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Making Mathematics Relevant to Engineering Students

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Making Mathematics Relevant to Engineering Students

Abstract: In 2004, the author, an Associate Professor of Mathematics and a truly nontraditional student, started an adventure by going back to school and majoring in Mechanical Engineering. The author, along with his advisor and co-author, describe and relate in this paper some of the pedagogical and curriculum discussions brought about by this adventure. In particular, some of the questions that arose during these discussions include questions about when vectors should be introduced and the reasoning behind why mathematical proofs are taught. Finally, the authors tell of the changes they have seen in their own units and classrooms because of this interaction between the Mathematics and the Mechanical Engineering Departments.

Introduction

During the 2011 Spring Commencement at the local university, the presenter read the next name on the list of graduates, "Dr. John Doe, Bachelors of Science in Mechanical Engineering". Actually, no prefix was attached when the name was read but the graduate was well known in the university community having sat many times with the faculty during previous commencements. But this time he was with the students. This is the story, in the student's and his advisor's words, of this adventure and how it influenced those involved.

Motivation

Ever since I could remember, I have had a wrench in my hand. I am a "grease monkey", a "gear head", or whatever other pet name is given to someone who loves cars. Also, I tinker with all sorts of mechanical and electronic equipment. If I see something broken, I usually try to fix it before sending it to the rubbish pile. But it is my love of automobiles that got me into this adventure.

While walking through the halls of my building one day, I noticed a flyer mentioning the start of a new organization on campus which was going to build and take to competition a ½ scale race car. The organization was Formula SAE. I went to the meeting with the intention of helping once in awhile and left as the advisor of the new organization with the understanding that as soon as they could recruit an engineering professor who would like to be the advisor, I would gladly hand over the reins. Three years later I was finally replaced, but not without having been heavily influenced by the Formula SAE students and also the Baja SAE students with whom we shared a shop. Although the students respected me as advisor, I was reminded by a few that I was not an engineer even though I had a PhD in Statistics including a bachelors and masters degree in mathematics. This challenged me. I had the mathematical expertise but not the experiential expertise. I was sure I could do engineering mathematics, but I had never really applied the mathematics and statistics I knew anywhere else than in the classes I taught.

Also, by earning this second bachelors degree, I would be completing a program I started in 1985. I had started school with the intention of becoming an engineer but changed majors to education and then to mathematics. After talking this over with my wife, we agreed I should go back to school.

My motivation changed significantly as I went through my program of study. It became more of a goal to learn the material rather than just have the degree. Even though I have now finished my program of study and have taken on more responsibility at the university, I still have the desire to go back into the classroom. Curiosity became more of the driving factor than anything else. Also, I found it invigorating to be able to teach Calculus, Statistics, and Differential Equations with solid, concrete examples I understood. I believe going back into the classroom changed my teaching style for the better.

Classroom Experiences

When I entered my first basic engineering class, it was a strange feeling. Here I was taking AutoCad along with some of my Calculus students. I knew what they were thinking, "There goes the curve!" But after a few weeks I became one of the students. In fact, there were many times when the students would start to complain about their professors and forget I was a professor as well. There are no student evaluation instrument to compare with unbridled classroom conversation.

On the other hand, I did notice my presence in the classroom changed student work ethic. Whether it was just my presence, or whether they saw how much I studied I do not know. But I could tell they studied more around me. I hope I was a good influence in that respect. This trend continued in the lower level classes I took, but by my "senior" year, most of the students were accomplished engineering apprentices and would, many times, outperform me on classroom tasks.

My statics class was a different matter. The professor was new to campus and did not know me as a faculty member. Anonymity was refreshing to me. I do not know if it released some of the implicit pressure on me to perform like a professor should, what ever that is, but I was more relaxed in that class than any other. Of course, the first test was grueling. I started studying two weeks before the test and then studied three hours the night before just to make sure I got it right. One would think a mathematics professor would know how to find the cosine and sine of an angle and be able to keep the forces balanced in the horizontal and vertical directions. Well, I can now.

