MathinSite: web-based support for deepening the mathematical insight of engineering undergraduates

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Abstract

Many engineering undergraduates have problems with mathematics. Even areas of school mathematics – invariably including algebra - sometimes have to be reinforced at undergraduate level. A bar to learning is often a lack of an understanding and this is where visualisations sometimes help - either by setting problems in an engineering context, or by using graphical visualisations. In the latter case, the maxim, "A picture is worth a thousand words" is most appropriate.

Even if students have problems rearranging mathematical equations, they can, almost always, "read", understand and draw graphs. Now a graph is basically a visualisation of a mathematical equation, be it as simple as the straight-line equation or as complicated as the solution of a second-order partial differential equation. Consequently, displaying the graph of (i.e. visualising) an equation can help deepen student understanding of the mathematics behind that equation. During the early 1990s, the author wrote and presented for student use some graphical mathematics software[1] using Visual Basic. Through its use, students began to realise what was happening with the equations they were investigating - and realised that engineering mathematics could be enjoyable (evidenced, in part, by students talking and enthusing about mathematics, and using the software in their own time).

With the bursary accompanying a UK National Teaching Fellowship[2], the author is currently developing the above-mentioned work into the ***MathinSite***[3] web site using interactive Java applets with a strong graphical content.

This paper will discuss the rationale and philosophy behind the use of ***MathinSite*** in deepening engineering students' mathematical understanding - a rationale and philosophy that could be adopted in other areas of engineering education.

Background

It is fairly obvious to anyone who has been present at an undergraduate engineering examination board recently that analytical subjects, mathematics in particular, cause more students to fail than non-analytical subjects. This seems to have worsened over the last ten years or so and the problem has been well documented in the UK through papers such as those by Howson[4], James[5] and Sutherland and Pozzi [6].

There are a variety of reasons for the continuing reduction in the level of student capability in mathematics. Note here the use of the word "capability" here rather than "ability". Students' ability is not in question. Students may well be *able* (intelligent) enough to successfully study mathematics at undergraduate level but may not *capable* (competent) of so doing.

Amongst the reasons for the reduced mathematical capabilities of first-year undergraduate engineers in the UK is the marked decline in the number of students taking mathematics and physics to the end of their high school studies. With not enough studying higher mathematics there are fewer potential engineering undergraduates offering analytical subjects at the required level prior to university entry. And for those who have studied mathematics to the end of high school, there is no longer the same guarantee of depth of study. There appears to have been a continuing decline over several years in the standard of content of high school mathematics syllabuses as measured by student attainment on completion (Hunt and Lawson[7]). A further factor is the increase in the number of mature students returning to study who will necessarily have substantial gaps in their education – perhaps 10 years or more. At the same time there has been an increase in the number of UK universities from about 30 in 1985 to over 100 in 2002. This, together with a UK government requirement to increase considerably those staying on for further and higher education, has left course tutors trawling their nets wider to fill the increased number of places available - often with less-capable students; students who are not properly prepared to undertake such a course of study.

Universities have a number of possible solutions, including:

- Disband Engineering Departments and discontinue engineering provision. Arguably there is already an overprovision nationally; so jettisoning engineering in favour of other (less expensive to run) courses in which a university may have a good reputation makes economic sense – unfortunately for the engineers. This option has been rarely adopted in engineering, although it has happened to a number of Mathematics departments within UK universities in which there are just not enough applicants to fill the places on mathematics degrees.
- "Water down" the content of engineering degrees. Hopefully this has not been, nor ever will be, the option to take.
- Introduce a Level 0, or Foundation Year, to bridge the knowledge gap (usually for students whose end of high school grades were not high enough, or for mature students with a gap in their learning).
- Change the emphasis of the engineering degrees. For example, some universities have already converted courses from heavily analytical Mechanical Engineering degrees to less analytical Design Engineering degrees. Not a "watering down", but a "shift of emphasis".
- Accommodate the analytically less-capable student.

The last three points above have been adopted at Bournemouth University but it is the last point that forms the basis for discussion of this paper.

Diagnostic Testing and Follow-up Support

With the recent widening of access to Higher Education, it became imperative to ensure that incoming engineering students with weak mathematical competence should be accommodated. So, in order not to turn away students who, apart from their mathematical shortcomings, looked like potentially good engineering undergraduates, it was decided during the early 1990s to introduce a "Diagnostic Quiz" to be taken by *all* of Bournemouth

University's engineering students during the first week of their course. The mathematics in the quiz was set at a pre-calculus level equivalent to that studied up to about the age of 15. The reason for such a low level was that if a student's foundation mathematics was weak then some follow-up support was needed at this level – teaching anything at a higher level, without the foundation support, would be equivalent to "building on sand". An extra one-hour session per week of "Extra Maths" for the low scorers in the diagnostic quiz was run concurrently with the main lectures. The outcome of this proved successful, with a number of mathematically weak entrants going on to obtain high-classification degrees at the end of their university courses. Two papers[8, 9] were published analysing, amongst other things, the effectiveness of Bournemouth's scheme and a subsequent handbook[10] on the *modus operandi* of instituting mathematics diagnostic testing was written for the UK's Open Learning Foundation.

