AC 2005-1039: MECHANICS OF MATERIALS: AN INTRODUCTORY COURSE WITH INTEGRATION OF THEORY, ANALYSIS, VERIFICATION AND DESIGN

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Mechanics of Materials: an Introductory Course with Integration of Theory, Analysis, Verification and Design

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Abstract

This paper presents a description of a first undergraduate course in mechanics of materials. Although many of the features of this course have been used by other faculty and presented formally in textbooks, the authors believe they have united them in a way that produces a course that is unique and innovative. The paper is titled "Mechanics of Materials: an Introductory Course with Integration of Theory, Analysis, Verification and Design". The subtitle has been included to emphasize the unification of four strategic elements: Theory, Analysis, Verification and Design. The course leads the student through a traditional exposure to theory, but a non-traditional progressive approach to analysis that uses a modern engineering tool. Introduction of verification develops the student's discipline to question and test 'answers'. If a problem solution can be formulated in general symbolic format, and if specific solutions can then be obtained and carefully verified, the extension from analysis for one set of variables to the design for different sets of specifications can be done quickly and easily with confidence. Three examples are included to demonstrate the approach and one example considers design.

Introduction

In a homework assignment, the ultimate goal for a majority of undergraduate engineering students is simply to obtain the 'answer' in the back of the book. A common approach is to search the textbook chapter for the applicable formula or equation and immediately insert numbers and calculate an answer. This approach is often successful with problems that require few equations, especially if the equations can be solved sequentially or are easily manipulated to isolate the unknown variable. The unfortunate aspect of this is that students may spend very little time focusing on the basic fundamental physics of the problem and, generally, no time at all on the very important verification of the 'answer'! As problems become more complex, with increased numbers of simultaneous equations and/or nonlinear equations, such as with statically indeterminate problems, this approach is laborious and fraught with opportunities for equation manipulation errors. As a result, introductory course instruction and textbooks do not involve these types of problems. In reality, many engineering problems contain multiple unknowns, coupled equations and complex nonlinear equations.

Problem statements in introductory mechanics of materials textbooks[1-40] are presented with known variables defined numerically, symbolically or in combination. The authors have found from experience that students clearly prefer problems where the known variables are defined numerically versus symbolically. Current textbook illustrative examples predominately combine the fundamental equations to isolate the unknowns yielding sequential solutions in symbolic form. Next, if supplied, known numerical values are inserted and unknowns determined.

The authors propose that all variables be retained symbolically, and all equations be written symbolically in natural form without any algebraic manipulation. Once all equations are developed, they are solved by the method of choice, i.e., by hand and/or, preferably, a modern engineering tool. For all but the simplest problems, the authors strongly endorse the use of a commercial program equation solver, supported by verification of the result. This approach allows the students to focus on the basic fundamental physics of the problem rather than on the algebraic manipulation required to isolate the required solution variable(s).

The paper will first discuss the paper subtitles, Theory, Analysis, Verification and Design, to emphasize the focus of our approach to teaching mechanics of materials and to indicate how it differs from past and current textbooks. The paper then considers three simple mechanics of materials examples, one of which considers design, to demonstrate our approach.

Theory

The theory and topic coverage is typical of a traditional one semester introductory mechanics of materials course. Considerable attention is focused on concepts and procedures which the authors have found to be difficult for the student, such as:

- Free body diagram construction.
- The distinction between applied forces and couples on a body and internal forces and couples on an exposed internal plane.
- Construction of diagrams for internal force, stress, strain and displacement for axial and torsion problems as well as the traditional shear force, bending couple and displacement diagrams for beams.
- Use of coordinate axes and careful sign control for all problems involving displacement.
- The use of compatibility diagrams.

Theory is presented and followed with example problems throughout the course. The examples include an explanation of every step with stated governing principles.

The ten topics considered in our course are presented sequentially in the following order:

1. *Planar Equilibrium Analysis of a Rigid Body.*
2. *Stress.*
3. *Strain.*
4. *Material Properties and Hooke's Law.*
5. *Centric Axial Tension and Compression.*
6. *Torsion.*
7. *Bending.*
8. *Combined Analysis: Centric Axial, Torsion, Bending and Shear.*
9. *Static Failure Theories: a Comparison of Strength and Stress.*
10. *Columns.*

A design case study of a hoist structure is included at the conclusion of each topic to reinforce the concepts presented.

