access to the author's homepage, publisher homepage and publisher textbook homepage. A link is also provided at the end of the section, to the search engine for the Library of Congress, with this link their database can be searched for more references, including more specific areas of FEM.

Theory / Examples					
Chapter(s)	Theory Section(s)/ Page Number	Example Section(s)/ Page Number	Author/Title	Publisher	Copyright ISBN Call #
One	*Section 1-2* pp. 10-16	*Examples 1.1 & 1.2* pp. 16-19	*Hartley Grandin, Jr.* Fundamentals of Finite Element Method	Macmillan Publishing Company New York	1986 0-02-345480-6 TA347.F5G73

Figure 10. References Page from the Bar Element Module.

3. Format of the Finite Element Course

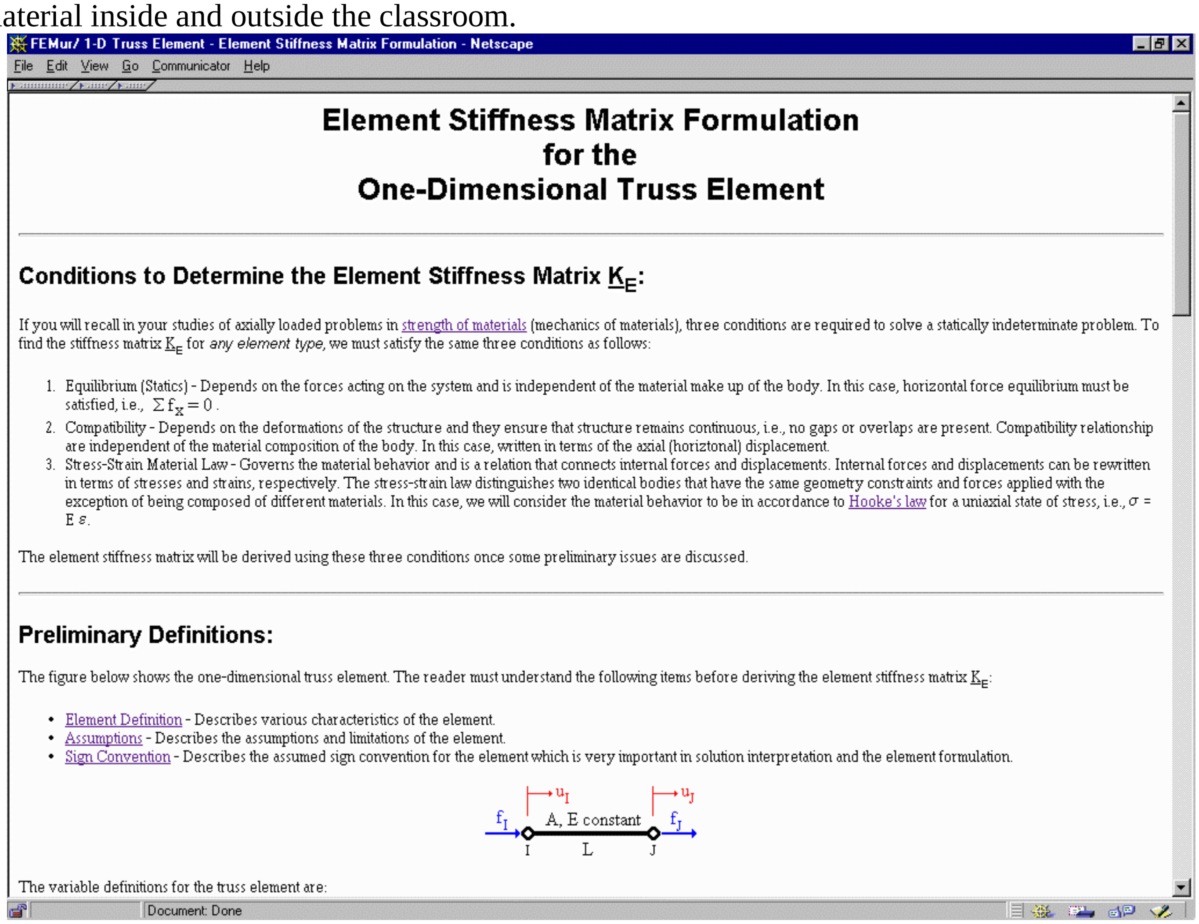
The Introduction to Finite Element Method course of this discussion, (ME3512) at WPI is a seven week junior/senior level course that meets four class hours (fifty minutes) per week and has two, one-half hour design laboratories per term. The course typically enrolls forty to sixty students and consists of approximately ninety-percent mechanical engineering majors and the rest come from civil engineering. The textbook by Logan [8] was used and the topics covered Chapters 1 through 10, Chapter 12, and the first half of Chapters 11 and 15. The course grade is based on two exams (50%), homework (25%), and two design experiences (25%) involving design insights and application of the commercial FEM code Algor (http://www.algor.com/). A course syllabus can be found at http://jjrencis.wpi.edu/teaching/me3512/.