Test Anxiety

Because of my experiences in Statics, I gained a new understanding of test anxiety. I felt it a couple of other times during my plan of study. I had worked all the homework assignments and had studied the night before the exams, yet I felt unprepared in some cases and just not ready. I realized something was missing. In the Math classes I have taught, I was the one to visit for assistance with homework and to clear up any misconceptions. The reason I could help was because I was mathematically confident. I knew what I was doing. But mechanical engineering was, in many ways, new material to me. I had to gain confidence. But confidence comes with competence. I knew I had to become proficient in the material to the point I could teach it. Here is where all of Higher Education could take a lesson from Engineering. Because of the nature of the material and the laboratory classes, I learned and learned well. I could see and touch and hear the results of my calculations. "A picture is worth a thousand words" can also be stated as "a concrete example is worth a thousand minutes of lecture". Of course, for some courses the abstract and theoretical nature of the material makes it a little harder to present such examples, but it really helps the learner.

Teaching Style

One major influence of this adventure was on my teaching style. I started using motivational examples in my classroom. These were not just examples taken from the end of the chapter we were covering in class but examples taken from my own engineering classes. Because most of the students in my classes are from other colleges and a majority of them are engineering students who were to see this material in a couple of semesters, why not show them the application of the mathematics right now? Here are just a few of the problems and examples I have added to my repertoire.

Law of Sines and Cosines

In my precalculus class we studied the law of sines and cosines. What better way to give the students an exercise in using these two laws than surveying. I found a survey map of a plot of land, as seen below, and asked the students to tell me the area.

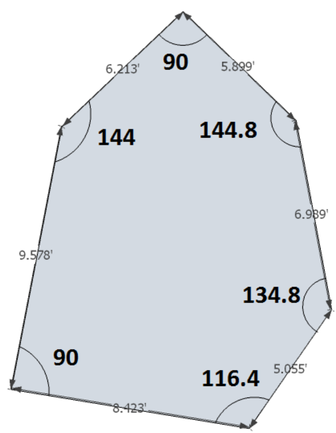


Figure 1: Survey Map used in Precalculus

We discussed how to break up the area into triangles and then, in groups, the students found the area. On the next test this type of problem was correctly solved by 95% of the class.

Vibration and Damping

For sinusoidal functions like $f(t) = A\sin(\omega t + \phi)$, the precalculus class needed to study the parameters of amplitude, frequency, and phase. Luckily, I was teaching this course in our STEM Center, a facility built to help train K-12 teachers in the disciplines of Science, Technology, Engineering and Mathematics. In the stockroom I found accelerometers, hand held data loggers, yardsticks, and tape. I had the students, in groups of four, tape the accelerometer to the end of the yardstick, hold the yardstick on the table and then set in vibrational motion. Based on their empirical data, the students were asked to solve for A, ω, and ϕ. ϕ estimation became a point of interest for the class. Many groups were getting similar frequencies and could visualize the different amplitudes, but the phase angle was tougher to understand. After some time, the groups started to realize that you could manipulate ϕ by when you started recording the data. Again, the pass rate on this question on the precalculus exam was above 90%.

Dynamics and Related Rates

Right after taking dynamics class in 2005, I had an idea for a set of videos to assist students in calculus I with related rates problems. A colleague and I obtained funding and produced videos covering related rates problems on a sliding ladder, a lighthouse, and a piston on a crankshaft. Here are some snapshots of the piston problem as it is being solved.