Analysis

A primary goal in this course is to show the student that force and elastic deformation analysis of single or multiple connected bodies is based on the application of only three fundamental sets of equations:

- rigid body equilibrium equations,
- material load-deformation equations derived from Hooke's Law and
- equations defining the known or assumed geometry of deformation.

The commonality of a general approach to all problems is emphasized, an approach that is identical for determinate and indeterminate structures containing axial, torsional and/or bending loads. This general approach is formulated to emphasize:

- identification of applicable fundamental independent equation set(s) being written,
- formulation of the necessary governing equations in symbolic form, with no algebraic manipulation to isolate unknowns,
- matching the number of unknowns with the number of independent equations and
- entering the known numerical data and solving for the unknown variables.

For the general problem involving deformation, our proposed non-traditional structured problem solving format contains eight analysis steps. The students are required to follow the appropriate steps listed below for every in-class and homework problem they solve.

1. *Model.* The success of any analysis is highly dependent on the validity and appropriateness of the model used to predict and analyze its behavior in a real system, whether centric axial loading, torsion, bending or a combination of the above. Assumptions and limitations need also be stated. This step is not explicitly emphasized in any mechanics of materials textbook.

2. *Free Body Diagrams.* This step is where all the free body diagrams initially thought to be required for the solution are drawn. The free body diagrams include the complete structure and/or parts of the structure. Very importantly, all dimensions and loads, even those which are known, are defined symbolically.

3. *Equilibrium Equations.* The equilibrium equations for each free body diagram required for a solution are written. All equations are formulated symbolically. There is no attempt made at this point to isolate the unknown variables. However, every term in each equation must be examined for dimensional homogeneity.

4. *Compatibility and Boundary Conditions.* One or more compatibility equations are written in symbolic form to relate the displacements. A compatibility diagram is used when appropriate to assist in developing the compatibility equations. All equations are

formulated symbolically and there is no algebraic manipulation. Every term in each equation must be examined for dimensional homogeneity. Although compatibility equations are commonly written for indeterminate problems, the authors emphasize their use for determinate problems just as is done in the textbooks by Craig[7], Crandall[8] et al., Shames[30], and Shames & Pitarresi[31].

5. *Material Law.* The material law equations are written for each part of a structure based on the Model in Step 1. All equations are formulated symbolically and there is no algebraic manipulation. Every term in each equation must be examined for dimensional homogeneity.

6. *Complementary and Supporting Formulas.* Steps 1 through 5 are sufficient to solve for the (primary) variables force and displacement in a structures problem. Step 6 includes complementary formulas for other (secondary) variables such as stress and strain, variables which may govern the maximum allowable in service values of force and displacement, but which do not affect the governing equilibrium or deformation equations. Supporting formulas are those which might be required to supply variable values in the material law equations and complementary formulas; formulas such as area, moment of inertia, centroid location of a cross-section, volume, etc.

   The complementary and supporting formulas are written symbolically and are necessary to develop a complete analysis. The complementary formulas might involve solution governing variables such as stress, strain and stiffness. Supporting formulas may be necessary to completely define variables in Steps 3 through 5 and in the complementary formulas. These formulas might include cross-sectional area, polar moment of inertia, centroid location, moment of inertia, section modulus, effective length, radius of gyration, etc.

7. *Solve.* The independent equations developed in Steps 3 through 6 solve the problem. The students compare the number of independent equations and the number of unknowns. The authors emphasize that the student should not proceed until the number of unknowns equals the number of independent equations.

   The solution may be obtained by hand, and this generally requires algebraic manipulation. Alternatively, the solution of any number of equations, linear or non-linear, can be obtained with a modern engineering tool. With intelligent application of verification (Step 8), the computer program is a much more reliable calculation device than a calculator. (ABET[41] criterion 3(k) states that engineering programs must demonstrate that their students have the "ability to use the techniques, skills, and modern engineering tools necessary for engineering practice".) The students are allowed to select the modern engineering tool of their choice, and this might include Mathcad[42], Matlab[43] and TKSolver[44]. The authors have not seen this solution procedure in any mechanics of materials textbook.

8. *Verify.* This important step is a critique of the answer, and is discussed in depth in the next section. This step is considered only in the mechanics of materials textbook by Craig[7].

Problems in statics require only Steps 1, 2, 3, 6 and 7. These five steps have not been employed in the treatment of statics problems in any statics or mechanics of materials textbook. Furthermore, Steps 1 through 8 have not been suggested in any mechanics of materials textbook.

