Matlab numerical method application in student research

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Abstract

Matrices Laboratory is a versatile package that performs a multitude of mathematical calculations involving signal-processing systems and control systems, and it has toolboxes for image processing, neural networks and communication applications. This "hands-on" student research introduces a method of capturing the luminance of roadway scenes using a charge-coupled device (CCD) camera, and later, analyzing these images to calculate the spatial frequency content in the scene. This numerical method research was introduced in a 3-credit Special Problems course.

Introduction

This paper introduces an imaging concept of Digital Signal Processing for measuring visibility at a scene using spatial analysis. Spatial analysis describes the Frequency content at the scene of interest. The scene taken into consideration throughout this study is a road intersection. The study uses a prototype road scene to prove the principle within a laboratory. The prototype is placed in a lab setting that uses two luminaries. One luminary produces incandescent light and the second produces light of a different frequency spectrum. Light emanating from the first source is white and the second is yellow. The main reason for choosing such lighting is to study the effect of these luminaries on the scene of interest. The prototype road scene is scaled to obtain near real results. The images are processed using MATLAB and the results, along with an analysis are presented in this paper. This concept is implemented in the Design of Digital System Processing (DSP) course - introduction to digital signal processing emphasizing biomedical imaging and digital audio applications. Students learn topics such as phasors, the wave equation, sampling and quantizing, feedforward and feedback filters, periodic sound, transform methods, and filter design. This course uses intuitive and quantitative approaches to develop the mathematics critical to understanding DSP techniques.

Fourier Series

Physicist Joseph Fourier developed this analysis to study heat transfer problems where he recognized that a function, $f_p(x)$, whose graph displays a periodicity, T, could be considered to be an infinite sum of sinusoidal functions. The Fourier series may be represented as the sum of a series of sine functions, cosine functions, and complex exponential functions or any of several other sinusoidal representations (Wilson (1995), Baher (1990), and Lathi (1989)) [1-6]. The $f_p(x)$ is a uniform and finite periodic limited function with period, p, defined over a range of its variable x between x_0 and x_1. The main function breaks into a series of circular functions where n is an integer at any point over its range, $p = x_1 - x_0$, and $\mathrm{w} = \frac{2\mathrm{p}}{p}$

$$f_p(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos(n\mathrm{w}x) + B_n \sin(n\mathrm{w}x))$$

$$A_0 = \frac{2}{p}\int_{x_0}^{x_1} f_p(x)dx \quad , \quad A_n = \frac{2}{p}\int_{x_0}^{x_1} f_p(x)\cos(n\mathrm{w}x)dx \quad , \quad B_n = \frac{2}{p}\int_{x_0}^{x_1} f_p(x)\sin(n\mathrm{w}x)dx$$

$$f_p(x) = \sum_{n=-\infty}^{\infty} D_n e^{in\mathrm{w}x} \quad , \quad D_n = \frac{1}{p}\int_{x_0}^{x_1} f_p(x)e^{-in\mathrm{w}x}dx$$

Where x is a length, ω is an angular frequency in radian per x-length, θ_n, is a phase shift angle, and A_n B_n C_n & D_n are amplitudes of the frequencies at $\omega_n = n\omega x$. Function $f_p(x) = \sin 2\mathrm{p}x$ shows a sinusoidal waveform that can be simulated in Matlab [8].

$$f_p(x) = \sum_{n=0}^{\infty} (C_n \cos(n\mathrm{w}x - \mathrm{q}))$$

The Fourier series of any periodic function may be represented in the spatial or time domain as a function of f (x), or in the frequency domain as a function of F(ω).

$$f_p(x) = \sum_{n=0}^{\infty} F_n(\mathrm{w})$$

When the function is represented in the time domain, the function is usually a continuous function. When in frequency space, the function is represented as an infinite series of amplitudes, A_n, B_n, C_n, or D_n, at discrete frequencies. Figure 1 shows one discrete frequency component.

