AC 2007-802: MATHCAD FUNCTIONS FOR THE THERMODYNAMIC PROPERTIES OF MOIST AIR, AMMONIA, PROPANE, AND R-22

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MathCAD Functions for the Thermodynamics Properties of
Moist Air, Ammonia, Propane, and R-22

Abstract

MathCAD functions were constructed to evaluate the thermodynamic properties of moist air, ammonia, propane, and Refrigerant 22. The functions were constructed to ease the need for time-consuming interpolation using tabularized thermodynamic data while reinforcing the functional representation of traditional thermodynamic property tables. Three examples are provided to demonstrate the use of the functions in an undergraduate applied thermodynamics course. The examples include the analysis of an evaporative cooling system, the analysis of a cascade, reversed-Rankine cycle refrigeration system, and the design of a propane based heat pump system. The examples are presented in their entirety in appendices to demonstrate the report-quality worksheets possible using MathCAD. The efforts described are an extension of earlier efforts to develop a complete set thermodynamic property functions needed to teach an undergraduate applied thermodynamics course. While the function development was directed at the applied thermodynamics course, which is commonly taught early in the undergraduate mechanical engineering curriculum, the functions are excellent tools for upper-level electives such as HVAC, internal combustion engines, thermal systems design, and turbomachinery.

Introduction

Learning with a combination of a textbook and a software package is a contemporary engineering-thermodynamics pedagogy. Many software tools are available for evaluating thermodynamic properties of engineering fluids. Many of these software tools are proprietary packages sold by textbook publishers, such as "Interactive Thermodynamics: IT" [1]. In fact, finding a thermodynamics text that does not come with a software package is difficult. Some textbooks are now built around using a software or web-based internet package [2]. While many educational software packages are available for evaluating thermodynamic properties, evidence that shows that practicing engineers continue to use these thermodynamic-property software packages after entering the workforce is not readily available.

MathCAD, MatLab, and Engineering Equation Solver (EES) are all powerful computational and analytical packages [3,4,5]. Many schools teach and require the use of a computational tool such as MathCAD, MatLab, or EES [6]. From informal conversations with engineers who learned to use one of these computational tools, many of them continue to use these tools after graduation. Developing extensions or toolkits for software that the students will use after graduation seems more appropriate than developing complete software packages that will only be used by students in an educational environment. Because of the need for thermodynamic property functions for the widely used computational tools, functions were generated to evaluate the thermodynamic properties of water, R-134a, air, and twelve species of the CHON system in MathCAD. Equilibrium functions for eight independent reactions involving species of the CHON system are also included. The function set created provides the minimum number of thermodynamic functions required to teach a two-course sequence in undergraduate engineering thermodynamics.

Each computational analysis package has strengths and weaknesses when compared to the others. EES also has the thermodynamic functions discussed here (and functions for many other fluids). While EES is not a traditional structured programming language, the appearance of EES programs is similar to the appearance of C or FORTRAN programs, and some users find the unit conversion procedures awkward in EES.

Recent efforts to implement thermodynamic property functions in spreadsheet programs such as MS Excel have also been published [7,8]. These efforts were initiated because MS Excel is inexpensive when included with a standard Microsoft Office installation. Property functions for MS Excel would certainly be useful for corporate engineers after graduation. However, the educational benefits of using MS Excel are arguable. The cellular nature of spreadsheets limits functionality and readability of engineering calculations, the syntax required is just as complicated and difficult to read as C or FORTRAN, and MS Excel provides only limited automatic unit conversions.

As the case with all thermodynamic property evaluation software, the intent was 1) to take some of the effort and distraction away from working problems requiring extensive interpolation in the tables and 2) to allow the students to analyze more complicated problems. MathCAD was chosen for this project because of its mathematical report appearance, because of its ability to perform calculations with automatic unit handling and conversion, and because of its wide use in the Department of Mechanical Engineering at the University of Alabama at Birmingham.

While developed for a two-course sequence in undergraduate thermodynamics, the MathCAD function sets are useful for many other engineering courses. The functions presented in this paper and the thermodynamic functions presented in previous works have been found to be very useful in courses such as internal combustion engines, propulsion, combustion, compressible flow, and heating ventilating and air conditioning [9, 10, 11, 12].

Referencing the Function Sets in MathCAD

The functions are contained in four MathCAD worksheets. Table 1 presents the files and the information contained in each file. To use the functions in a new MathCAD worksheet, the information in the property worksheets does not have to be copied into the new worksheet. The function worksheet may be referenced by using the <Insert, Reference> command, and identifying the file required. When this is done correctly, a statement similar to

> Reference:C:\ThermoII\SteamProps.mcd

will appear in the worksheet. All functions generated in referenced file will then be available for use in the new worksheet.

All of the functions generated are available to the educational public on the condition that if the functions are used for instruction, the instructor agrees to have his students complete a survey form at the end of the course. The function sets may be obtained by emailing the author at *smcclain@uab.edu*. Please include the subject line "MathCAD: Thermo Function Set Request" in your email. The reader should note that *all* functions presented in this paper are for

educational use only, and that the user assumes all responsibility for calculations performed using the functions if used for an industrial application.