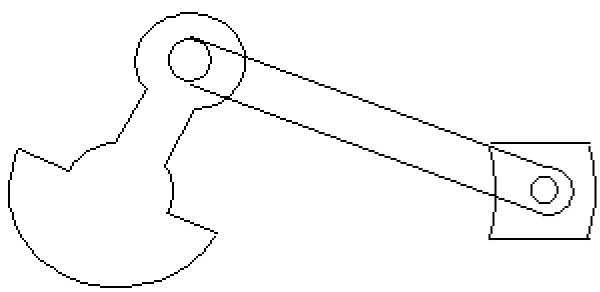


Figure 2: Beginning of related rates piston problem

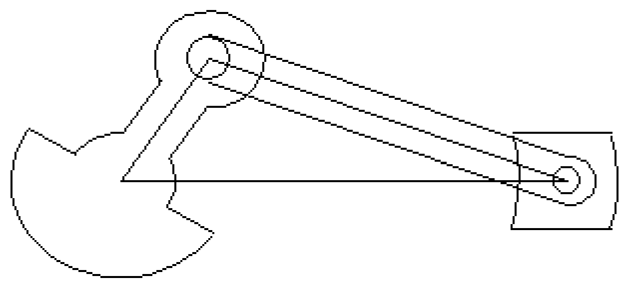


Figure 3: Screen capture when triangle is superimposed on crank-piston system

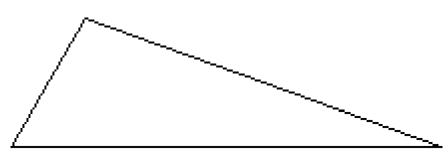
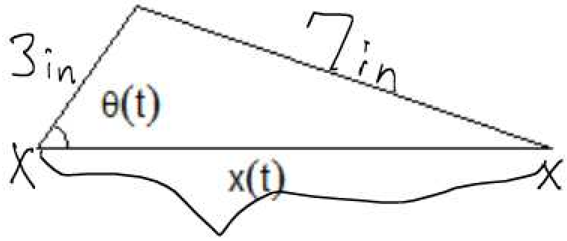


Figure 4: Screen capture when crank-piston system is removed but the moving triangle remains



Let $\theta(t)$ be the angle between crank and the horizontal line through piston.
Let $x(t)$ be the distance from fixed point of crank to piston.

Given: $\frac{d\theta}{dt} = 2000 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{rads}}{\text{rev.}} = 4000\pi$ rads/min
Find: $\frac{dx}{dt}$ when $\theta = \frac{\pi}{3}$

Figure 5: Beginning of mathematical explanation and solution

After differentiating $49 \;=\; 9 + x^2 - 6x\cos\theta$ with respect to t, we obtain

$$
\begin{aligned}
0 &= 0 + 2x\frac{dx}{dt} - 6\left[x(-\sin\theta)\frac{d\theta}{dt} + \frac{dx}{dt}\cos\theta\right] \\
0 &= 2x\frac{dx}{dt} + 6x\sin\theta\frac{d\theta}{dt} - 6\frac{dx}{dt}\cos\theta \\
6\cos\theta\frac{dx}{dt} - 2x\frac{dx}{dt} &= (6x\sin\theta)\frac{d\theta}{dt} \\
[6\cos\theta - 2x]\frac{dx}{dt} &= (6x\sin\theta)\frac{d\theta}{dt} \\
\frac{dx}{dt} &= \frac{(6x\sin\theta)\frac{d\theta}{dt}}{6\cos\theta - 2x}
\end{aligned}
$$

Figure 6: Middle section of mathematical solution

So when $\theta = \pi/3$, we have $x = 8$ and the velocity of the piston can be determined by substituting into our related rates formula.

$$
\begin{aligned}
\frac{dx}{dt} &= \frac{6x\sin\theta\frac{d\theta}{dt}}{6\cos\theta - 2x} \\
\text{So } \frac{dx}{dt} &= \frac{6(8)\sin(\pi/3)4000\pi}{6\cos\pi/3 - 2(8)} \\
&= \frac{48(\frac{\sqrt{3}}{2})4000\pi}{6(1/2) - 16} \\
&= \frac{96000\pi\sqrt{3}}{-13} \\
&\approx -40182 \text{ inches per minute}
\end{aligned}
$$

This can be easily converted to the units feet per second as follows:

$$-40182\frac{\text{inches}}{\text{minute}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} = -55.8\text{ft/sec}$$

Figure 7: Final page of related rates piston problem solution

Although this approach may have been innovative for us in 2005, today you can find numerous videos on various websites, like MyPhysicslab and Wolfram, that have far superior quality and sophistication. But, at the time, this one video really gave our students an example of where and how mathematics could be applied.

