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Math in Engineering: Beyond the Equations

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Math in Engineering: Looking Beyond the Equations

Abstract

In this paper, perceived student shortcomings that inhibit a student's acceptance, development, and lifelong recognition of mathematics usage are discussed. Observations made in calculus and engineering statics regarding student attitudes towards mathematics, the use of mathematics, modern computing, and learning in general, are presented and discussed. Interventions are proposed to help students develop a lifelong appreciation for and awareness of the mathematics they will encounter and use, even if subconsciously, every day in professional practice. The paper concludes with a summary of student recognition of the impact of the interventions in their lives.

Introduction

Engineering students begin working with simple mathematical models in their first math and science courses. As they progress in school, the models become more involved, as does the mathematics. By the time a student graduates and enters engineering practice, they should be experts in, or at the very least comfortable with, the development of mathematical models and capable of solving many physical problems. Wankat and Oreovicz suggest that obtaining 'expert' status takes a decade of consistent work assembling increasingly complicated models to accumulate that level of knowledge [1]. Math, science, and engineering courses are where tomorrow's experts begin their development.

Calculus and differential equations are standard prerequisite courses in engineering programs. Significant time, typically fifteen semester credit hours, is dedicated to teaching mathematics to engineers, but how is this math really used? In the spring of 2019, Dr. Brooks, a calculus professor, enrolled in Dr. McDonald's engineering statics class. She wanted to see how the math she taught in Calculus I, a prerequisite to the statics course, was employed. The resulting experience was enlightening for both professors, the authors of this paper. Math is generally perceived as 'being hard' and that you have to be 'really smart' to be successful at it. Often, math classes act as the initial culling of engineering school candidates, and passing that first calculus class can be quite rewarding. By the time a student completes differential equations, they either feel invincible or simply relieved to have made it through.

In the engineering classes that follow, math is used to develop models and describe the behavior of stress, strain, fluid flow, electric current, heat, and even traffic on highways. Everything engineers design involves the modeling of observed physical behaviors *using math*. Why then, when practicing engineers are asked how often do they use higher order math, do they often reply "*Never*"?

An Example from Strength of Materials

Students designing a timber ‘T’ beam for a quiz question exemplified how a model and the accompanying mathematics is, or is not, used in engineering. The beam is constructed using two planks by setting one on edge (the stem of the T) and nailing the other down the middle to the top edge. The quiz question asked students to specify s, the maximum nail spacing required to safely fasten the two planks together for the given loading. Students learn about internal forces in statics and then shear flow and shear stress in strength of materials courses. They typically work several fastener spacing problems during class and in assigned homework. The quiz problem described a situation where $V(x)$, the internal shear force in the beam evaluated along the length, x, of the beam, is constant in three regions of the beam but varies between those regions. Since s is dependent on V, the instructor anticipated that students would specify a maximum spacing for each region of the beam. The model is not complicated, nor is the math, even though some basic calculus is involved. The entire class missed this problem, with many of the students earning zero credit. Most of the practice problems worked prior to the quiz specified V on the cross-section of the beam located at an unspecified x, where s was to be determined. When given V, the students could calculate s. The students had also demonstrated earlier that they were accomplished at plotting $V(x)$, the internal shear force diagram, and determining V at any specific section, or x, along the length of the beam. In the quiz question, the students were unable to connect the two processes without explicit directions. This resulted in an ‘aha moment’ for the instructor. While grading this question, the instructor realized that the students knew the equations, and they could do the mathematical operations; however, they did not fully understand the model or the mathematics. They studied to obtain an answer, a numerical value, rather than understand the fundamental behavior and solve the problem. They were not looking beyond the equations.

After graduating, students don’t magically change their approach to problem solving. The methods they develop and exercise while in school carry over into their professional practice. If they don’t learn to look beyond the equations while they are in school, they won’t go beyond the equations as engineers. The expert designer recognizes situations that do not conform to the standard model, typical solution, and tabulated handbook values. Then they use their mathematics skills to adjust the model, update the equations, and arrive at a satisfactory solution.

An Example from Algebra

In algebra, when solving for the roots of polynomials, students are taught to move all terms to one side of the equation so that the other side of the equation is zero, factor completely, and then set each factor equal to zero. This method utilizes the zero-product property of the real numbers: one cannot multiply two (or more) non-zero numbers to obtain zero. However, it is not uncommon for students to misinterpret this method. Indeed, upon having two or more factors whose product is equal to one, for example, they will set each factor equal to one. Procedurally, this makes sense, but logically and mathematically, it does not.

