MATHEMATICS AND CHEMICAL ENGINEERING EDUCATION

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ABSTRACT

This paper is based on the report developed by the authors at the Mathematics Association of America (MAA) Curriculum Foundations Engineering Workshop held at Clemson University in May 2000. The objectives of this paper are to identify the mathematics needed by chemical engineering undergraduates, to stimulate a dialog between mathematics and chemical engineering educators on this topic, and to determine the most effective way of providing the necessary mathematics. The focus is on subject matter and not on pedagogy.

The broad categories of mathematics essential to chemical engineering are pre-calculus foundations (provided by the K-12 school system or the by first-year university mathematics program), linear algebra, calculus, differential equations, and probability/statistics. Important topics within each area can be identified. The best place and time to teach this body of knowledge is open to discussion i.e.. what topics are best taught by the mathematics department, what topics should be incorporated into chemical engineering courses, what topics should be covered in the first and second years, and to what extent should the mathematics be spread out over four years?

The effective use of "mathematics technology" in mathematics and chemical engineering courses is discussed. Also, various ways are presented for exposing students to chemical engineering applications in the mathematics courses.

Introduction

The Mathematics Association of America (MAA) has begun a major analysis of the undergraduate mathematics curriculum through the Committee on the Undergraduate Program in Mathematics (CUPM). Two subcommittees of CUPM are involved in this study: Calculus

Reform and the First Two Years (CRAFTY) and Mathematics Across Disciplines (MAD). The National Science Foundation is involved in this study through Project INTERMATH as part of the Mathematics Across the Curriculum (MATC). These organizations sponsored an interdisciplinary workshop (engineers, mathematicians, and physicists) at the United States Military Academy in November 1999 (Arney and Small, 1999).

A workshop, "MAA Curriculum Foundations Engineering Workshop," for engineers (chemical, civil, electrical, and mechanical) was sponsored and hosted by Clemson University in May 2000. Mathematicians also participated. At some of the sessions all of the disciplines met to address common problems, and then the engineering disciplines met separately to address problems common to that specific discipline. At least one mathematician participated in these discipline-specific discussions. The participants addressed questions on (1) concepts, problem solving skills, and the desired balance between them; (2) mathematics technology; (3) mathematics education reform and education reform in the specific discipline; and (4) instructional techniques.

Each of the engineering disciplines developed a report that addressed these questions. These reports and reports from workshops for the sciences and for mathematicians will be used by the MAA to produce detailed curricular recommendations for the first two years of undergraduate mathematics instruction.

What follows is the report developed by the authors in the chemical engineering sessions at the Clemson University MAA Workshop.

What Chemical Engineers Do

Since this is a report for mathematicians, we thought an appropriate introduction would be to try to say what chemical engineers do, why we need mathematics, and how we use it. A reasonably broad definition of what we do is that *chemical engineers design materials and the processes by which materials are made*. Traditionally, chemical engineers have been associated with the petroleum and large-scale chemical industries, but especially in recent years, chemical engineers have been involved in pharmaceuticals, foods, polymer processing, microelectronics and biotechnology. The core subjects that underlie and unify this broad field are thermodynamics, chemical reaction processes, transport processes i.e., the spatial and temporal distribution of mass, momentum and energy, and process dynamics and control. On top of this fundamental framework, a central emphasis of chemical engineering education is *model building and analysis*. Good chemical engineers bring together the fundamentals to build a model of a process that will help them understand and optimize its performance. To be good at model building and analysis, students must have the mathematical background to understand and work with the core scientific areas, as well as to find solutions to the final model that they build.

Here's an example. A starting point for understanding any process is writing down the conservation laws that the process satisfies: for conserved quantities, accumulation=input – output. Depending on the level of detail of the model, this equation might be, for example, a large system of linear algebraic equations that determine the relationships between fluxes of chemical species throughout the process (a species balance), or it might be a parabolic partial differential equation governing the temperature of fluid in a reactor. In the thermodynamics of multiphase systems, energy is conserved but takes on a variety of forms; a good knowledge of multivariable differential calculus is essential here to keep track of everything.

Math for Chemical Engineering

We do not view our role here as one of prescribing the mathematics curriculum – we do not want to see taught only what students can "get by" with knowing. Nor do we want to come down on either side of the "traditional" vs. "reform" debate – it is likely that both sides are right, to an extent. The following represent our general thoughts on subject matter and emphasis:

Pre-calculus Foundations

By foundations, we mean basic knowledge of

- families of functions (polynomial, exponential…) in terms of data, graphs, words and equations, basic trig identities, properties of logarithms, etc.
- equations and inequalities
- basic logic and algorithms
- small linear systems of equations
- coordinate systems
- basic arithmetic and manipulation skills.

Mastery of these areas is crucial. Probably the most important role that the mathematics education community can play is to actively critique the pedagogy of K-12 education – to help sort out which "reforms" are productive from those that are merely ed-school fads – and to encourage schools not to neglect the education of the more mathematically inclined students by focusing the curriculum too narrowly on the average performer. Another important role here is to provide programs that help K-12 math teachers understand something about the applications of the math that they teach (engineers should do more here).

Linear Mathematics

We feel that our students would benefit from earlier exposure to the basics of linear systems in R^N, particularly

- the geometry of linear spaces
- vector algebra (especially in 3D)
- existence and uniqueness, Gaussian elimination, geometric interpretation, over determined and under determined systems and least squares problems for $\mathbf{Ax}=\mathbf{b}$
- characteristic polynomial and diagonalization, Jordan form, range and null space of $\mathbf{A}$, and geometry for $\mathbf{Ax}= \lambda\mathbf{x}$.