Because of the success I had with hands-on applications and relevant videos, I started adding motivational examples from Biology, Chemistry and Computer Science to assist those majors in our classes.

Effect on the Respective Departments

Conversations with my colleagues in the Mathematics Department about my classroom experiences in engineering appeared to have an effect. The department chair and the faculty were beginning to see how their courses served other departments. These were not just mathematics courses for math majors anymore. My colleagues started to use motivational examples as well. Calculus was becoming, for the students, not just a hoop to jump through but material to be learned. When we had a meeting to discuss a curriculum change, we started asking how it would influence other departments and not just our own majors.

On the flip side, my presence in the College of Engineering started a dialog which is still continuing. In one class I was taking, the engineering professor was astounded when the class could not solve a differential equation, including the author. It was not that we, the class, had not had differential equations, it was the level of expectation by the professor. With the Differential Equations text and syllabus available, a discussion helped the professor understand more fully the level at which students exited the Differential Equations class.

Why do you teach them proofs?

I decided to write this as a one act play. Setting: Two professors from two different colleges are eating lunch in the cafeteria.

Engineering Professor: So why do you teach proofs in Calculus? I think they just need to know about proofs.

Math Professor: No, they need to know how to do some proofs.

Engineering Professor: Why?

Math Professor: Because knowing how to work on an air conditioner is different than knowing why it works. The how will come with the why. Knowing about proofs is like telling your engineering students that stresses exist and Mohr's circle is just a quick ten minute lecture. Proofs allow students to see for themselves that the theorems they use in Calculus class are really just general solutions to a specific problem. It is like the formula for heat transfer through a wall. The formula is derived through a differential equation proof, i.e., a general solution to a specific problem that can be used again and again.

Student Respect

By being in the classroom with the students as a student, I gained a different respect than the usual respect given to an instructor. It was more collegial and fraternal. Students became more open to ask questions both in class and out of class. I used such opportunities to encourage and help the students make the connections with the mathematics I was teaching and the engineering principles they were to learn. Sometimes I had to build a large bridge for them to see the connection but my engineering experiences helped me with the trusses needed to make that connection.

Advisor's Perception

I guess it is now my turn as the advisor. I heard compliments from some of my advisees about this unique math professor who was taking engineering classes and who presented engineering examples in his calculus class to better help the students understand the material. I knew who this was. He was my advisee. I realized very early in our advisor-advisee relationship that this was going to be a fruitful undertaking for all the parties involved.

Of course, having a colleague as an advisee was a different experience. He knew the policies and procedures of the university like any other faculty member and showed concern when I presented him with a program of study that would require twenty-three extra hours more than the hours required for a second bachelors degree. This led to our conversation regarding ABET accreditation. I explained about ABET accreditation for a professional degree. The Department of Mathematics is in the College of Arts and Sciences and is under only SACCS accreditation. His departmental concerns are different than ours and are more about preparation for graduate school and teaching than industrial guidelines and standards. He had to start at the beginning to get a true ABET accredited Mechanical Engineering degree. A result of this discussion was that my colleague and advisee was better prepared to discuss curriculum changes in his department that might influence our engineering programs.

Conclusion

As with any recommendation letter, the last sentence or two is an emphatic recommendation of the individual. We do likewise. We highly recommend faculty return to the classroom. As in our case, we observed that this situation improves collegiality and understanding of the curriculum and philosophies of separate departments. This is a good path for any professor who has an aptitude for engineering. Going back to school re-ignites the passion for learning. We hope that administrators in other institutions will be as supportive as those at our institution.