Table 1. Property Function Files

File	Information Contained
R22Props.mcd	Thermodynamic properties of R-22
AmmoniaProps.mcd	Thermodynamic properties of Ammonia
PropaneProps.mcd	Thermodynamic properties of Propane
SteamProps.mcd	Thermodynamic properties of water and moist air

Phase Change Substances

A consistent naming scheme was chosen for the phase change fluids. The function calls for *R-22, ammonia, and propane* begin with the variable to be determined, followed by an underscore, followed by the independent property(s), followed by a fluid identifier (*R22, NH3, or C3H8*), followed finally by the values of the independent property(s) in parenthesis. Figure 1 presents the format of function calls for the phase change substances. The functions require that all independent properties be entered with the appropriate absolute units.
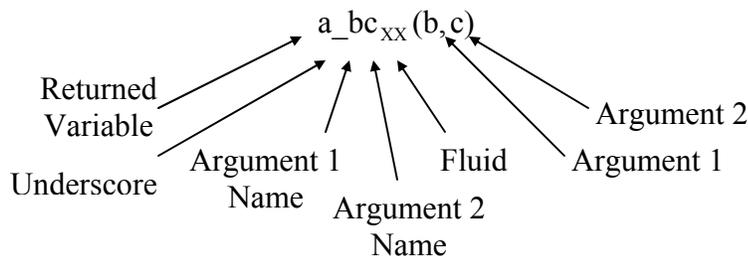

Figure 1. Scheme for Naming Property Functions

The functions were constructed to reflect and reinforce how property information is provided in traditional property tables. Functions are provided for the saturation pressure based on temperature and for the saturation temperature based on pressure. Saturated liquid and saturated vapor functions are provided for specific volume (v), specific internal energy (u), specific enthalpy (h), and specific entropy (s) based on either temperature or pressure. Functions for v, u, h, and s of saturated mixtures are also provided as functions of quality and either temperature or pressure. Functions that return quality are provided. Functions that return temperature or enthalpy as functions of pressure and entropy, often required in the analyses of Rankine and reversed-Rankine cycles, are provided. Finally, a function that determines temperature based on pressure and enthalpy, often used when analyzing reversed-Rankine cycles, is provided. Table 2 presents a summary of the functions provided for the phase-change substances. The following sections describe the source of the equation of state and any corrections or adaptations that were made to the property formulations.

Table 2. Functions for the Phase Change Substances

Function or Constant	Description
Tsat_P$_{XX}$(P)	Returns the saturation temperature as a function of pressure
Psat_T$_{XX}$(T)	Returns the saturation pressure as a functions of temperature
zf_T$_{XX}$(T)	Returns the z property (v, u, h, or s) for saturated liquid as a function of temperature
zf_P$_{XX}$(P)	Returns the z property (v, u, h, or s) for saturated liquid as a function of pressure
zg_T$_{XX}$(T)	Returns the z property (v, u, h, or s) for saturated vapor as a function of temperature
zg_P$_{XX}$(P)	Returns the z property (v, u, h, or s) for saturated vapor as a function of pressure
z_Tx$_{XX}$(T,x)	Returns the z property (v, u, h, or s) for a saturated mixture as a function of temperature and quality
z_Px$_{XX}$(P,x)	Returns the z property (v, u, h, or s) for a saturated mixture as a function of pressure and quality
z_TP$_{XX}$(T,P)	Returns the z property (v, u, h, or s) for either a compressed liquid or a superheated vapor as a function of temperature and pressure
x_Tz$_{XX}$(T,z)	Returns the quality of a saturated mixture as a function of the temperature and the z property (v, u, h, or s)
x_Pz$_{XX}$(T,z)	Returns the quality of a saturated mixture as a function of the temperature and the z property (v, u, h, or s)
T_Ps$_{XX}$(P,s)	Returns the temperature as a function of pressure and entropy
h_Ps$_{XX}$(P,s)	Returns the enthalpy as a function of pressure and entropy
T_Ph$_{XX}$(P,h)	Returns the temperature as a function of pressure and enthalpy

R-22

The equation of state used in the formulation of the R-22 function set is that proposed by Wagner, Marx, and Pruβ [13]. The R-22 formulation is based on the dimensionless Helmholtz free energy and covers the entire fluid region from 116 K to 550 K at pressures up to 200 MPa.

The R-22 function set was only tested at maximum pressures and temperatures of 2.4 MPa and 450 K, respectively, as these are the high extreme values found for R-22 in most thermodynamics textbooks. The range of validity of the R-22 function set is slightly smaller than that identified by Wagner et al. Problems evaluating properties as functions of pressure and entropy were identified around the critical point.

Ammonia

The equation of state used in the formulation of the ammonia function set is that proposed by Haar and Gallagher [14]. It is also based on the dimensionless Helmholtz free energy and covers temperatures from 195 K to 750 K and pressures from "dilute gas" to 500 MPa.

The ammonia function set was also tested at lower maximum pressures and temperatures, 2 MPa and 550 K, respectively, as these are the high extreme values for ammonia found in most thermodynamics textbooks. As with the R-22 function set, significant deviations from the experimentally measured ammonia property values were identified near the critical point.

The functions for ammonia were adapted because of inconsistencies found in the auxiliary equations for the saturated vapor and liquid densities. Despite many hours of effort, the source

of the problems could not be identified, hence new auxiliary equations for the saturated vapor and liquid densities were developed. Demonstrating MathCAD's powerful analysis tools, the new auxiliary equations were developed using MathCAD's "linfit" function. The "linfit" function generates the best-fit coefficients of a linear combination of functions [15]. Equation (1) presents the auxiliary equation for the saturated vapor density, and equation (2) presents the auxiliary equation for the saturated liquid density.

$$\frac{\rho_g}{\rho_c} = \exp\left[-\left(Cg_0\theta^{\frac{1}{2}} + \sum_{i=1}^{9} Cg_i\theta^i\right)^{\frac{1}{2}}\right] \tag{1}$$

$$\frac{\rho_f}{\rho_c} = \exp\left[\left(Cf_0\theta^{\frac{1}{2}} + \sum_{i=1}^{9} Cf_i\theta^i\right)^{\frac{1}{2}}\right] \tag{2}$$

Where

$$\theta = 1 - \frac{T}{T_c} \tag{3}$$

The constants for Eqs. (1) and (2) are listed in Table 3. Figures 2 and 3 present a comparison of the developed auxiliary equations to the values reported by Haar and Gallagher [14].