Much as a child clings to their parent’s hand and blindly follows in a crowded street, students obey the commands given by their math instructors without knowledge of why they are doing so and where it will take them. When students understand the reasoning behind the methods of problem-solving, that is, when their understanding reaches beyond the superficial equations and procedures, they are much less likely to make conceptual mistakes and more likely to retain the concepts being taught.

Observations

High school graduates entering college-level engineering, mathematics, and science courses often demonstrate a culture of rote learning [2]. They follow a prescribed approach to obtain an answer, often times never pausing to think if or why the approach works. They can calculate *s*, and don't question what the given *V* represents. While having a prescribed approach can aid in demystifying problem solving for apprehensive students and may increase productivity (in that no time is spent thinking), such procedural approaches perpetuate the general mindset that math and other STEM subjects are a 'bag of tricks' rather than a means of critical thinking. This complication became apparent while Dr. Brooks was formally enrolled in engineering statics for credit under the instruction of Dr. McDonald. In particular, high-performing students in calculus were struggling to identify appropriate solution pathways in statics. When the procedural path was obscured, the necessity to think critically seemed to be an inconvenience for the students.

Specific to science degree-seeking students taking mathematics courses, the lack of applied problems perpetuates rote learning. In mathematics courses, students are tasked with solving mathematical problems in a generic setting. However, when the time comes to apply mathematics in external courses, many students act as if they have never encountered the mathematical concepts; their knowledge retention is near zero without the aid of a prompt review. This phenomenon was observed by Dr. Brooks while attending a statics lecture in Dr. McDonald's class. Dr. McDonald stated that since an angle θ was small, he was going to replace $sin(\theta)$ with θ. When Dr. Brooks pointed out that Dr. McDonald was using the local linear approximation for the sine function, a concept introduced in Calculus I the previous semester, the students had a look of bewilderment on their faces. By not exposing students to mathematics 'in the wild', that is, utilizing mathematics as it occurs in external disciplines, instructors are doing their students a great disservice. The mathematical rules and steps that students want to blindly follow need the support of reason and the utility of application to become memorable and pertinent.

Engineering students work hard to learn calculus and differential equations. These are the tools used to develop the models of physical phenomena that engineers use to solve problems. In their engineering courses, the instructors often brush over the mathematical development of the models and, in the interest of time, simply jump into applying the resulting equations. When the model development isn't explained, emphasized, and practiced, the students only become proficient in solving the equations and computing answers. They aren't required to use their calculus skills, which they then soon forget; they fail to develop the conceptual understanding that builds their confidence, allows them to solve more difficult problems, and provides context for the computed answer. Recently, a diligent statics student followed the author's prescribed steps to locate the centroid of the area under a parabola using equations found on the back cover of the text. The correct answer was computed; however, the student had no idea how, why, or what the meaning was. Figuring out how to find the *abc*'s in $y=ax^2+bx+c$ and then integrate xdA to get the answer without using the equation on the back cover was not considered, even though the integration process had been the focus of a lecture two hours earlier and the student wanted to know how the equation was developed. Was the math hard? No. Because the student couldn't get beyond the equation in order to understand the phenomena (the moment of an area about an axis), comprehending the solution was impossible.

The past 50 years has ushered in an era of technology unparalleled in history. If not guided in its proper usage, students are in jeopardy of becoming so dependent upon it that they cannot function without it. When this happens, they aren't solving problems: an engineering trait. Instead, they are simply computing values: a technician trait. Blindly entering values and reporting a numerical result does not constitute solving a problem. Finding roots on a graphing calculator by scrolling the birdie to zero doesn't qualify as factoring equations. One day, a student actually became curious and brave enough to ask the professor, "Exactly what does the EXP key do?" Technology exists to support, not replace, mathematics. Engineers, whether students or professionals, need to remember that problem solving involves understanding the entire process; a black box solver should never be trusted! It is shameful that students and graduates alike pick up a calculator to work simple sums, products, and functions that they should be exercising their minds to determine. Calculators, spreadsheet templates, computer programs, and other technological devices save a great deal of time. They aren't bad -- they just shouldn't be used blindly. Users need to understand the basis and limitations of any technology before relying on it.