Each frequency is ω_n and discrete phase shift is θ_n. The discrete frequencies are determined by the period, T, of the periodic function. Each frequency contains a portion of the total energy or power in the function f(x). The total energy in the function f(x) is the sum of the amplitudes in each discrete frequency.

$$E_{f(x)} = \sum_{n=0}^{\infty} |C_n|^2 \quad \text{, or,} \quad E_{f(x)} = \sum_{n=-\infty}^{\infty} \left|D_n e^{i(n\mathrm{w}x+\mathrm{q}_n)}\right|^2 = \sum_{n=-\infty}^{\infty} |D_n|^2$$

A Fourier function can be moved back and forth between frequency and time domain. Fourier series of periodic functions have discrete frequency content and an infinite sum of the discrete frequencies where the discrete frequencies a have period of $\omega_n = n2\pi/T$ and $n = 0, 1, 2, 3, \ldots\infty$. The constant, non-oscillating term is n=0. The fundamental frequency, first oscillating term is n=1. The first harmonic, second oscillating term is n=2, etc. When the period, T, decreases, ω_1 becomes larger. Then the offset between two neighboring frequencies ($\omega_2 - \omega_1$) grows larger, and the distance between ω_{n+1} and ω_n increases. When the period, T, increases, ω_1 becomes smaller and ω_1 and ω_2 grow closer together. When the period becomes very large, ω_1 and ω_2 move close together until, as T goes to infinity and the frequency becomes continuous.

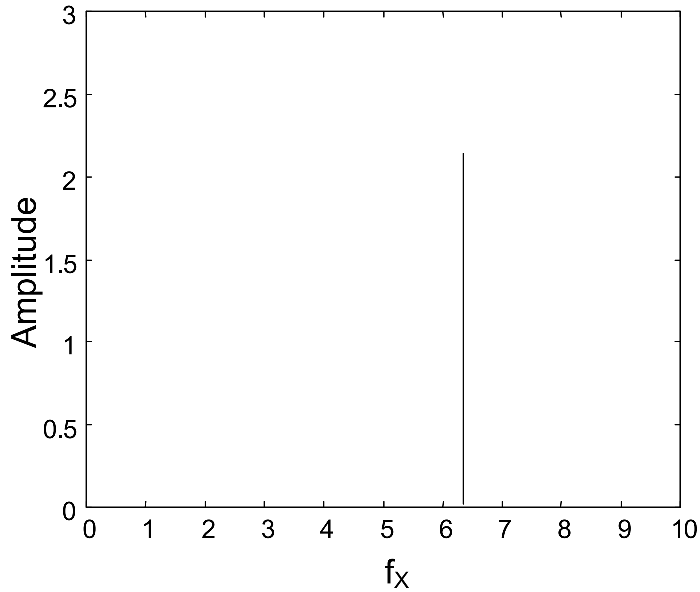


Figure 1: The spectrum of the single frequency wave.

Fourier analysis is widely used in the study of electrical networks, digital signal processing, bioengineering, and communication systems. Any function, $f(x)$ (not limited to periodic function) can be composed of the superposition of a series of continuous periodic functions of suitable amplitudes and frequencies. A periodic function u can be represented by $F(u)e^{jxu}$. The original function can be considered as a summation of periodic functions,

$$f(x) = \int_{-\infty}^{\infty} F(u)e^{jxu}du, \text{ and } F(u) = \frac{1}{2p}\int_{-\infty}^{\infty} f(x)e^{-jxu}dx$$

The function which provides the amplitudes, *F(u),* of the periodic terms of frequency u, is called the Fourier Transform. Then *f(x)* or *F(u)* can be determined and the other computed from the Fourier Transform relationship [1-9].

The Spatial-Frequency Content Method of analysis of digital images decomposes (separates) a waveform or function into sinusoids of different frequency, which sum to the original waveform. This waveform identifies the different frequency sinusoids and their respective amplitudes (Brigham (1988)).

A complex periodic signal is formed by the combination of different integral frequencies that were required to produce the multisine. In the frequency spectrum of such a complex signal, all the component sinusoids can be viewed. Fourier Transform is a tool that is used to convert the signal from the time domain to the frequency domain[1-6]. The two major algorithms used to transpose a signal from its time domain to frequency domain counterpart are 'Discrete Fourier Transform' and 'Fast Fourier Transform'. The software package MATLAB, has an inbuilt function that will perform such transformation [8].