Table 3. Constants for the Ammonia Auxiliary Equations

Constant	Equation (1)	Constant	Equation (2)
Cg_0	1.49423985997742	Cf_0	1.26229794683888
Cg_1	11.5348327868205	Cf_1	0.2942327870764
Cg_2	171.453114991705	Cf_2	31.2736018575536
Cg_3	-1.34712691210935E+3	Cf_3	-480.651583063728
Cg_4	1.09383974999497E+4	Cf_4	3.82312159369239E+3
Cg_5	-5.06303055361903E+4	Cf_5	-1.80222499821142E+4
Cg_6	1.48581146651944E+5	Cf_6	5.20379044624135E+4
Cg_7	-2.60029819302634E+5	Cf_7	-9.04024162719172E+4
Cg_8	2.51215980424554E+5	Cf_8	8.68081267630708E+4
Cg_9	-9.98576487272334E+4	Cf_9	-3.54164892213865E+4

Propane

The equation of state used in the formulation of the propane Mathcad function set is that proposed by Miyamoto and Watanabe [16]. It is also based on the dimensionless Helmholtz free energy and covers temperatures from 85.48 K (triple point) to 623 K, and pressures to 103 MPa.
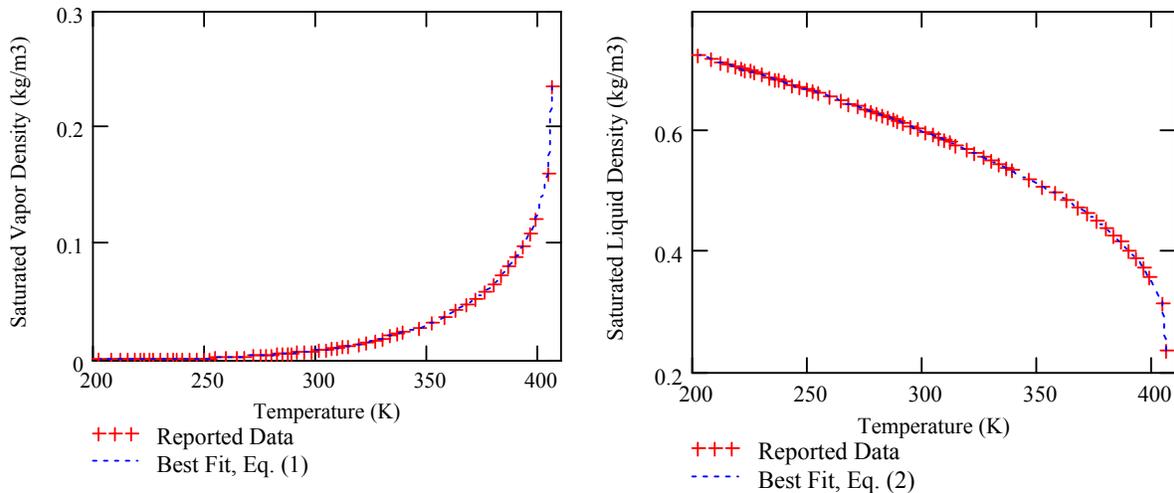
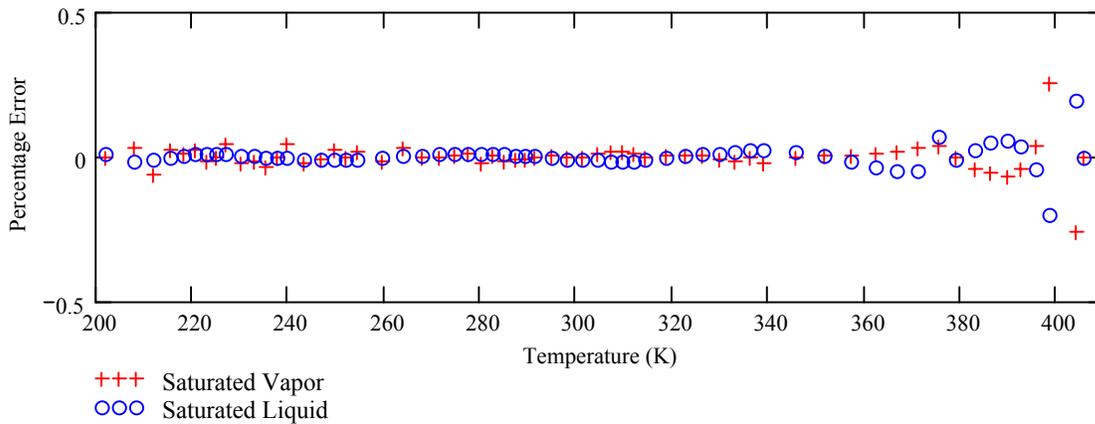Figure 2. The Ammonia Saturated Densities and Auxiliary Functions

Figure 3. The Percentage Error Associated with the Auxiliary Functions for Saturated Densities