Within any STEM field, a skill that requires careful development is that of effectively communicating solutions. In high school math and science courses, the work that students are often required to show in their solutions is minimal. For full credit, high school students are accustomed to simply writing their answers down in a list. In college-level math, science, and engineering courses, they quickly learn that showing their work is not just encouraged, it is required! Some students have never had to show any work, and they really don't know how. In practice, just knowing how to find the answer is not enough. Presenting and defending a solution requires that the solution be supported with dialogue explaining what was done and why it was done. Students cannot create that dialogue without looking beyond the equations. They have to understand the model and the mathematics in order to explain it, and without an explanation, most solutions are of little value.

Interventions

After Dr. McDonald and Dr. Brooks' experience in the spring semester of 2019, they recognized that they could address many of the aforementioned issues by adjusting their instructional styles in the core lower-level engineering courses. Indeed, in their article entitled "Why They Leave: Understanding Student Attrition from Engineering Majors," Geisinger and Raman note that "teaching styles were more important in predicting student success in the classroom than was the students' amount of precollege preparation, a finding that suggests that engineering instructors can play a crucial role in increasing retention. [3]" What follows are some of the more significant pedagogical adjustments Dr. Brooks and Dr. McDonald made.

Dr. Brooks' intention for enrolling in the statics course for credit was to gain a better understanding of how calculus and differential equation concepts were utilized in engineering courses, and, ultimately, she wanted to tailor her teaching of mathematics to engineering students. One of the first things that she changed within her classroom was her emphasis on geometric interpretations. Research has shown that when students are able to visualize mathematics, they are able to achieve a deeper understanding [4]. Being able to 'go through the mathematical motions' to achieve the correct answer is one thing, but understanding the meaning behind concepts is quite another. Throughout her experience in statics, it became evident early on that geometric interpretations are the key to identifying solution paths in many engineering problems. As a result, whenever possible, Dr. Brooks made sure to not only explain new mathematical concepts but also show them using cartoons and sketches. For example, when introducing double integration as a tool to compute areas in multivariable calculus, Dr. Brooks would illustrate that *dA* represents an infinitesimally small patch of area on a surface, and the integral serves to add up all of the patches of area. This concept is employed in statics when computing centroids. Similarly, within single variable calculus, Dr. Brooks would repeatedly refer to Riemann sums to illustrate that integration is merely the sum of areas of infinitesimally small rectangles. In guiding students to think about seemingly abstract concepts, such as integration, with a geometric mindset, students are better able to apply these concepts to coursework later on in other disciplines. This intervention discourages the familiar method of rote learning and focuses on critical thinking and conceptual understanding.

Prior to the spring semester of 2019, Dr. Brooks always included applied mathematical problems within her calculus and differential equations courses; she recognized the value of students making connections between new mathematical concepts and how they relate to the world. However, after her humbling experience as a student, she also began emphasizing including units with answers, when applicable, as the units often convey as much information as the numerical value. In addition, she would discuss whether the numerical value obtained for that question made sense in the context of the problem. All too often, students simply write down an answer because 'their calculator said so'. Discussing the validity of a particular answer aids in training students to pause, assess their answer, and determine whether the problem merits closer examination. The skill of self-assessment is so important that authors of university textbooks, such as Beer and Johnston, have begun to explicitly define and instruct it in their textbooks [5]. Undoubtedly, this particular skill is one that is necessary for all aspiring engineers to master as they prepare to enter the workforce -- where applied problems exist in the wild and have different flavors like 'pounds' and 'newtons'! For some students, the units and the equations are different processes -- unrelated to each other. The reality is that in every solution, they are related, and when students begin to understand the relationships and/or differences, especially between mass and force, then they begin to look beyond the numerical value and investigate the meaning of their answer.

Beginning in the fall of 2017, Dr. Brooks began incorporating learning assistants (LAs) in her Calculus I and Calculus II classrooms. The concept of LAs was first introduced at the University of Colorado in Boulder in 2012, and the LA model has now been adopted by institutions across the globe [6], [7]. The main goal of incorporating LAs into classroom instruction is to aid in transforming the setting to an active learning environment. This transition, in conjunction with interactions with LAs, has been shown to increase student performance [8]. In Dr. Brooks' LA-assisted courses, rather than lecturing and expecting students to mindlessly copy down solutions and memorize steps, students work on problems in class on their own or in groups, with assistance from the LAs and Dr. Brooks. This pedagogical approach discourages rote learning while encouraging critical thinking and collaboration. Additionally, as a result of her experience in statics, Dr. Brooks began having the LAs speak to the students about appropriate study habits, balancing work loads, and other essential topics with which students often struggle. Such interactions among students and peer mentors aid in improving student attitudes and their sense of belonging at the institution.