"Feeling" Mathematics

Despite all the Extra Maths support, it was obvious that some students still did not "feel" the mathematics. Based on the maxim, "A picture is worth a thousand words" (especially one that is interactive), some Visual Basic (VB) programs[1] were written during the mid-1990s. As a very simple example, consider the equation of the straight line, $y = mx + c$. Probably all mathematicians reading this will have already a picture in their minds of a two-dimensional plane with a set of perpendicular axes and a straight line embedded somewhere within (and did it have a positive slope?!). Unfortunately not all students will see such a picture – at all. When introducing this topic, a lecturer will usually draw on the board several examples showing different lines for different values of m and c. These pictures will be worth only 500 words! They are *static*. With these, students who do not feel the mathematics may not be confident enough to answer "What if …" questions, e.g. "What if I change m from 2 to 3?" This is where interactive *dynamic* visualisations become worth the full 1000 words. Common to all the VB software that was produced in this series, the 'Straight Line' software provided a standard Windows-type user interface with a graphics area containing the axes and the graph, a text area giving all relevant information about the equation represented in the graphics area and scroll bars to change the values of the parameters (here m and c) within a predetermined range of values. By changing parameter values with the scrollbars it was found that most students became more engaged with their own learning. The interaction with, and the dynamics of, the graphics and text gave instant feedback and helped students to obtain a better understanding of the equations under investigation. Interacting this way helped most students to "see" and "feel" the underlying mathematics, so raising their self-confidence in the subject and giving them greater mathematical insight. This approach seemed to add weight to the saying:

"I hear and I forget,
I see and I remember,
I do and I understand."
(Anonymous)

That this was a valuable approach in the teaching and learning of mathematics became readily apparent, for example, during tutorials in which the software was used when occasionally students could be heard exclaiming (when changing m, say), "Oh, is *that* what it does!"

Java Mathematics

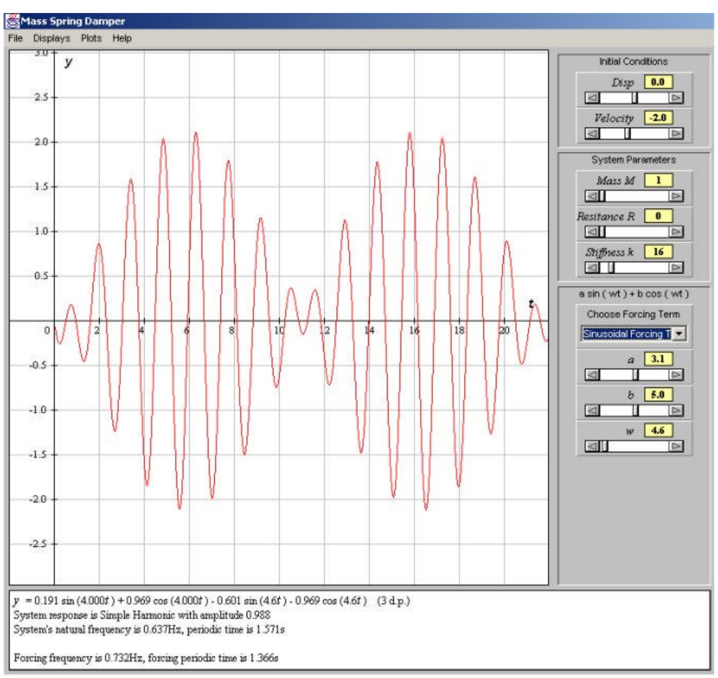
During this time the VB software and all its associated run-time files were loaded on to PCs within the university. However, students indicated that they would like to access the programs in their own time away from the university. Without wishing to give the software away and bearing in mind that more students were becoming internet-enabled at home (and with the introduction of the Java programming language), a web presentation using applets became the way forward.

Java is a platform-independent programming language that allows programs, called ‘applets’, to run through Internet browsers such as Internet Explorer and Netscape Navigator. ‘Platform independence’ means that applets, although written perhaps on a PC, should run successfully when viewed through any Java-enabled internet browser on any computer with any operating system, e.g. PC with Windows, Mac with MacOS, Unix systems, so facilitating universal access.