The propane Mathcad function set was tested to maximum pressures and temperatures of 4.0 MPa and 473 K, as these are common maximum values for propane found in thermodynamic property tables of typical thermodynamic textbooks [17, 18, 19]. As with R-22 and ammonia, the propane functions significant errors around the critical point. The propane function set also exhibited substantial and unexplained errors in the compressed liquid region. To circumvent the errors in the compressed liquid region, users are encouraged to approximate specific volume, internal energy, and entropy as the saturated liquid specific volume, internal energy, and entropy at the given temperature. Enthalpy can be evaluated similarly, but requires accounting for the difference in pressure from the saturation pressure. Equations (4) through (7) present the compressed liquid approximations suggested for the propane function set.

$$v_{\substack{comp.\\liquid}}(T,P) \approx v_f(T) \tag{4}$$

$$u_{\substack{comp.\\liquid}}(T,P) \approx u_f(T) \tag{5}$$

$$s_{\substack{comp. \\ liquid}}(T,P) \approx s_f(T) \tag{6}$$

$$h_{\substack{comp. \\ liquid}}(T,P) \approx h_f(T) + v_f(T) \cdot (P - P_{sat}(T)) \tag{7}$$

Moist Air

Functions were developed for the properties of moist air above the triple-point temperature of water. The function equations are those presented in any undergraduate thermodynamics textbook [17, 18, 19]. However, the function results for enthalpy will differ from the ASHRAE psychrometric charts when using English/USCS units [20]. This difference is caused by the omission of the reference temperature in the enthalpy expressions used in the ASHRAE formulations. While the absolute values of enthalpy will not agree, differences (deltas) in enthalpy will be the same when evaluated using either the functions or the psychrometric charts.

Table 4 presents the twenty functions available for the thermodynamic properties of moist air. Because of the many parameters associated with moist air (T, P, P_v, ω_s, ω, ϕ, μ, h_{ma}, T_{dp}, T_{wb}, v_{ma}), a comprehensive set of functions for each parameter was not developed. The functions created are functions that are most commonly required for engineering processes such as moist air heating, cooling, and mixing.

While the set of thermodynamic functions for moist air is not comprehensive, the available functions enable users to create new functions as needed by nesting the distributed functions. For example, to create a function to evaluate the specific volume as a function of the temperature, enthalpy, and pressure, the user could nest the function ω_Th_{ma} into the $v\_T\omega P_{ma}$ function. The Mathcad construction would appear as:

$$v\_ThP_{ma}(T,h,P) := v\_T\omega P_{ma}\big(T, \omega\_Th_{ma}(T,h), P\big)$$

The user ability to nest available functions is a very powerful feature of Mathcad.

Examples

Three example problems are provided demonstrating the use of the function sets in the analysis of thermodynamic processes. The examples provided include 1) the analysis of a R-22 and ammonia cascaded refrigeration cycle, 2) the analysis of a propane based heat pump cycle, and 3) the analysis of an evaporative air cooling device. The complete example solutions are presented in the appendices.

R-22 and Ammonia Cascaded Refrigeration Cycle

Appendix A presents the analysis of a cascaded ammonia and R-22 refrigeration cycle with ammonia operating in the topping cycle. The parameters of the cascade cycle provided are the cold reservoir temperature, the ambient temperature, the ammonia mass flow rate and evaporator pressure, and the R-22 condenser pressure. The mass flow rate of R-22, the rate of heat transfer

Table 4. Functions for the Properties of Moist Air

Function	Description
$\omega\_PvP_{ma}(Pv,P)$	Returns humidity ratio as a function of vapor pressure and pressure
$\omega\_T\omega_{ma}(T, h)$	Returns humidity ratio as a function of temperature and enthalpy per unit mass of dry air
$\omega\_TTwbP_{ma}(T,Twb,P)$	Returns humidity ratio as a function of temperature (dry bulb), wet bulb temperature, and pressure
$\omega\_T\phi P_{ma}(T,\phi,P)$	Returns humidity ratio as a function of temperature, relative humidity, and pressure
$\omega\_TdpP_{ma}(Tdp,P)$	Returns humidity ratio as a function of the dew point temperature and the pressure
$\omega s\_TP_{ma}(T,P)$	Returns saturation humidity ratio as a function of temperature and pressure
$Pv\_\omega P_{ma}(\omega,P)$	Returns vapor pressure as a function of humidity ratio and pressure
$Pv\_T\phi_{ma}(T, \phi)$	Returns vapor pressure as a function of temperature and relative humidity
$\mu\_TPvP_{ma}(T,Pv,P)$	Returns degree of saturation as a function of temperature, vapor pressure, and pressure
$\phi\_TPv_{ma}(T,Pv)$	Returns relative humidity as a function of temperature and vapor pressure
$\phi\_T\omega P_{ma}(T, \omega,P)$	Returns relative humidity as a function of temperature, humidity ratio, and pressure
$h\_T\omega_{ma}(T, \omega)$	Returns enthalpy per unit mass of dry air as a function of temperature and humidity ratio
$h\_TTwbP_{ma}(T,Twb,P)$	Returns enthalpy per unit mass of dry air as a function of temperature (dry bulb), wet bulb temperature, and pressure
$h\_T\phi P_{ma}(T,\phi,P)$	Returns enthalpy per unit mass of dry air as a function of temperature, relative humidity, and pressure
$h\_TTdpP_{ma}(T,Tdp,P)$	Returns the enthalpy per unit mass of dry air as a function of the temperature, the dew point temperature, and the pressure
$T\_\omega h_{ma}(\omega,h)$	Returns temperature as a function of humidity ratio and the enthalpy per unit mass of dry air
$Tdp\_Pv_{ma}(Pv)$	Returns dew point temperature as a function of the vapor pressure
$Tdp\_\omega P_{ma}(\omega,P)$	Returns dew point temperature as a function of the humidity ratio and the pressure
$v\_T\omega P_{ma}(T,\omega,P)$	Returns the specific volume per unit mass of dry air as a function of the temperature, the humidity ratio, and the pressure
$v\_T\phi P_{ma}(T, \phi,P)$	Returns the specific volume per unit mass of dry air as a function of the temperature, the relative humidity, and the pressure