Having Dr. Brooks attend the statics class as a student caused a great deal of reflection on Dr. McDonald's part. It quickly became clear that many mathematical processes were only lightly touched upon, in favor of dwelling on ‘this is how you work the problem’ discussions. Having a mathematician in the audience makes one pay attention to how mathematics is presented and applied. Engineers often take mathematical shortcuts without providing much in the way of explanation; this was quickly pointed out!

One of the first opportunities that engineering students have to apply their newly acquired knowledge of calculus is in statics determining centroids. As an instructor, it is tempting to believe that the initial fundamental concepts are already mastered in prerequisite courses and that they only need to be lightly covered. Those initial topics, regardless of how fundamental, need to be well-developed. The time spent explaining each concept pays dividends later when students are able to smoothly solve problems because they fully understand the basic principles. Furthermore, by fully explaining those first concepts, students come to expect full explanations of all the following concepts. Dr. McDonald now spends more time in the details of those introductory concepts such as unit vectors, vector products, and simultaneous equations, which he previously assumed to be prerequisite knowledge. Later on, he doesn’t shy away from employing calculus, even for the really simple problems. When introducing the centroid of an area determination, integrate the square, then a triangle and eventually a parabola. When students master the simple problems and really understand the development, then they are prepared to work on more complex problems. They need to practice on a lot of simple problems, building their understanding and confidence, before moving on to difficult problems.

All of the class time cannot be spent deriving equations. Demonstrating problem solving at the board is also required; however, the students are better served if they solve the problems on their own rather than simply watching the instructor. The ‘flipped’ classroom [9] is an excellent example of where students are mentored through solving problems rather than being ‘shown’ how to work problems. Repeatedly showing students a solution method leads to rote memorization of the steps and little understanding of what each step accomplishes. Having students answer their own how-to questions amongst themselves leads to more experimentation and results in greater understanding. They begin looking beyond the equations and into the behaviors.

Today’s technology enables lightning fast, precise calculations. Students need to learn how to exploit the incredible computational power available to them without becoming dependent on it. Using spreadsheets to do homework exposes students to one of today's most commonly used office tools and opens the door to an astonishing assortment of computational tools while engaging them in the development of fully explained (using English) and legibly presented solutions [10]. Spreadsheet use is introduced in most primary school curriculums, and engineering professors generally assume that their students are competent users. Typically, they are not; however, engineering graduates should be, and the way they become competent is through practice. Dr. McDonald assigns one problem each week to be completed and fully explained using a spreadsheet. He assigns an additional ten ‘study’ problems that only require work notes. Many students catch on quickly and complete all of the problems using spreadsheets. Filling in a spreadsheet template doesn’t expand a student’s knowledge of a subject very much, but programming a solution in a spreadsheet does. Programming requires the student to thoroughly investigate the problem, understand the mathematical formulation, and explore all of the possible avenues toward the solution until the correct pathway has been identified. Requiring students to document and explain the solution further develops their knowledge because they have to communicate what they are thinking about and what they are doing. They are developing creative problem-solving skills. Even if they elect to plagiarize, documenting answers from a solutions manual is surprisingly difficult since the publishers seldom provide more than calculations. To explain the calculations, students have to look beyond the equations. Students should implicitly understand that units must be associated with input values and the results. Unfortunately, this often isn’t the case, and they may require an explicit instruction. By preparing spreadsheet solutions, students develop organization skills, better understand the mathematical procedures, and discover that they can experiment with solutions -- something they never did before. The detailed preparation requires more time and that results in greater retention. The transition from turning in a list of values, which they called answers in high school, to submitting well-presented and defended senior-level engineering problem solutions is not instantaneous. It takes time to develop, so start early and be persistent.

Exams are traditionally thought of as assessment tools rather than learning opportunities. With COVID-19 and the global pandemic, the ability to give a traditional exam was essentially eliminated and take-home exams became much more desirable [11], [12], [13]. Dr. McDonald’s take-home exams utilize a measure of trust in student integrity. Questions are developed so that answers cannot be easily found via Google, and students must explain their work rather than simply showing an equation and computing a result. In a take-home exam, the mathematics must be developed and explained as well as worked! Take-home exams take more time for students to complete and that additional time spent may result in greater retention.