4. Implementation of Learning Modules in the Classroom

The learning modules were used in the classroom at WPI for approximately one and half weeks of the seven-week term. The industrial applications of FEM, single degree of freedom spring, bar element and beam element learning modules were staggered throughout the seven week term since other topics were introduced, e.g., once the bar element learning module was completed the plane truss element was introduced via the traditional white (chalk) board. The linear algebra module was not used in the classroom due to time restrictions; however, students used it as an outside resource to review the material. A majority of the students enrolled in the class have been exposed to linear algebra concepts in mathematics and/or engineering course(s). The instructor as needed in lecture to reinforce basic FEM concepts reviewed linear algebra fundamentals.

A computer screen projection system was used to introduce the learning modules in the classroom. The learning modules also resided locally on the computer to eliminate any connection problems to the Web site during class time. Furthermore, the learning modules resided not only on http://femur.wpi.edu/, but a mirror site to eliminate student frustration with network and server problems outside the class. Students were provided with photocopies of the Web pages that were discussed in class. The students overwhelmingly liked having hard copies of the modules (they refer to them as class notes) so they could focus their attention on the lecture and could easily write down any additional notes without missing much of the lecture.

The introduction of each component of a module in the classroom was not done in a sequential manner since hyperlinks are used throughout the Web pages. The first author used the 'element formulation' component to introduce the bar element and then considered the 'example problem' component. Figure 11 contains the initial screen for the formulation of the element stiffness matrix for the bar element (http://femur/Learning-Modules/Stress-Analysis/One-Dimensional-Elements/Truss-Element/form1.htm). Hyperlinks provided access to the other components of the bar learning module at an appropriate time in the lecture. Even though the computer was used extensively in the classroom, the instructor found that a white board is a necessity to address unanticipated questions and to fill in any voids in module content. The students and instructor felt that the quality of the colored graphics enhanced their understanding of the material inside and outside the classroom.


FEMur/ 1-D Truss Element - Element Stiffness Matrix Formulation - Netscape

File Edit View Go Communicator Help

Element Stiffness Matrix Formulation for the One-Dimensional Truss Element

Conditions to Determine the Element Stiffness Matrix $\underline{K}_E$:

If you will recall in your studies of axially loaded problems in strength of materials (mechanics of materials), three conditions are required to solve a statically indeterminate problem. To find the stiffness matrix $\underline{K}_E$ for *any element type*, we must satisfy the same three conditions as follows:

1. Equilibrium (Statics) - Depends on the forces acting on the system and is independent of the material make up of the body. In this case, horizontal force equilibrium must be satisfied, i.e., $\sum f_x = 0$.
2. Compatibility - Depends on the deformations of the structure and they ensure that structure remains continuous, i.e., no gaps or overlaps are present. Compatibility relationship are independent of the material composition of the body. In this case, written in terms of the axial (horiztonal) displacement.
3. Stress-Strain Material Law - Governs the material behavior and is a relation that connects internal forces and displacements. Internal forces and displacements can be rewritten in terms of stresses and strains, respectively. The stress-strain law distinguishes two identical bodies that have the same geometry constraints and forces applied with the exception of being composed of different materials. In this case, we will consider the material behavior to be in accordance to Hooke's law for a uniaxial state of stress, i.e., $\sigma = E\,\varepsilon$.

The element stiffness matrix will be derived using these three conditions once some preliminary issues are discussed.

Preliminary Definitions:

The figure below shows the one-dimensional truss element. The reader must understand the following items before deriving the element stiffness matrix $\underline{K}_E$:

- Element Definition - Describes various characteristics of the element.
- Assumptions - Describes the assumptions and limitations of the element.
- Sign Convention - Describes the assumed sign convention for the element which is very important in solution interpretation and the element formulation.

The variable definitions for the truss element are:

Document: Done

Figure 11. Web page for formulation of bar element stiffness matrix.

The instructor observed a significant increase in student participation during lecture in comparison to a traditional one. He was also able to spend more class time addressing questions and providing more in-depth explanations. In addition, the instructor felt that the students asked more substantial questions since they did not have to take down their own notes.

5. Student Feedback

The first author handed out a survey (similar to the one in [4]) to the WPI students at the end of the seven week term. The first question that was asked, “How comfortable are you with computers?” The students response indicated that over 90% were quite comfortable and the remainder was somewhat comfortable with computers. The second question addressed, “How much have you used the World-Wide Web?” The response yielded 65% a lot, 25% some and 10% a little. The students were also asked, “Do you like having course material on-line?” Approximately 90% said yes and 10% were uncertain. The next question inquired about, “How often have you used the on-line materials?” and 90% responded every day, 5% said every week and the remainder said infrequently. The next question asked, “Was the emphasis on using on-line materials, too much, just right, too little or don’t care?” which yielded a response of over 95% just right and the remainder doesn’t care. The next question, "For what did you use the on-line materials most of the time?" In this case 10% responded reading material prior to class, 80% looking up information and 10% preparing for exams. The students were also asked “Which learning module did you like the most?” and the approximate breakdown was 50% beam, 40% bar, 5% linear algebra, and 5% the single degree of freedom spring. Another question asked, “Should the course should be more challenging.” and approximately 5% agreed 60% disagreed and 35% strongly disagreed. The next question addressed, “Do you like having a course with on-line material? Please explain.” About 95% of the responses said yes and felt that it provided more than one source of examples and information. Approximately 3% felt that the computer made the lecture less personable. The final question was, “Please comment on the usefulness of the course in the future.” All students felt that the course would be very useful in future courses/projects and on the job. The seniors commented that job recruiters were very impressed that the course not only emphasized a fundamental understanding of FEM theory, but validation of the solutions obtained from a commercial FEM program.