Analysis of an Image

Fourier Transform, in essence, breaks a multisine into sine waveforms of different frequencies that sum to the original waveform. The separate components of the composite waveform can be individually visualized and used for further inspection to determine the nature of the multisine and hence obtain information from it. Any arbitrary object that can yield an image may be represented by a series of simple or multiple Fourier integrals. The amplitudes of these terms pertaining to the series can be regarded as describing the spatial frequencies, which leads to a complete representation of the object of concern in a different domain than the usual interpretation. An image is broken down into a combination of sines and cosines. Each point in the spatial domain of the image represents a particular frequency pertaining to the image. These two listed domains are interchangeable. In other words an image can be represented in the time and frequency domain.

MATLAB

This package, Matrices Laboratory, is a versatile package that performs a multitude of mathematical calculations involving matrices. This software is used in modeling signal processing systems and control systems and has toolboxes for image processing, neural networks and communication applications. This package is structured like C++, and provides functions for every process. These functions can be called in the program to achieve the end that is expected of the process. The main functions that MATLAB provides in the accomplishment of this project are FFT, FFTSHIFT and IFFT. These functions help in transforming an image between the time domain and frequency domain. The function FFT (x) provides the discrete Fourier Transform (DFT) of a vector 'x'. If the length of 'x' is a power of two, a radix-2 FFT algorithm is used. Otherwise a slower non-power algorithm is used to calculate the FFTs. The function FFTSHIFT is used to flip the quadrants of the image such that the DC component of the image falls at the center of the transformed matrix [2]. The Fourier Transform of an image would produce the following 2D image and graph, Figure 2.

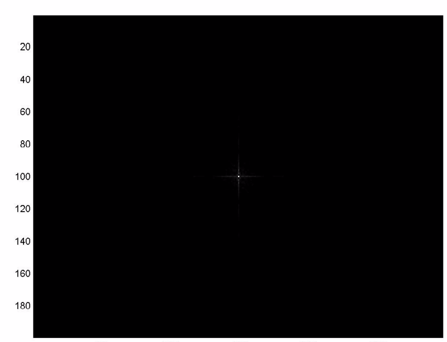


Figure 2 Gray Scale Image of an arbitrary 'Road Scene' and FFT representations (0th component is placed in the center and it is seen as a dot)

Interpreting volume under the FFT curve is important for image spatial analysis. The tallest spike indicates the zeroth [0^{th}] frequency component of the image. This spike is a direct indication of the brightness of the image, which would translate into the luminance at the scene of the object. Remaining frequencies are divided into lower, mid and higher regions. The number of amplitude spikes should constantly diminish as the frequency increases [3]. Expanding on the idea presented above, the volume under the curve of FFT's at a certain frequency will divulge required information of the picture which can be stated to be its spatial content. The DSP algorithm used to obtain this spatial information from a picture is as follows[3]:

- Image is acquired using a CCD Camera. The CCD camera is chosen for the unique properties it provides, especially for 'night' image acquisition.
- Acquired image is converted into 256 gray levels.
- FFT of the image is computed.
- Frequency components in the FFT of the image are segregated. This segregation is achieved by separating the FFT of the image into bands, or regions, that would approximately contain the frequency components starting with the 0^{th} component in the center[9].

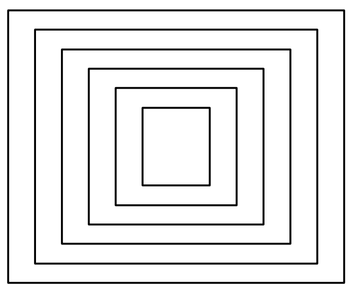


Figure 3 Rectangular Frequency Bands [9]

- Sum of the FFT's of the pixels local to each ring is calculated.
- The sums are plotted against the frequency band using any graphing software package.
- The data is ready for analysis.

Choosing the thickness of the frequency band is critical to the exercise of spatial content analysis, as this method separates the FFT frequency components. The distribution of different frequency components can be uniformly spaced throughout the FFT's image array, or can be non-linearly distributed from the center, radiating outwards. Care should be taken in determining the optimal size of the frequency band to obtain correct results.