While teaching in the COVID-19 alternative delivery mode, Dr. McDonald uploaded a take-home statics quiz, and Dr. Brooks, now a statics graduate, accidentally dropped into one of her virtual meetings that several of her calculus II students were using to collaboratively prepare for the statics quiz. Dr. McDonald had posed the general problem configuration on the course web page ahead of the quiz, but the actual questions were only available when the quiz was opened and a twenty-minute timer began. The students were diligently deciphering the 3-D geometry of a frame and the applied forces, checking coordinates, working out position vectors, and trying to predict what they could pre-work that would save time once they began the quiz. This kind of collaboration, under the pressure of a pending quiz, can be beneficial. As a result of their focus, those students likely learned more in that one study session helping each other than in the prior week solving the ten study problems. During the study session, they desired knowledge instead of answers. Since they didn’t know the questions, they wanted to understand why and how specific mechanisms worked. The quiz became a learning experience as well as an assessment tool, hopefully it resulted in improved knowledge retention, and it encouraged students to demonstrate their math knowledge to a greater extent than working the same question in a classroom exam.

Results

The Western Illinois University Quad Cities campus in Moline, Illinois is a branch of the main campus located 90 miles away in Macomb, Illinois. The total Quad Cities campus enrollment, as of Spring 2021, is just shy of 800 [14]. The School of Engineering was formed in 2009 and is located on the Quad Cities campus. It currently offers degree programs in general, mechanical, civil, and electrical engineering and consists of 138 undergraduate students -- 25% of the undergraduate population on the branch campus. The small enrollment at the Quad Cities campus results in small class sizes. Within the School of Engineering, lower division (freshman/sophomore) core courses typically enroll twenty to thirty students and the upper division (junior/senior) discipline specific courses enroll five to fifteen students.

After three and a half semesters of changes in teaching strategies at Western Illinois University's Quad Cities campus, Dr. McDonald and Dr. Brooks have gleaned both quantitative and qualitative data that indicates the aforementioned interventions have made a difference in student awareness and appreciation of mathematics. To begin, we will analyze the trend in declared math minors on the Quad Cities campus.

Prior to 2016, it was not possible for students on the Quad Cities campus to earn a minor in mathematics because mathematics courses outside of those required for the engineering majors were not being offered. During the summer of 2016, linear algebra was offered as an online summer course, and as a result, Quad Cities students were able to complete a minor. Interest in the minor has remained strong and is growing. The following table displays the number of engineering students who declared mathematics as a minor, as well the corresponding percentage of the students enrolled in the School of Engineering.

Year (Spring Semester)	**2017**	**2018**	**2019**	**2020**	**2021**
Number of Math Minors on Campus	16	20	26	25	21
Cumulative Number of Engineering Students	154	108	128	144	138
Percentage of Engineering Students Declaring Math as a Minor	10%	19%	20%	17%	15%

Table 1: Mathematics Minors on the Quad Cities Campus. [15]

While the percentage of engineering majors who have declared math as a minor has slightly decreased, the fact that it has held relatively steady despite the global pandemic is telling. Indeed, as a result of COVID-19, many students have reduced their course load or even withdrawn entirely. The fact that the percentage of engineering students who have declared mathematics as a minor is within 5% of the historical high indicates that engineering students remain interested in mathematics as well as recognize its value as a supplemental (and not required) minor to accompany their engineering major.

Perhaps more telling than the trend in declared mathematics minors is that in student retention. It is not uncommon for a percentage of engineering students to drop out or switch majors during their freshman year. At Western Illinois University, the standard courses taken by engineering students during their freshman year include Calculus I, Physics I, and a CAD course in the fall followed by Calculus II, Physics II, and Statics in the spring. These courses heavily emphasize mathematics and/or mathematical modeling. The table below displays the number of new freshman engineering students (both full and part-time) that began in the fall and were retained in the spring, as well as the corresponding percentage of retained students.