Pedagogically the step-by-step solution format allows a student to build a structure in their minds of how to efficiently approach a problem and solve it. The authors believe that this step-by-step procedure will help students build logic, promote analytical thinking, provide a true physical understanding of the subject and, hopefully, extend the same disciplined process to other courses.

Verification

One of our educational goals is to convince students of the wisdom to question and test solutions to verify their 'answers'. We do this by integrating verification as part of the structured problem solving format discussed in the previous section. There are very few textbooks that have addressed verification. It has been considered in statics by Sandor[45] and by Sheppard & Tongue[46], and in mechanics of materials by Craig[7]. Verification is new to almost all undergraduates, but it is critical and really must be formally integrated into the solution process! Once our students graduate and become professionals, they must be prepared to stand behind their 'answers'.

In our approach, verification Step 8 is carried out after solution Step 7 is performed once. The power of our proposed use of the modern engineering tool rests in the ability to quickly and easily run many cases to verify the problem solution. How does one test the problem solution? Listed below are some suggested questions that students may apply for the purpose of verification of their 'answers'.

- *A hand calculation?* A longhand analysis for the complete solution, a partial solution and supporting calculations, e.g., geometric properties. The pitfall here is that a longhand solution of incorrect equations might check the computer solution (of the same incorrect equations) leaving a false impression of verification of the 'answer'.

- *Comparison with a known problem solution?* A known problem solution may be found in references, e.g., handbooks, appendices, textbooks, etc.

- *Examination of limiting cases with known solutions?* Limiting cases are constructed which establish a problem with a known solution. For example, removing the static indeterminacy by reducing the stiffness (lowering of the elastic modulus) of structural components yields an example that may be tested with a hand calculation or compared to other known solutions. Altering the placement of load(s) is another example. Known problem solutions may be found in handbooks, appendices, textbooks, etc.

- *Examination of obvious known solutions?* These are problems that are simple and which yield quick, very apparent known solutions. For example, zero applied loads must yield no response. Other examples, a concentrated applied load positioned at a rigid support would result in zero response, a load reversal would yield the same magnitudes but opposite signs.

- *Your best judgment?* This is where an examination of the answer points to obvious quantitative and/or qualitative errors. In a quantitative sense, are answers of the correct order of magnitude? From a qualitative perspective, do the applied loadings produce reactions and displacements in directions obvious from a physical understanding of the problem? Are the signs correct?

- *Comparison with experimentation?* Experimentation gives substance to theoretical concepts and provides a means of augmenting insights gained from analytical studies. Furthermore, it can also be used to verify results. Due to time limitations in our course, experimentation is not considered.

As indicated above, attempts at solution verification may take many forms, and, although in some cases it may not yield absolute proof, it does improve the level of confidence. The authors believe verification Step 8 will help students build logic, promote analytical thinking and provide a better physical understanding of the subject.

Design

Engineering design defined by ABET EC2000[41] is “the process of devising a system, component, or process to meet desired needs. It is a decision making process (often iterative), in which the basic sciences, mathematics, and the engineering sciences are applied to convert resources optimally to meet these stated needs.” Another educational goal of our course is to introduce design through homework problems and short, simple and well-defined projects. As the student progresses to more advanced courses, i.e., machine design, structural design, etc., projects become lengthier, open-ended and difficult, leading to the major design experience.

In accordance to ABET EC2000[41], an engineering program must demonstrate that the graduates of a program have satisfied Criteria 3(c) “an ability to design a system, component, or process to meet the desired needs…”. The approach proposed in this paper can be used to demonstrate Criteria 3(c) applied to individual structural components. Furthermore, if the approach is used in other courses, i.e., statics, machine design, structural design, etc., then this can be used to demonstrate ABET EC2000[41] Criteria 4 as follows: “Students must be prepared for engineering practice through the curriculum culminating in a major design experience based on the knowledge and skills acquired in earlier course work…”.

Some mechanics of materials textbooks that introduce design include Beer & Johnston[2,3], Craig[7], Pytel & Kiusalaas[25], Shames[30], Shames & Pitarresi[31], Ugural[37] and Yeigh[40]. In general, the presentations involve homework problems or special problems identified under the category of computer application. The problems tend not to have a structured format and request a single solution for a single set of specific requirements. In other words, the solutions are not developed in general symbolic form. This certainly limits the opportunity for solution verification testing and extension to iterative design studies.