During 1999 funding was obtained from Bournemouth University’s Learning and Teaching Development Initiative to allow the transcription of some of the VB software into Java applets and to have them placed on the University’s Intranet. Since this was funded locally and, consequently, placed on the university’s Intranet, students were still not able to access the applets from outside the university. However, with the award of a Fellowship[2] from the Higher Education Funding Council for England’s National Teaching Fellowship Scheme, the author obtained three years’ further funding in 2000 for the project and its applets to become a worldwide Internet resource (***MathinSite***[3]).

MathinSite

MathinSite is an on-going project whose mathematics applets are its mainstay. ***MathinSite***’s primary aim is to deepen student understanding of mathematics through the use of interactive, stimulating, visually dynamic software. An example is shown in the following figure - a screenshot from the ‘Mass / Spring / Damper’ applet.



The scrollbars on the right here are the only means of data entry. Changing scrollbar values effect changes in system parameters such as initial velocity, spring stiffness and the angular velocity of an applied force. The graphics change in real time, showing system response visually. The text box below the graphics area shows the system’s differential equation, analytical solution and other *important* quantities, such as periodic times of the natural and forcing frequencies. It is a prime intention to keep on-screen text to a minimum – the emphasis here is on dynamic interaction.

However, just allowing students to load and run applets and play with scrollbars willy-nilly will not achieve the primary aim, so each applet is accompanied by a tutorial worksheet – sometimes two or more – and most with a theory sheet. ***MathinSite***'s applets and accompanying tutorial sheets are not intended to be a replacement for face-to-face tuition. Their purpose is to *supplement* the taught element of a course with student-centred learning – made possible with the tutorial sheets guiding the student through a structured use of the applets. These tutorial sheets are highly contrived since they lead the student through a set of predetermined tasks using the applet ("I know what I want my students to see!"). However, students are allowed to proceed at their own pace and, every so often, space is left on the tutorial sheet for reflection, e.g. 'Draw the graph shown on the screen in the space provided', 'Write here how the graph changed when …', 'Why do you think that the graph…?', 'What was the equation in this case?'

An extract from the 'Mass / Spring / Damper' applet's worksheet is shown in the following frame. Here, students are guided into discovering for themselves the phenomenon of 'Beats'.

- Use the slider bars to set the following values: *Disp* = 0, *Velocity* = 0, M = 10, R = 1, k = 90 and a sinusoidal forcing term with a = 20, b = 0 and ω = 10. Write down here the differential equation relating to the values.

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- Write down the analytical solution here.

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- You should see a very lightly damped system with $R^2 << 4Mk$ so the transient response is almost simple harmonic motion (SHM). Superimposed on these oscillations are the oscillations due to the forcing term.
- From the analytical solution determine the frequency in Hz of the natural and forced oscillations.

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- Now press and hold down the arrowhead at the left hand end of the ω slider and watch as ω reduces from 10 to about 3.7. Stop here. Now step down through 3.6, 3.5 to 3.4. At ω =3.4 describe what you see in the response (you may need to scroll the graphics window to the right to see this effect more clearly).

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- Keep a record of this case by sketching the resulting response curve, noting also the analytical solution and the list of parameter values, etc. Label this as ***Figure 2***, ***"Beats"*** so that you can reference it to these notes and cross-reference it with the discussion on Beats in the Theory Sheet.

Some of the tutorial sheets finish with a set of exercises for which the answers are *not* provided (on the tutorial sheet) since the answers can be obtained using the applet itself. Examples taken from the 'Mass / Spring / Damper's' worksheets are shown in the following frame. Note how with these particular examples that the student is using this applet in its secondary role, as a design tool.

1. There are three ways in which you can help solve the problems of near resonance of the Millennium Bridge over the River Thames in London (see accompanying Theory Sheet). You now know that light damping can lead to such problems in mechanical systems; so one solution would be to increase the resistance to motion. What else can you do to eliminate such problems? [Hint: think $R^2 – 4Mk$]. Discuss how any of this could be done in real world terms - and consider sending your solution to the Millennium Bridge Commission, London!

2. A second order linear system of the mass-spring-damper type has system parameters M = 1, R = 4, k = 1. The initial conditions are *Disp* = 2 and *Velocity* = 0 and the system is excited by $F(t)$ = sin $4t$, an oscillating force. Enter this data into the applet and use either the analytical result or the graph to estimate the transient time. Although the manufacturer requires steady state oscillations with angular velocity 4 rad/sec, the transient response is not fast enough. Change the value of the resistance factor, R, until critical damping is achieved. Sketch the resulting response curve, noting also the list of parameter values and the analytical solution.