Academic Year	Starting Number	Number Retained	Percent Retained
2016 - 2017	17	14	82%
2017 - 2018	12	9	75%
2018 - 2019	16	11	69%
2019 - 2020	19	13	68%
2020 - 2021	12	11	92%

Table 2: Retention Rates of New Freshman Engineering Majors on the Quad Cities Campus. [16]

As was mentioned earlier, the global pandemic brought on unprecedented challenges within higher education. In spite of the associated complications, the freshman retention rate held steady during the 2019-2020 academic year. Furthermore, it increased to an all-time high for the 2020-2021 academic year. Although the number of new freshmen within the School of Engineering dropped overall, the increase in retention resulted in an equivalent raw number of students retained in the spring of 2021 when compared to the spring of 2019 and an increase from the spring of 2018. This drastic increase in the freshman retention rate illustrates an increase in freshman student success in mathematics courses (calculus) as well as applied mathematics courses (physics and statics). In comparison, the ASEE "Engineering by the Numbers" survey indicated that the overall student persistence to the second year was around 80% in 2014 [17].

What the above quantitative data fails to effectively convey are students' attitudes toward mathematics. In Dr. Brooks' course evaluations at the end of the fall semester of 2020, one of the sophomore Calculus III students, an engineering major, commented "I really enjoyed this class, and [Calculus] has been my favorite series of classes here at WIU." This student's comment illustrates that he/she does not view mathematics as a 'necessary evil' dictated by a degree plan; rather, this student recognizes its worth and expresses his/her appreciation for the three-semester sequence. Students also recognize the value of understanding the 'why' behind the procedures for solving calculus problems. In the Calculus I course evaluations in the fall of 2019, one student wrote, "She is always able to provide an explanation of why things in Calculus must be done a certain way, which really helps to further my understanding." By encouraging students to focus not only on the 'how' but also the 'why', they begin to understand mathematics on a deeper level. An increased mathematical awareness will help students retain their understanding throughout their life.

A recent engineering graduate sent a gratifying note to Dr. McDonald thanking him for the forced introduction into using mathematics in spreadsheets to solve engineering problems. He now uses spreadsheets that he develops every day at work! He has advanced ahead of other junior level engineers because he documents and explains his calculations and solutions. Four years ago, when Dr. McDonald first started requiring spreadsheet work, he observed students in the computer lab grinding numbers through their calculator and entering the results into the spreadsheet they were preparing for his assignment! That hasn't happened recently. Students now are quicker to adopt the spreadsheet solution concept in other courses when not required, mostly for the improved legibility, but also for the calculation abilities.

Encouraging students to collaborate with each other has been productive. Many of the interventions promote collaboration and peer-to-peer learning, such as the LAs and even the take-home exams. For some students, spreadsheets are tough to initially figure out, but their peers that demonstrate some proficiency early on are easily identified and usually willing to help. The initial help sessions are great to observe. Although they are guiding this cell to that cell operations, the students are also discussing the mathematics taking place in each cell and 'what the mathematics is doing' rather than 'it has to be this value'.

Conclusion

Engineers are problem solvers. They provide the means for clean water, safe transportation, economical energy, and other processes or devices that improve the way our modern society and civilization functions. They use physical models described by mathematical equations to communicate and solve problems; therefore, they should be using mathematics within their profession.

While studying engineering, the most successful students often share similar traits; they are careful, innovative, and reflective. They understand how equations are derived, they recognize constraints and limitations, and they identify how to best apply this knowledge. These students are also proficient in estimating solutions and recognizing unrealistic results. Conversely, less successful students are often complacent; they rely on rote memorization and procedural approaches. They can calculate an answer but lack a deep understanding of the mechanics supporting the answer, and they fail to recognize improbable results. They are more likely to 'abuse' equations by using ill-conditioned or improper input as opposed to recognizing that the equation is but one small part of a larger and more intricate solution path.

By intentional and mindful development of their teaching pedagogy, professors of mathematics, science, and engineering can improve student awareness of mathematics in problem solving by continuously exposing not only the design equations but also their development and the physical behaviors they represent to their students. In particular, when instructors adapt their teaching style to encourage students to look beyond the equations, teaching them to look *for* places to use mathematics within their engineering courses, the students will rise up to the challenge. When unique problems arise, these students are then able to recognize the limitations of a coded solution, common errors in assumptions, or the potential for failure when a published solution is not properly applied. As a result of their diligence, these students have a better understanding of the application of mathematics and an ability to incorporate those applications into their professional work throughout their life. These students become the engineers that expand the boundary of human knowledge. They look beyond the equations.

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