from the R-22 to the ammonia, the total power of the compressors, and the system COP_R are required results. The most significant aspect of this example solution is the use of multiple property function sets in the same worksheet. To obtain the required answers, the ammonia and R-22 functions must be used concurrently.

Propane Heat Pump Cycle

In Appendix B, a heat pump using propane as the working fluid is analyzed. In the problem statement, the evaporator temperature and the condenser pressure are specified. The required results are the mass flow rate of propane required to provide a set heating load, the pressure of the evaporator, the compressor exit temperature of the propane, the total power required by the compressor, and the COP_H of the heat pump. Appendix B demonstrates the usefulness of the propane functions.

Evaporative Air Cooling

An example analysis of air through an evaporative cooler system is presented in Appendix C. In the example, moist air enters a device at a steady rate. Cold liquid water is injected into moist air. The process cools the moist-air stream while increasing the relative humidity. The usefulness of the moist air functions is demonstrated in determining the temperature and relative humidity of the air exiting the device.

Discussion and Conclusions

As part of a continued effort to develop thermodynamic property function sets for use with Mathcad in applied thermodynamics courses, functions were developed to evaluate the properties of ammonia, propane, R-22, and moist air. The development of the function sets for use with Mathcad and differences in the function results compared to common textbook tables and charts were discussed. Examples demonstrating the use of the function sets related to applied thermodynamic problems were also provided.

The educational effectiveness of the current function sets has not been directly evaluated. However, the results of student surveys assessing the ease of use of Mathcad, the thermodynamic evaluation capabilities of Mathcad, and the overall student response to Mathcad employing the previously developed property function sets are presented in Ref. [12]. The primary results of the assessment were that 1) the functions enabled complex thermodynamic analyses that could not be performed by hand calculations, 2) the use of the functions eased the time an effort required to complete assignments, and 3) the students required very little preparation time for using Mathcad. The addition of function sets for moist air, R-22, ammonia, and propane was an effort to build on the strengths of the Mathcad function sets identified in previous publications and assessments.

Acknowledgements

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Appendix A - Cascaded Refrigeration Cycle Example

→ Reference:C:\ThermoII\AmmoniaProps.mcd

→ Reference:C:\ThermoII\R22Props.mcd

Problem Statement: An ideal cascade refrigeration cycle is used to maintain a cold reservoir temperature of -45 C. The ambient temperature is 25 C. Ammonia and R22 are used in the topping and bottoming cycles, respectively. The ammonia exits the heat exchanger at a pressure of 100 kPa. The R22 exits the heat exchanger at a pressure of 200 kPa. The mass flow rate of the ammonia is 0.2 kg/s. Assuming the heat exchanger is adiabatic and the compressors are isentropic, answer the following questions.

1. What is the mass flow rate of the R22?
2. What is the rate of heat transfer from the R22 to the ammonia?
3. What is the total work required by the compressors?
4. What is the COP of the system?

Sketches: A system schematic is presented below.
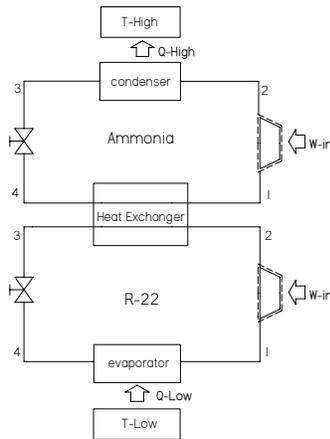
Solution: Required engineering units are defined. The design parameters from the problem statement are entered to begin the solution.

$$kPa := 10^3 \cdot Pa$$

$$T_H := (273 + 25) \cdot K \qquad T_L := (273 - 45) \cdot K$$

$$P_{L\_NH3} := 100 \, kPa \qquad P_{H\_R22} := 200 \, kPa$$

$$mdot_{NH3} := 0.2 \cdot \frac{kg}{s} \qquad \Delta T := 10 \, K$$

The minimum condenser temperature and evaporator temperature for the ammonia and R22, respectively, are then evaluated using a reasonable temperature difference to achieve the required heat transfer.

$$T_{cmin\_NH3} := T_H + \Delta T \qquad T_{cmin\_NH3} = 308K \qquad \frac{T_{cmin\_NH3} - 273 \cdot K}{K} = 35$$

$$T_{evap\_R22} := T_L - \Delta T \qquad T_{evap\_R22} = 218K \qquad \frac{T_{evap\_R22} - 273 \cdot K}{K} = -55$$

The condenser pressure and evaporator pressure for the ammonia and R22, respectively, are then set as the saturation pressures at the minimum condenser and evaporator temperatures, respectively.

$$P_{cond\_NH3} := Psat\_T_{NH3}(T_{cmin\_NH3}) \qquad P_{cond\_NH3} = 1.344 \times 10^3 \, kPa$$

$$P_{evap\_R22} := Psat\_T_{R22}(T_{evap\_R22}) \qquad P_{evap\_R22} = 49.073 kPa$$

With the important temperatures and pressures specified, the cycle analysis follows. The ammonia cycle is analyzed first. The ammonia is assumed to be a saturated vapor at the exit of the heat exchanger (state 1).