At Wisconsin, for example, there is a course on "linear algebra," which introduces these notions and applies them to systems of ordinary differential equations (see below). Many chemical engineering students take this in lieu of the traditional differential equations class.

Calculus and Differential Equations
In our discussions of calculus, the importance of visualization repeatedly arose, especially as a guide to differential and vector calculus in multiple dimensions, plotting (e.g. what function is linear on a log-log plot?), working in cylindrical and spherical coordinate systems and how to convert between coordinate systems. Somewhat less time could be spent on techniques for evaluating complicated integrals, with the time spent instead on, for example, visualizing the application of the chain rule in multiple dimensions. Understanding of truncated Taylor series for local approximation of functions is very important and should be seen early and often. In differential equations, a thorough knowledge of linear constant coefficient systems (IVPs and BVPs; see above) is preferable to emphasis on existence theory and series solutions for non-constant coefficient problems. Some qualitative theory for nonlinear systems would be nice.

Probability and Statistics
Alumni surveys generally show that this is the most common application of mathematics for the practicing BS chemical engineer.

Students interested in graduate school should be encouraged by their math professors as well as their engineering advisors to take additional math courses.
A final general comment: for engineers, concepts are more important than proofs, but students should have some idea of the power of a theorem. In other words, we are comfortable with students learning mathematical facts without necessarily having seen the proofs.

Technique and Technology
A fair amount of the discussion at the workshop, within our group and others, centered around the use of "technology" in the math courses for engineers. In the discussions, "technology" meant a number of different things, from numerical methods to graphing calculators to symbolic manipulation packages. We'd like to emphasize here some points to be kept in mind when thinking of the introduction of these tools into math courses. Here are some questions and our responses:

"Why should I learn to do it by hand?"
- sense of form of mathematical expressions, understanding of what manipulations are available, facility with these manipulations
- fluency in the language of mathematical concepts[1]
- appreciation and recognition of mathematical rigor
- discipline, maturity, confidence of mastery
- closed form results are best, if available
- recognition of limitations of closed form results, where things get difficult
- knowledge of what computers do...

[1] Yes students should know the integration by parts formula!

"Use of computers 'dumbs down' the math course – why use them?"

- solution of complex problems -- in upper level courses, extensive use is made of programs like MATLAB, MathCAD, Mathematica, Polymath
- exploration of solution and design space
- visualization, especially in multidimensional and vector calculus
- relief from tedium
- confidence in results derived by hand.

Ultimately, we feel that the technology should take a back seat in mathematics courses until it becomes necessary to solve interesting problems. For example, in a linear algebra course, students should be able to do LU decomposition of a 3x3 system by hand before they are shown that MATLAB does it in one command. At the same time, it is useful to point out the relationship between numerical techniques and exact ones (e.g., a Riemann sum can be evaluated numerically to approximate an integral).

A Suggestion for Coupling Math and Engineering Education

One set of issues that arose repeatedly in the workshop discussions was the concern that students don't see connections between mathematical tools, concepts and principles and their utility in engineering. A related concern was the time lag between exposure to mathematics and its application. The notion of "just-in-time" learning arose repeatedly, and the suggestion was made that the math courses be more application- or example-driven and be more evenly spread through the curriculum, rather than "front loaded" into the first two years. Our group shares these concerns, but thinks that the above suggestion is impractical and undesirable, for the following reasons: (1) part of the beauty and power of mathematics is that it is example-independent – calculus applies to economics just as it does to mechanics, (2) the time spent developing the background for engineering applications is time not spent on mathematical principles and tools, and (3) the "just-in-time" approach will not satisfy all the engineering majors – electrical engineers do not need Laplace transforms at the same time as chemical engineers.

We propose that an alternate structure be considered for addressing these concerns, which are essentially about how to connect math and engineering in the students' minds. We suggest the introduction of **discipline specific supplements**, especially for the calculus sequence. These could be workbooks or web pages, for example containing

- engineering background material e.g., some basic thermodynamics, and specific mathematical principles and/or tools e.g., total differentials and partial derivatives
- exercises or projects integrating mathematics and engineering
- additional discipline-specific emphasis e.g., sine and cosine for electrical engineering students.

These could be used independently by the students, or used in a one-credit course running in parallel with the calculus courses, or simply be resources for math instructors wishing to gain perspective on engineering applications or bring engineering applications into the math classroom. This is perhaps overly ambitious, but we believe it is worth considering. Within chemical engineering, there is an organization called CAChE (Computer Aids for Chemical Engineering) that may take a role in studying this possibility. Michael B. Cutlip of this working group is active in that organization.

Concluding Remarks

Without exception, this group felt that the workshop was a very productive way to promote dialogue between the mathematics and engineering education communities and we would like to see workshops of this type continue to be held. Another venue that mathematics educators may want to explore is the American Society for Engineering Education, which has a Mathematics Division. On the other hand, it may be productive for engineering educators to attend MAA meetings. Perhaps most importantly, mechanisms need to be implemented to promote interaction between engineering and mathematics faculty within individual universities – good relationships at this level will enable math instructors to understand what material the engineering faculty would like to see reinforced and emphasized.

References

Arney, D. C., and Small, D., *Proceedings of the Interdisciplinary Workshop on Core Mathematics: Considering Change in the First Two Years of Undergraduate Mathematics*, Mathematics Association of America, West Point, NY, (November 4-7, 1999)

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