The proposed approach in this paper is based on implementation of symbolic equations and therefore allows easy extension to design. With equations written in symbolic form, they are entered into a modern engineering tool (equation solver) and validated through thorough testing in Step 8. The equations then may be used not only for repetitive analysis of a structure, but also for design of a similar structure, where the dimensions and materials must be selected for a given loading. Incorporating a computer equation solver with the 'raw' fundamental symbolic equations, as proposed in our approach, not only leads to easy design applications, but also has the added benefits of reduced opportunity for algebraic errors and increased engineering productivity.

Introduction of Examples

The first example to be considered is a statically determinate axial composite bar subjected to concentrated loads. After this problem is solved, we will make the structure statically indeterminate and show that the governing equations are identical to the statically determinate case and may be solved with only a change in the recognition of known and unknown variables. The third example considers a design application of the second example. The example problems are presented with discussion as one might find in a textbook. The examples will focus on three elements of our approach that includes Analysis, Verification and Design and it is assumed that the reader has the appropriate background in Theory. The problems will be solved using the structured problem solving format discussed in the Analysis section.

Example 1 *Two Segment Determinate Bar with Concentrated Loads.*

The composite round bar in Fig. 1 consists of two segments. Each segment has a specified length, cross section diameter and material. The bar is rigidly supported ($u_A = 0$) at the left end, point A, and two forces are applied as shown; P_B at the junction of the sections, point B, and P_C at the end, point C.

Derive the governing symbolic equations that will yield the displacement of the bar cross sections at locations B and C, and solve for the displacements using the following input:

P_B = - 18.0 kN,	P_C = 6.0 kN,
L_1 = 0.508 m,	L_2 = 0.635 m,
d_1 = 40 mm,	d_2 = 30 mm,
Steel: E_1 = 207 GPa,	Aluminum: E_2 = 69 GPa.

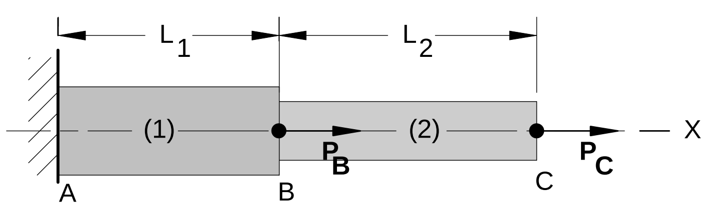


Figure 1. Two segment determinate bar with concentrated loads.

SOLUTION:

1. *Model.* Figure 2(b) shows the full composite bar with the reference coordinate x axis origin located at the wall. This x axis is common to both segments (1) and (2). The displacements u_B and u_C are shown in Figs. 2(a) and (b) as vectors indicating the change in position from the undeformed state. In Figs. 2(c) and (d), the bar is separated with a cut just to the left of point B, the point where the force P_B is applied. The separated bars are uniform with end loads only. Since each segment is a uniform bar with end loads, we will apply to each segment the material law derived in class. The assumptions of this model are consistent with a uniform bar with end loads.

2. *Free Body Diagrams.* The free body diagrams of the individual segments are shown in Figs. 2(c) and (d). The individual segments, FBDs I and II, are the full lengths of the two segments of the bar because we want to involve the displacements only of points A, B and C. Note that the separating cut has been made just slightly to the left of point B so that the force $F_B^{(1)}$ is internal to segment (1). If the cut had been made to the right of point B, we would show a force $F_B^{(2)}$ that would have a different magnitude because it would be internal to segment (2), not segment (1). Note also, as a standard practice, all unknown internal bar forces are, and will continue to be, drawn in the positive sense (tensile), i.e., directed outward from the surface.

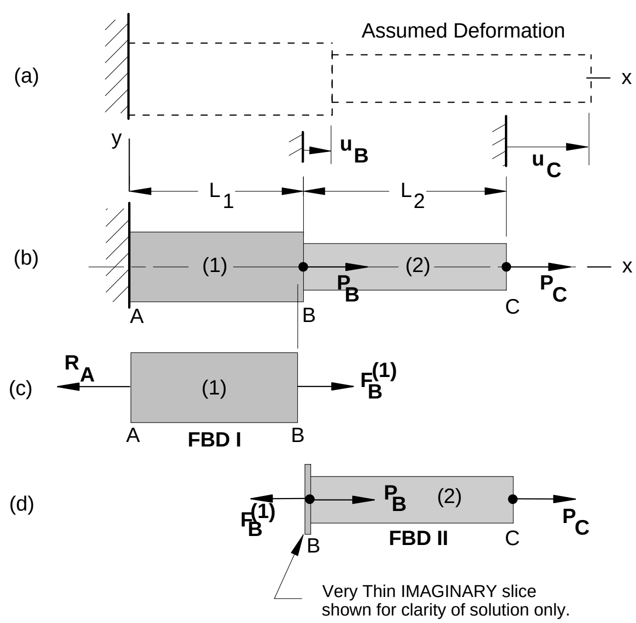


Figure 2. Assumed deformation and free body diagrams of structure and segments.