Although students may use the applets initially in the university's computer laboratory in a formal tutorial, the work may be finished in the student's own time on the Internet from home since the applet / worksheet combination is self-contained. Some students have even initiated the use of other applets themselves as they become more engaged with the ***MathinSite*** culture. The overall effect here is of empowerment – students become the owners of their own learning process.

Why use *MathinSite*?

Why should this material be developed? Surely many lecturers who are involved with the learning and teaching of engineering mathematics may already be doing the same thing, perhaps using computer algebra packages such as Derive, MathCad or Mathematica, or graphical calculators?

There are a variety of reasons why this approach is popular with students, not least accessibility. In the ten years that this software / paperware combination has been in development at Bournemouth, feedback from students, amongst others, has contributed towards the following list (not necessarily in any order of importance):

- ***MathinSite***'s applets visualise mathematics – the interactive visualisations can enhance and deepen the learning process.
- Each applet gives immediate interactive feedback to users so enabling "What if …?" investigations to be carried out.
- Each applet is a self-contained single aspect of mathematics – bite-size chunks. It is not possible to be sidetracked into other areas.
- Distracting on-screen text is kept to a bare minimum. All main tutorial text is to be found on the students' (takeaway) worksheets.
- Each applet uses a standard Windows-type display (with which most students are familiar) and, further, all the applets have a common user interface and require the same interaction. Here, ease of use is the key. To use computer algebra packages or graphical calculators involves a learning curve just to access software packages or calculator functionality even before the mathematics can be tackled. (Of course, for those already familiar with these, this would not be a problem.)
- The use of the applets does not have to be initiated by the lecturer. Their use does not even require supervision by a lecturer since the accompanying tutorial worksheets guide students through the applets' use.
- Since each applet is accompanied by a tutorial worksheet, lecturers can introduce them directly into their own tutorials without any further work. Obviously the worksheets provided are not mandatory and there will be those who want to write their own.
- The software will not crash due to inappropriate data since scrollbars allow only parameter values input from prescribed, valid ranges.
- The working of an applet cannot be changed by the end-user. It has a single task to do, points to make, and users cannot alter the program to perform other tasks.
- The applets do not need proprietary software to run them apart, that is, from freely available Java-enabled web browsers (for example, the latest version of Netscape).
- The applets are immediately accessible anywhere in the world at any time from any computer with an Internet connection. And last, but by no means least,
- Their use is free.

So is *MathinSite* the mathematician's "Holy Grail"?

To say that ***MathinSite*** is the mathematical panacea that many lecturers have craved would be crass. Such a panacea does not exist. Some students may not even need ***MathinSite*** since their depth of mathematical understanding is already rock-solid. For other students, the visualisations provided by ***MathinSite*** may even complicate and confuse. Hegedus[11] mentions that, "Whilst dynamic visual imagery can assist students in visualising and possibly conceptualising the mathematics being introduced, there are studies, which report that this can often be at the loss of procedural understanding." In particular, the work of Aspinwall *et al*[12] is referenced.

Be that as it may, the vast majority of students who have used ***MathinSite*** applets have mentioned that, in some way or other, the software has improved their mathematical understanding (and their confidence to *talk* mathematics in class). ***MathinSite*** should not be seen, then, as the Holy Grail, but as one further tool in the armoury of those who are trying to engage students in the learning of mathematics.

A disadvantage with ***MathinSite*** is that its contents are necessarily finite (at the time of writing, the web site contains about 12 applets) and, in particular, the contents are total self-indulgence on the part of the author since they are tailored to his students' requirements. However, the applets should find general use amongst others who teach and learn engineering mathematics. Production of further applets is planned but, even with the modularity and rapid application development capabilities of Java, this is a time-consuming exercise (but "watch this space"!).

Conclusion

It has not been possible yet to assess fully the efficacy of ***MathinSite*** in reducing failure rates on engineering courses due to mathematical subjects. This may be difficult to assess anyway since the diagnostic testing / follow-up support combination has already paid dividends in this area at Bournemouth University. A comprehensive review of ***MathinSite***'s use obtained from user feedback is planned for 2002 – 2003, the third year of the project. However, from classroom observation and questionnaire feedback already obtained, it is obvious that students like the approach adopted in the ***MathinSite*** applet / worksheet combination. Many students ask, "When are you going to write an applet on…?", indicating their eagerness to deepen their understanding in other areas of mathematics. That they should even ask such a question indicates a keenness to engage more in their own learning and improve their mathematical self-confidence. As can only be expected, not everyone becomes so engaged. Nevertheless, while the visualisations of ***MathinSite*** do engage, while they raise enthusiasm and add deeper insight in an area traditionally known for providing high failure rates on engineering courses, ***MathinSite***'s continued use (as a supplement to lectures, tutorials and seminars) is assured at Bournemouth University.

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Biographical Details

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