CCD Camera

The heart of a Digital Imaging Camera is a Charge-Coupled Device [CCD]. The CCDs replace the film and Vacuum tubes from previous cameras. A high-resolution camera is constructed by combining a highly sensitive chip with a set of sensitive lenses, a cooling method and additional electronics. Camera applications are widespread in scientific, astronomical, biomedical and other commercial areas. The CCD camera principle of operation is a MOS capacitor[10-11]. Imaging arrays usually consist of square or rectangular arrays of pixels. An M x N array may be thought of as a collection of M linear registers of N pixels each. The M linear registers are aligned vertically; side-by-side and separated by channel stop regions. Light is incident on the exposed thinned surface. The incident photons do not pass through the front surface electrodes and passivation layers. An enhancement layer is added to the back surface to create an electric field that forces photo-generated electrons toward the potential wells under the gates. An anti-reflective coating may be added to increase optical quantum efficiency. An additional independent linear register is placed next to the array with its charge transfer direction orthogonal to that in the array. The serial register is arranged so there is a single pixel adjacent to each of the M Columns and is terminated in a charge detection output amplifier. The resulting data stream is a pixel-by-pixel, and row-by-row representation of the image falling on the CCD.

Experimental Setup

The principle of this Imaging concept in Digital Signal Processing research is a prototype used within the bounds of a research laboratory. In order to make the research process as realistic as possible, the prototype is scaled to reality. The procedure used for scaling is as follows:

1. Vehicle (object under consideration) distance from the point of observation, (dimensions of the vehicle would have to be scaled).
2. Width of the road.
3. Reflectivity of the road.
4. Luminaries at the scene.
5. Spectral distribution of the luminaries, describing the color of emitted light.
6. Reflectivity of the curb or pavement.
7. Headlights, and their distribution, of oncoming vehicles.

There are several variables related to this field. This paper addresses the effect of two parameters: type of luminary available to provide illumination at the scene of interest and the headlights of the oncoming car. The two luminaries used are a source of incandescent light, and a light bulb that generated yellow light. The headlight of the oncoming car is mainly red light emanating from an LED. Hence the study relates more to the spatial response of the scene under the influence of illumination pertaining to a particular spectral region, or a combination of two such luminaries. Figure 3 describes the set-up used to collect data required as a proof of principle.

Image shown in Figure 3 is indicative of the set up that was used within the laboratory. The observer was a CCD camera. The Object was a prototype Chevrolet Monte Carlo. The roads were 12.5 cm in width. In order to scale the prototype to 50 times smaller than original, the camera simulating the original is placed 1.5 meters from the object. The luminaries involved in this experiment were directly above the prototype and are not shown in the experimental set up. The luminaries are of 2 types: a luminary that provides incandescent or white light, and a luminary that provides yellow light. In addition to these parameters, the car has LED headlight.

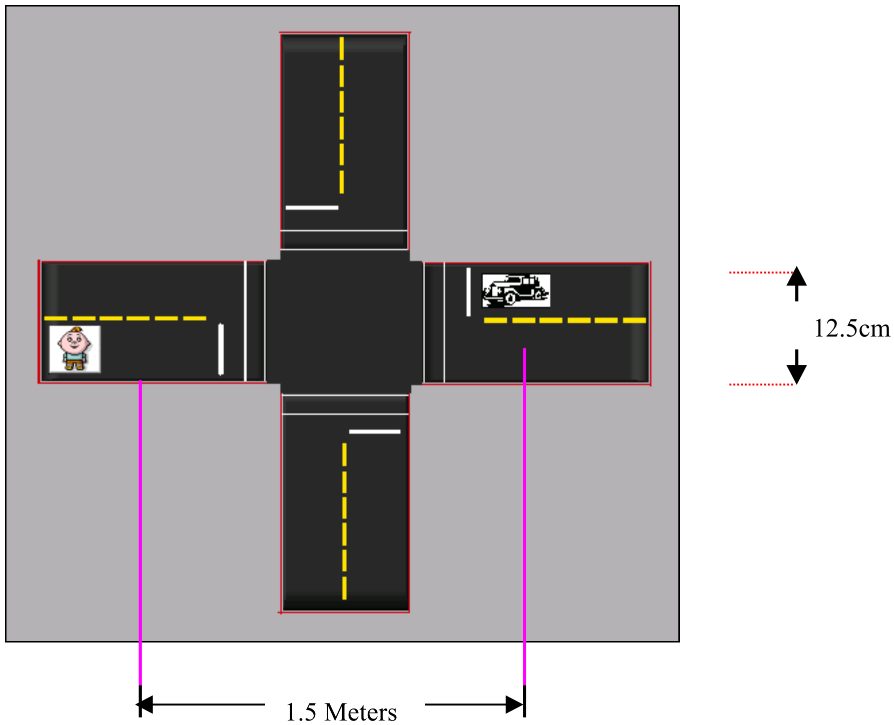
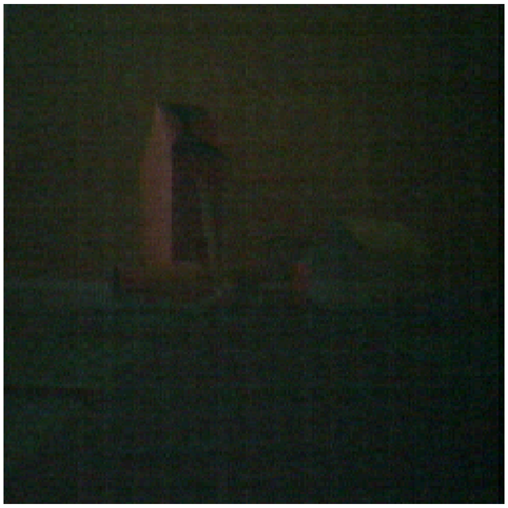