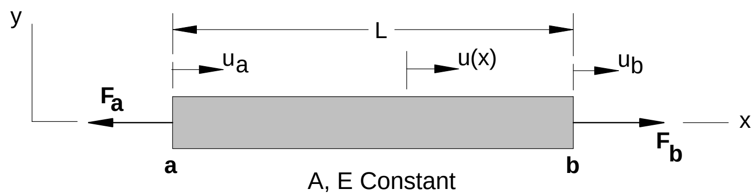
3. *Equilibrium Equations.* Writing the equilibrium equations for each segment in Fig. 2:

FBD I: $F_B^{(1)} = R_A$ (1)

FBD II: $F_B^{(1)} = P_B + P_C$ (2)

Note that if we are given the applied forces P_B and P_C, the internal forces $F_B^{(1)}$ and R_A can be calculated now. Since the forces can be calculated solely from the application of the Equilibrium Equations, we say that the force system is statically determinate.

4. *Material Law.* We apply the material law shown in Fig. 3 for the uniform end loaded bar to each of the individual segments (1) and (2). The common point B in Fig. 2(b) will be assigned to the end of each segment in Figs. 2(c) and 2(d) at the point where segments (1) and (2) are separated.



$$u_b = u_a + \frac{F_b L}{AE}$$

Figure 3. Material law and sign convention for a uniform, homogeneous, linear elastic bar with end loads.

Substituting the appropriate symbols and subscripts and adhering to the sign convention in Fig. 3 yields the following:

Segment (1): $$u_B = u_A + \frac{F_B^{(1)} L_1}{A_1 E_1} \tag{3}$$

Segment (2): $$u_C = u_B + \frac{P_C L_2}{A_2 E_2} \tag{4}$$

5. *Compatibility and Boundary Condition(s).* Compatibility is intended to define how the individual separated segments deform relative to one another in the assembled structure. For this case where displacements occur only along a straight line, we simply require the displacement of identical points in the individual segments to be equal, otherwise, the solution could indicate a gap or overlap at that point. We force this compatibility by assigning the same displacement symbol to the common point in each segment. For example, in Figs. 2(c) and (d), the displacement of point B in segment (1) must equal the displacement of point B in segment (2). For this very simple compatibility condition, the common displacement symbol u_{point}, will always be used without the need to introduce a formal equation.

   The boundary condition is the known displacement of point A at the wall:

$$u_A = 0 \quad \text{for a rigid support}$$

6. *Complementary and Supporting Formulas.* In this problem no complementary formulas are needed. The supporting formulas relating the cross section areas to the segment diameters are as follows:

$$A_1 = \frac{\pi d_1^2}{4} \tag{i}$$

$$A_2 = \frac{\pi d_2^2}{4} \tag{ii}$$

7. *Solve.* Considering the boundary condition, $u_A = 0$, as known, we have 4 independent equations, Eqs. (1), (2), (3) and (4), for the 4 unknown variables:

$$R_A,\ F_B^{(1)},\ u_B \text{ and } u_C$$

The solution of the governing equations (1) through (4) and the supplementary equations (i) through (ii) is obtained with an equation solver program. The solution is the following:

$$F_B^{(1)} = -\ 12.0 \text{ kN}$$
$$R_A = -\ 12.0 \text{ kN}$$
$$u_B = -\ 23.4\ \mu\text{m}$$
$$u_C = 54.7\ \mu\text{m}$$

8. *Verify.* Here is the place to make a strong case for the use of a modern engineering tool (equation solver). Having entered symbolic Eqs. (1) through (4) in an equation solver along with the formulas, Eqs. (i) and (ii), for calculation of areas, we now have a tool for testing the solution obtained in Step 6. Listed below are some suggested tests for this problem:

- Substitute equal values of lengths, areas and elastic modulus, and let $P_B = 0$, the solution should be for a uniform, homogeneous bar of length $2L$ with end load P_C:
$$u_C = P_C(2L)/AE$$

- Substitute $P_C = 0$, the solution should be the deformation of segment (1) only:
$$u_C = u_B = P_B L_1 / A_1 E_1$$

- Substitute $E_1 \to \infty$, yields $u_B = 0$; $E_2 \to \infty$, yields $u_C = u_B$; $E_1 \to \infty$ and $E_2 \to \infty$, yields $u_C = u_B = 0$.