Appendix A - Cascaded Refrigeration Cycle Example (continued…)

$$h_{1\_NH3} := hg\_P_{NH3}(P_{L\_NH3}) \qquad h_{1\_NH3} = 1.399 \times 10^3 \frac{kJ}{kg}$$

$$s_{1\_NH3} := sg\_P_{NH3}(P_{L\_NH3}) \qquad s_{1\_NH3} = 5.84 \frac{kJ}{kg \cdot K}$$

The compressor is assumed to be isentropic. The temperature at state 2 (the maximum cycle temperature) and the enthalpy at state 2 may be determined since the pressure is known and the entropy at state 2 equals the entropy at state 1.

$$T_{max\_NH3} := T\_Ps_{NH3}(P_{cond\_NH3}, s_{1\_NH3}) \qquad T_{max\_NH3} = 432.732K$$

$$\frac{T_{max\_NH3} - 273 \cdot K}{K} = 159.732$$

$$h_{2\_NH3} := h\_Ps_{NH3}(P_{cond\_NH3}, s_{1\_NH3}) \qquad h_{2\_NH3} = 1.803 \times 10^3 \frac{kJ}{kg}$$

The maximum cycle temperature for the ammonia is 159.7 C. For an ideal Rankine cycle, state 3 is assumed to be a saturated liquid at the condenser temperature. Thus the enthalpy at state 3 is:

$$h_{3\_NH3} := hf\_T_{NH3}(T_{cmin\_NH3}) \qquad h_{3\_NH3} = 346.449 \frac{kJ}{kg}$$

Since the trap produces no work and is assumed to be adiabatic, the enthalpy at state 4 equals the enthalpy at state 3.

$$h_{4\_NH3} := h_{3\_NH3} \qquad h_{4\_NH3} = 346.449 \frac{kJ}{kg}$$

The R22 cycle is analyzed next. The R22 is assumed to b a saturated vapor at the exit of the evaporator (state 1).

$$h_{1\_R22} := hg\_P_{R22}(P_{evap\_R22}) \qquad h_{1\_R22} = 226.15 \frac{kJ}{kg}$$

$$s_{1\_R22} := sg\_P_{R22}(P_{evap\_R22}) \qquad s_{1\_R22} = 1.04 \frac{kJ}{kg \cdot K}$$

The compressor is assumed to be isentropic. The temperature at state 2 (the maximum cycle temperature) and the enthalpy at state 2 may be determined since the pressure is known and the entropy at state 2 equals the entropy at state 1.

$$T_{max\_R22} := T\_Ps_{R22}(P_{H\_R22}, s_{1\_R22}) \qquad T_{max\_R22} = 275.808K$$

$$\frac{T_{max\_R22} - 273 \cdot K}{K} = 2.808$$

$$h_{2\_R22} := h\_Ps_{R22}(P_{H\_R22}, s_{1\_R22}) \qquad h_{2\_R22} = 258.425 \frac{kJ}{kg}$$

Appendix A - Cascaded Refrigeration Cycle Example (continued…)

The maximum cycle temperature for the R22 is 2.8 C. For an ideal Rankine cycle, state 3 is assumed to be a saturated liquid at the condenser temperature, or in this case, at the exit of the heat exchanger. This temperature is determined by finding the saturation temperature associated with the pressure of the R22 through the heat exchanger. Thus the enthalpy at state 3 is:

$$T_{he\_R22} := Tsat\_P_{R22}(P_{H\_R22}) \qquad\qquad T_{he\_R22} = 247.967K$$

$$h_{3\_R22} := hf\_T_{R22}(T_{he\_R22}) \qquad\qquad h_{3\_R22} = 16.493\frac{kJ}{kg}$$

Since the trap produces no work and is assumed to be adiabatic, the enthalpy at state 4 equals the enthalpy at state 3.

$$h_{4\_R22} := h_{3\_R22} \qquad\qquad h_{4\_R22} = 16.493\frac{kJ}{kg}$$

With all of the states evaluated, the cycle analysis proceeds quickly. The mass flow rate is determined by performing a thermodynamic analysis on the heat exchanger.

$$mdot_{R22} := \frac{mdot_{NH3}\cdot(h_{1\_NH3} - h_{4\_NH3})}{h_{2\_R22} - h_{3\_R22}} \qquad\qquad mdot_{R22} = 0.87\frac{kg}{s}$$

The rate of heat transfer from the R22 to the ammonia is determined below.

$$Q_{R22\_to\_NH3} := mdot_{R22}\cdot(h_{2\_R22} - h_{3\_R22}) \qquad Q_{R22\_to\_NH3} = 210.498kW$$

The total work required by the compressors is determined by multiplying the mass flow rates and the enthalpy changes for each compressor, and adding the results.

$$W_{NH3\_comp} := mdot_{NH3}\cdot(h_{2\_NH3} - h_{1\_NH3}) \qquad W_{NH3\_comp} = 80.86kW$$

$$W_{R22\_comp} := mdot_{R22}\cdot(h_{2\_R22} - h_{1\_R22}) \qquad W_{R22\_comp} = 28.082kW$$

$$W_{total\_comp} := W_{R22\_comp} + W_{NH3\_comp} \qquad W_{total\_comp} = 108.941kW$$

The coefficient of performance, COP_C, is the rate of heat absorption across the R22 evaporator divided by the work of the compressors.

$$COP_R := \frac{mdot_{R22}\cdot(h_{1\_R22} - h_{4\_R22})}{W_{total\_comp}} \qquad\qquad COP_R = 1.674$$

The example successfully demonstrates the use of multiple property files in one Mathcad worksheet.