Figure 3 Experimental Set up

Experimental Data

An image of the scene is taken using the CCD Camera, Figure 4. The following images are contained to a size of 250*250 pixels. The Imaging algorithm is applied to evaluate the image.



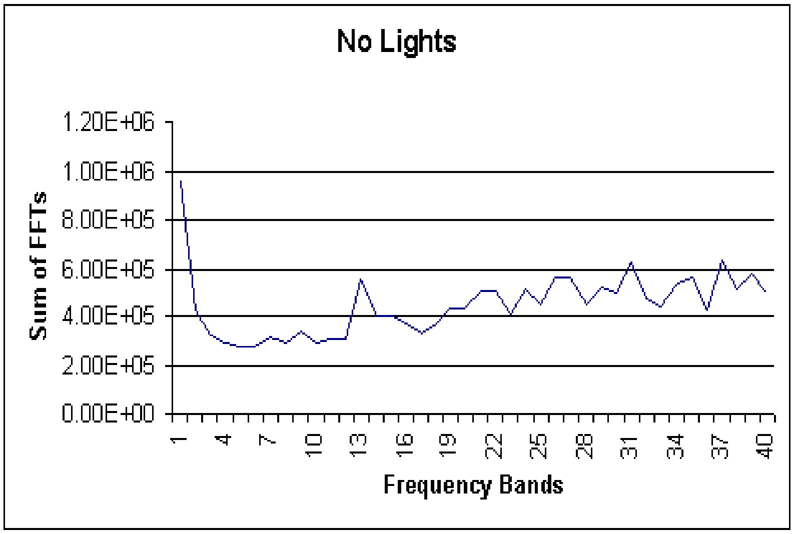
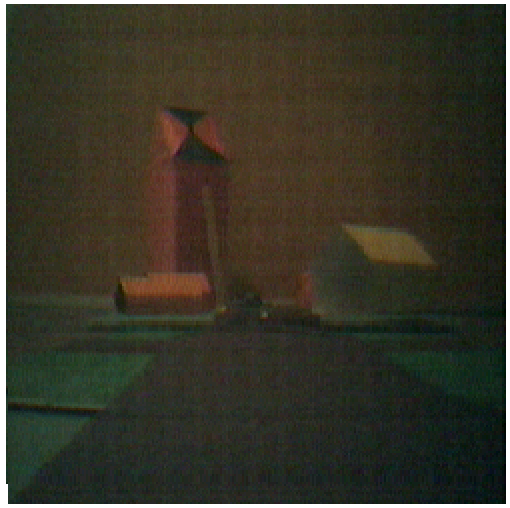


Figure 4 Image of Road Scene with 'No Lights ON' and Water Fall curve

The light incident on the scene is the spurious light that comes through in the room where the experimentation is performed. As a result we still have a minor D.C component that indicates a visibility of the scene.