The first author felt that the student feedback regarding the FEM learning modules was very positive. Furthermore, he thought that classroom attendance would drop since students had access to the Web pages (detailed neat and accurate notes), however, attendance averaged 95% during the learning module lectures. A majority of the seniors that missed the lecture had an on or off campus job interview. The instructor also used student suggestions to improve the learning modules.

6. Conclusion

The man-hours associated with the development, assessment and refinement of the learning modules was tremendous. In fact, the effort is comparable to writing a textbook since the creation of figures and equations, i.e., graphics, are very time-consuming tasks. For this reason the authors were not able to complete all the learning modules needed for an introductory FEM course. This is also why nearly all course Web sites include only the course description, schedule, guidelines, and homework assignments. Other examples of incomplete/detailed FEM Web sites can be found at [5, 6].

The authors feel that the only way to successfully implement high quality learning modules into a classroom setting is that everything must be completed before the course begins. Even though the students used the on-line learning modules extensively, the first author found that students overwhelmingly preferred to *study* from the hard copies rather than the screen. The main reason for this seems to be the poor resolution of the screen images that causes eye strain if used for extensive reading. This issue is addressed by the recently introduced screen technology known as "ClearType " by the Microsoft corporation [9]. This will allow the educators to create e-books that are more readable due to a more paper like look and feel. Furthermore, this will also accelerate the move of our society towards a paperless society. For now this response implies that using the Web to eliminate printing costs is not feasible. Overall, the undergraduate students at WPI were very pleased by the quality and content of the *FEMur-LAM* modules and are extremely pleased that valuable resources will be accessible to them in the future.

References

1. Shawki, T.G., "TAM 221 Mechanics of Materials," Department of Theoretical and Applied Mechanics, University of Illinois at Urbana-Champaign, Champaign, IL, 1995, http://e2.tam.uiuc.edu/TAM221/index/.
2. Kayser, J. and Massetti, M., "Statics Tutorial," Department of Civil and Environmental Engineering, Lafayette College, Easton, PA, 1995, http://www.lafayette.edu/kayserj/statics/cover.htm.
3. Martin, T., "The Development of Interactive World-Wide Web Courseware for Students of Engineering and Technology at Deakin University," Faculty of Science and Technology, School of Engineering and Technology, Deakin University, Australia, 1995, http://www.scu.edu.au/ausweb95/papers/education1/martin/.
4. Bourne, J.R., Brodersen, A.J., Campbell, J.O., Dawant, M.M. and Shiavi, R.G., "ES 130: Engineering Science 130: Introduction to Engineering," Center for Innovation in Engineering Education, Vanderbilt University, 1996, http://wwwfp.vuse.vanderbilt.edu:8888/es130-2/.
5. Liu, Y., "Introduction to Finite Element Method - I & II," Department of Mechanical, Industrial, and Nuclear Engineering, University of Cincinnati, Cincinnati, OH, http://urbana.mie.uc.edu/yliu/FEM-525/FEM-525.htm.
6. Gramoll, K., "ENGR 4510-026 Introduction to Finite Elements," School of Aerospace and Mechanical Engineering, University of Oklahoma, Norman, OK, http://eml.ou.edu/FE/home.htm.
7. Kwok, P., Flory, E., Felice, C., Mensching, J., Piquette, M., Rencis, J.J. and Hou, Z., "Bar Element, Beam Element, Single DOF Spring and Linear Algebra Learning Modules for Finite Element Method," Mechanical Engineering Department, Worcester Polytechnic Institute, Worcester, MA, 1996, http://femur.wpi.edu/Learning-Modules/.
8. Logan, D.L., <u>A First Course in the Finite Element Method Using Algor™</u>, PWS Publishing Company, Boston, MA, 1997.
9. Microsoft Corporation, "Microsoft ClearType", http://www.microsoft.com/typography/cleartype/default.htm.

JOSEPH J. RENCIS

Joseph J. Rencis is an Associate Professor in the Mechanical Engineering Department at Worcester Polytechnic Institute. His research focuses on the development of boundary and finite element methods for analyzing solid, heat transfer and fluid mechanics problems. He received his B.S. from the Milwaukee School of Engineering in 1980, a M.S. from Northwestern University in 1982 and a Ph.D. from Case Western Reserve University in 1985.

JAVED ALAM

Javed Alam is a professor of Civil and Environmental Engineering at Youngstown State University. He obtained his M.S. degree from Asian Institute of Technology and a Ph.D. degree from Case Western Reserve University. His research interests are in the area of Structural Simulations and the Use of Technology in Engineering Education.