Appendix B – Propane Heat Pump Cycle

Problem Statement: A novel concept for a heat pump system is required using propane as the working fluid. In the event of a power outage, the propane could be routed through a burner to provide the heating load. The heat pump would operate where the average outside temperature is 25 degrees F. It is desired to maintain an hot reservoir temperature of 72 degrees F. The propane would leave the evaporator as a saturated vapor at 15 degrees F and be compressed to 160 psia. It would then be condensed to a saturated liquid through the condenser. Assuming the compressor operation is reversible with negligible heat transfer, answer the following questions.

1. What is the required mass flow rate to provide a heating load of 100,000 Btu/h?
2. What is the pressure of the evaporator?
3. What is the temperature of the propane leaving the compressor?
3. What is the total work required by the compressor?
4. What is the COP of the system?

Bonus: Assuming the heating value of propane is 91,000 Btu/gal. Also assume the price of propane and electricity is $1.95/gal and $0.10/kWh, respectively. Compare the price of providing a heating load of 100,000 Btu/h using the heat pump system to the price of burning the propane directly.

Sketches: A system schematic and T-s diagram are shown below.
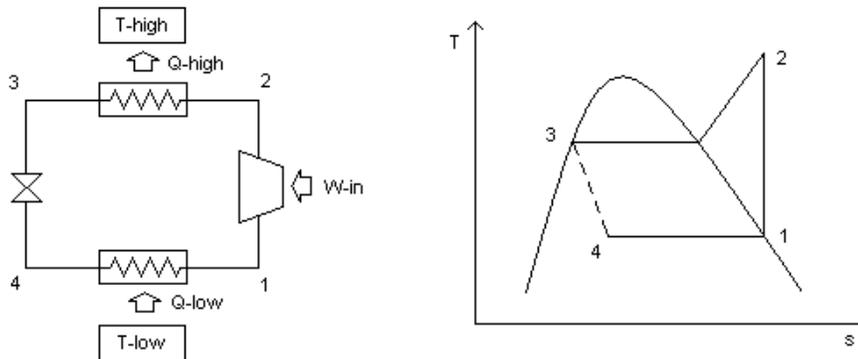
Solution: The known values are defined below.

$$T_1 := (460 + 15) \cdot R \qquad P_2 := 180 \, psi \qquad Qdot_H := 100000 \frac{BTU}{hr} \qquad \rho_{C3H8} := 31.7 \cdot \frac{lb}{ft^3}$$

$$C3H8_{HV} := 91000 \frac{BTU}{gal} \qquad C3H8_{cost} := 1.95 \cdot \frac{1}{gal} \qquad Elec_{cost} := 0.10 \cdot \frac{1}{kW \cdot hr}$$

The solution begins by analyzing each state in the cycle. At state 1, the pressure, enthalpy and entropy based on the assumption that the propane is a saturated vapor exiting the evaporator. The pressure at state 1 sets the pressure of the evaporator.

$$P_1 := Psat\_T_{C3H8}(T_1) \qquad h_1 := hg\_T_{C3H8}(T_1) \qquad s_1 := sg\_T_{C3H8}(T_1)$$

$$\boxed{P_1 = 51.236 psi} \qquad h_1 = 197.832 \frac{BTU}{lb} \qquad s_1 = 0.419 \frac{BTU}{lb \cdot R}$$

Since the compressor's operation is described as reversible with negligible heat transfer, the compression is isentropic, and the entropy at state 2 is equal to the entropy at state 1. Using this and the given pressure at state 2, the enthalpy and temperature are determined.

Appendix B – Propane Heat Pump Cycle (continued…)

$s_2 := s_1$

$h_2 := h\_Ps_{C3H8}(P_2, s_2)$ $\qquad$ $T_2 := T\_Ps_{C3H8}(P_2, s_2)$

$h_2 = 223.134\dfrac{BTU}{lb}$ $\qquad$ $T_2 = 566.221R$

The pressure at the exit of the condenser, state 3, is the same as that at the exit of the compressor. The propane is assumed to leave the condenser as a saturated liquid at the compressor exit pressure. The enthalpy and exit temperature are then determined.

$P_3 := P_2$

$h_3 := hf\_P_{C3H8}(P_3)$ $\qquad$ $T_3 := Tsat\_P_{C3H8}(P_3)$

$h_3 = 82.368\dfrac{BTU}{lb}$ $\qquad$ $\boxed{T_3 = 556.143R}$

The enthalpy at state 4 is approximately the same as that at state 3.

$h_4 := h_3$

To determine the required mass flow rate to achieve a 100,000 Btu/h heating load, the heating load is divided by the change in enthalpy from state 2 to 3.

$mdot_{min} := \dfrac{-Qdot_H}{h_3 - h_2}$ $\qquad$ $\boxed{mdot_{min} = 0.197\dfrac{lb}{s}}$

Using the calculated mass flow rate, the work required by the compressor is determined.