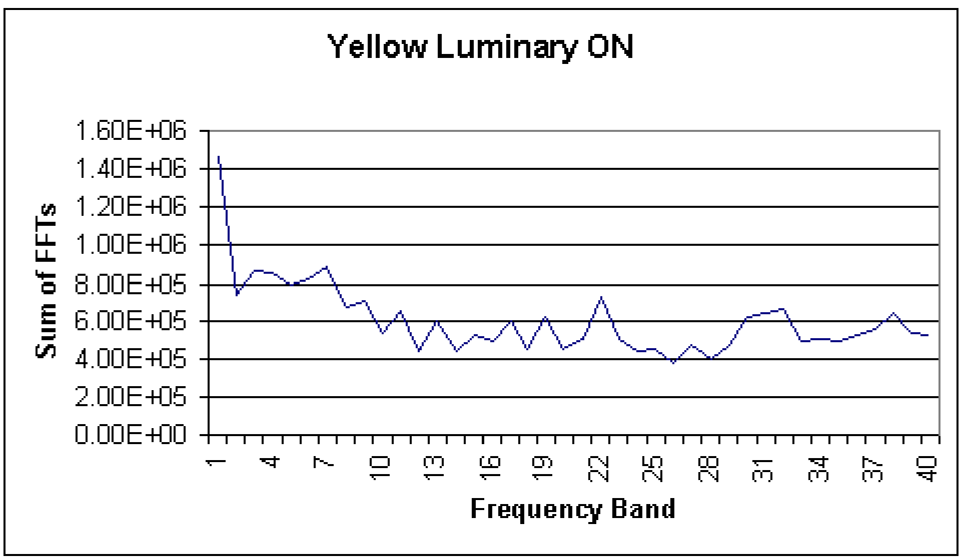
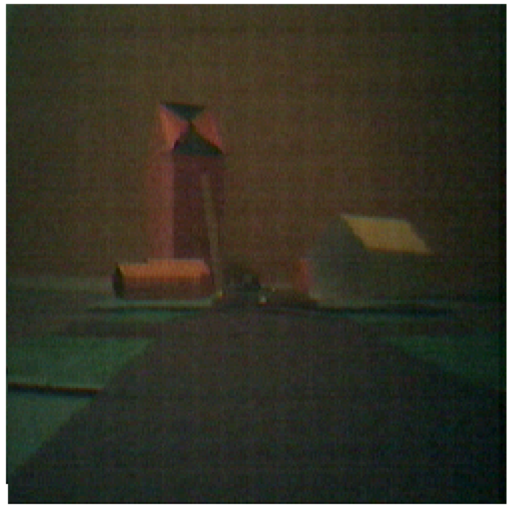


Figure 5 Image of Road Scene with 'Yellow Lights ON' and Water Fall curve

The lower frequency components, as shown in the above group have increased in magnitude indicating a higher background luminance. This luminance can be attributed to the yellow light emanated from the light source.



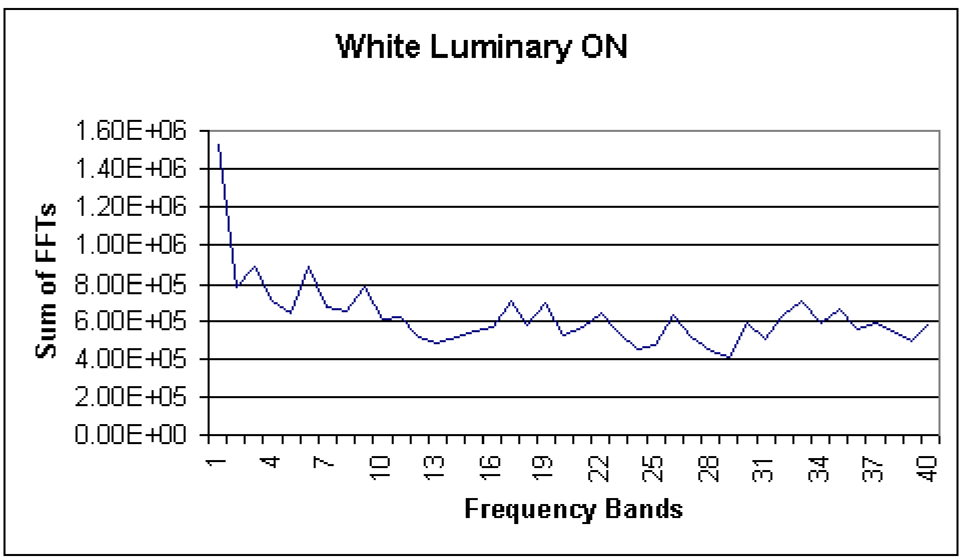
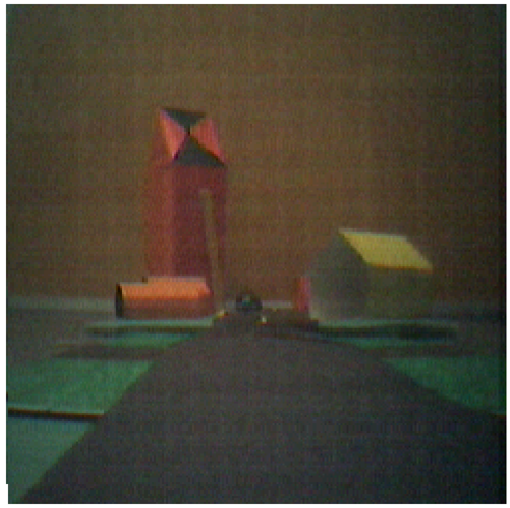