$W_{comp} := mdot_{min} \cdot (h_2 - h_1)$ $\qquad$ $\boxed{W_{comp} = 5.268kW}$

The coefficient of performance (COP) of the heat pump is determined as the rate of heat delivery to the high temperature space divided by the power of the compressor.

$COP_{HP} := \dfrac{Qdot_H}{W_{comp}}$ $\qquad$ $\boxed{COP_{HP} = 5.563}$

To determine the cost of burning propane to meet the heating load, the heating value is divided by the unit cost to get $/Btu. This is then multiplied by the heating load for the cost in dollars per hour.

$\dfrac{C3H8_{cost}}{C3H8_{HV}} \cdot Qdot_H = 2.143\dfrac{1}{hr}$

To determine the cost of the compressor work in the heat pump, the compressor work is multiplied by the cost per kWh of electricity.

$W_{comp} \cdot Elec_{cost} = 0.527\dfrac{1}{hr}$

Based on the stated costs of propane and electricity, the cost of burning the propane for heat is nearly four times as expensive as operating the heat pump.

Appendix C – Humidification of Moist Air

→ Reference:C:\ThermoII\SteamProps.mcd

Problem Statement: Air at 105 F and a relative humidity of 35% enters an evaporative cooler at a rate of 800 cfm. Liquid water at 55 F is injected into the air at a rate of 15 lbm/hr. Heat transfer ftrom the cooler to the surroundings is negligible. What are temeprature and relative humidity of the air exiting the evaporative cooler?

Sketch: A schematic of the evaporative cooler is provided below.
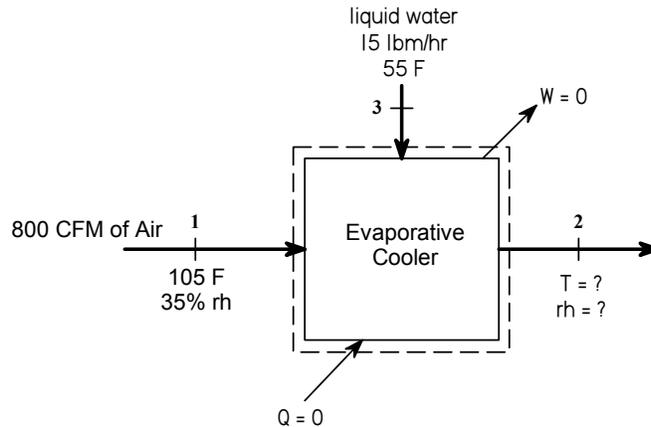
Solution: To begin the solution, the known quantities from the problem statement are entered.

$$T_1 := (105 + 460) \cdot R \qquad \phi_1 := 35\% \qquad Vdot_1 := 800 \cdot \frac{ft^3}{min}$$

$$T_w := (55 + 460) \cdot R \qquad m_w := 15 \cdot \frac{lb}{hr}$$

The properties of the moist air at the inlet and the properties of the liquid water are determined using the referenced property functions. Beginning with the inlet air properties:

$$v_1 := v\_T\phi P_{ma}(T_1, \phi_1, atm) \qquad v_1 = 14.63 \frac{ft^3}{lb} \qquad m_a := \frac{Vdot_1}{v_1} \qquad m_a = 0.911 \frac{lb}{s}$$

$$\omega_1 := \omega\_T\phi P_{ma}(T_1, \phi_1, atm) \qquad \omega_1 = 0.017$$

$$h_1 := h\_T\omega_{ma}(T_1, \omega_1) \qquad h_1 = 36.358 \frac{BTU}{lb}$$

The water properties are then evaluated:

$$h_w := h\_TP_{H2O}(T_w, atm) \qquad h_w = 55.233 \frac{kJ}{kg}$$

With all of the important inlet properties known, the system may be analyzed using conservation of species and conservation of energy. Since there is only one inlet and one exit for the moist air, the mass balance on the water reduces to

$$\sum_{in} m_a = \sum_{out} m_a \qquad \text{or} \qquad m_{a1} = m_{a2} = m_a$$

Performing the mass balance on the water vapor:

$$\sum_{in} m_w = \sum_{out} m_w \qquad \text{or} \qquad m_a \cdot \omega_1 + m_w = m_a \cdot \omega_2$$

Appendix C – Humidification of Moist Air (continued…)

Solving for the exiting humidity ratio yields:

$$\omega_2 := \omega_1 + \frac{m_w}{m_a} \qquad \omega_2 = 0.022$$

The steady flow energy equation is then:

$$Q - W = \sum_{out} m \cdot h - \sum_{in} m \cdot h$$

Since heat transfer is negligible and the cooler has no moving parts, the steady flow energy equation reduces to:

$$0 = m_a \cdot h_2 + m_w \cdot h_w - m_a \cdot h_1$$

Solving for the enthalpy of the moist air exiting the cooler yeilds:

$$h_2 := h_1 + \omega_2 \cdot h_w \qquad h_2 = 36.869 \frac{BTU}{lb}$$

The referenced functions are used to determine the exiting temperature and relative humidity.

$$T_2 := T\_\omega H_{ma}(\omega_2, h_2) \qquad \boxed{T_2 = 546.747R} \qquad \frac{T_2 - 460\,R}{R} = 86.747$$

$$\phi_2 := \phi\_T\omega P_{ma}(T_2, \omega_2, atm) \qquad \boxed{\phi_2 = 77.117\%}$$

Summary: 15 lbm of liquid water at 55 F are injected into 800 cfm of air at 105 F and a relative humidity of 35%. Moist air leaves the device at 86.7 F with a relative humidity of 77.1%.