Figure 6 Image of Road Scene with 'White Lights ON' and Water Fall curve

The DC component (background light) has visibly increased from none at all to the presence of white light produced by the incandescent light source. The higher frequency components have a more constant amplitude response.



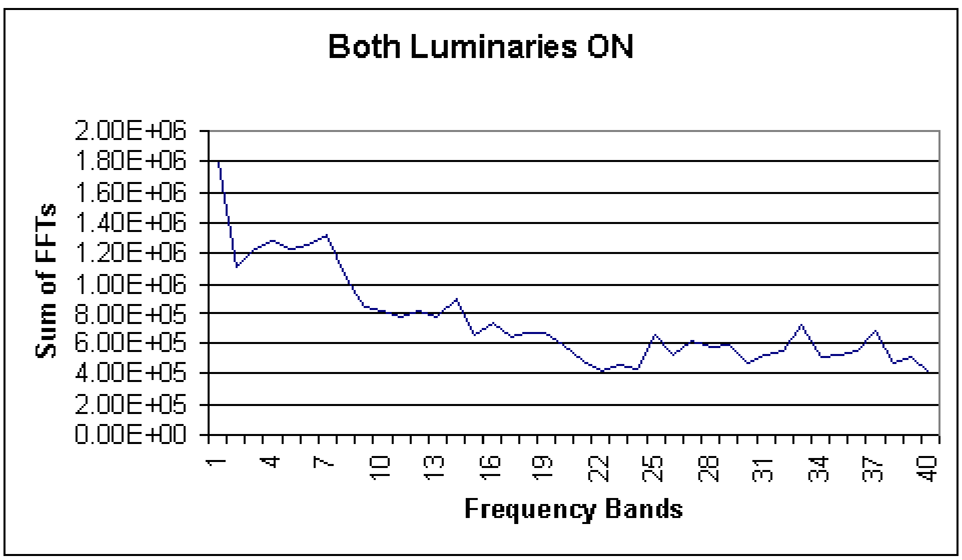


Figure 7 Image of Road Scene with 'Both Lights ON' and Water Fall curve

There is an increase in the background light from none at all to the presence of both lights, indicating a greater level of luminance at the scene.

Conclusion

The frequency component (information contents) of the image changes along with the type of lighting at the scene of interest. The low frequency bands are less significant when the lighting is at a low intensity level. A greater magnitude of low frequency bands is produced when both luminaries are switched ON. A larger illumination flux sharpens the edges resulting in greater information content at lower frequency bands. This result is significant when both luminaries are switched ON by determining the sum of its FFTs that amounts to a maximum of $SUM(ffts) = 1.8 \times 10^6$ for the 0^{th} frequency band and $SUM(ffts) = 5.82 \times 10^5$ for the 20^{th} frequency band of a total of 40 Frequency bands. The lower frequency bands information will be used to design a roadway lighting system.

An Imaging application of Digital Signal processing of measuring visibility using spatial analysis of an image at the scene of interest will be introduced to the Imaging course in bioengineering. Spatial analysis is another term describing the Frequency content at the scene of interest. The images are processed using MATLAB, and the results, along with an analysis, have been presented in this paper.

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