- Let P_B and P_C have the same magnitude, but opposite directions yielding $u_B = 0$.

- Compare output with a hand calculated solution, both the final results and intermediate values such as the segment cross-sectional areas.

- Find similar problems with answers in other texts. Substitute the new values and compare results.

- etc.

Example 2 ***Two Segment Indeterminate Bar with Concentrated Load.***

The composite round bar of Example 1 is modified by applying an additional rigid support at the right end as shown in Fig. 4, thus making the problem statically indeterminate.

The bar is subjected to the concentrated load P_B at point B. In this example, the right end displacement is known (u_C=0) and the reaction force at the right end support is unknown, whereas in Example 1, the displacement was unknown and the force was known.

Derive the governing symbolic equations which will yield the displacement of the bar cross section at location B, and solve for the displacement using the following input:

P_B = - 18 kN,	
L_1 = 0.508 m,	L_2 = 0.635 m,
d_1 = 40 mm,	d_2 = 30 mm,
Steel: E_1 = 207 GPa,	Aluminum: E_2 = 69 GPa.

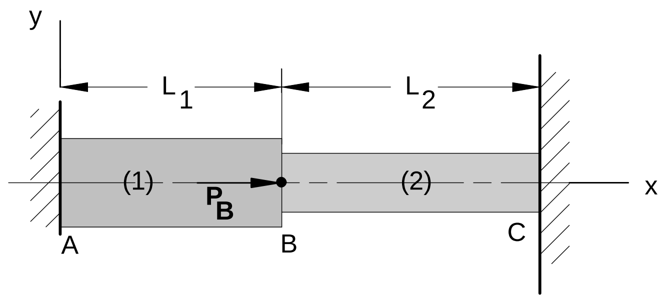


Figure 4. Two segment indeterminate bar with a concentrated load.

SOLUTION:

To solve this problem for the unknown reaction at the right end and the displacement of point B, one simply has to input the known displacement u_C of point C and solve for the unknown reaction force P_C. All governing independent symbolic equations are exactly the same; the free body diagrams are the same, equilibrium equations are the same, the material law equations are the same and compatibility is the same. All problems, statically determinate and/or indeterminate must satisfy the same fundamentals: equilibrium, compatibility and material law. Therefore, there is absolutely no change in the equations that have been entered into the equation solver. The only difference is in the specification of the force and displacement boundary conditions to achieve a particular solution.

It should be noted that the solution of the governing equations for this problem has been subjected to the verification Step 7 in Example 1. The model is the same, the governing equations are the same, only the boundary conditions have been changed.

Substituting the knowns supplied in the problem statement and the boundary conditions yields the following results:

$$F_B^{(1)} = -\ 15.65 \text{ kN}$$
$$R_A = -\ 15.65 \text{ kN}$$
$$P_C = 2.35 \text{ kN}$$
$$u_B = -\ 30.6\ \mu\text{m}$$

Example 3 ***Design Application of Example 2.***

The solution of the composite round bar of Example 2 yields a displacement of point B which is determined to be excessive. This displacement can be modified with permissible change of the diameter of segment (2). Solve for the diameter of segment (2) which will limit the displacement of point B to - 20 μm.

SOLUTION:

There certainly are different approaches to solving this design problem as follows:

- *Solution Alternative 1.* Input a list of independent diameter variable d_2 and solve for the list of corresponding displacements u_B at point B. Select the diameter d_2 satisfying the displacement design criteria.

- *Solution Alternative 2.* Create a plot of diameter d_2 versus displacement u_B. Select the diameter d_2 satisfying the displacement design criteria.

- *Solution Alternative 3.* With the governing equations in an equation solver, the solution of this problem is very easy. Establish the diameter d_2 of segment (2) as the unknown and the displacement u_B of point B as the known of the stated magnitude. The solution yields the following for the diameter d_2 of segment 2 based on the displacement design criteria:

  $$d_2 = 67.42 \text{ mm}$$

  The solution, although coupled and non-linear, is obtained directly with no intermediate analyses as required in Solution Alternatives 1 and 2.

Solution Alternatives 1 and 2 were the typical approach taken when structured programming languages, e.g., Basic, C, FORTRAN, Pascal, etc., became available. These languages require isolation of the knowns from the unknowns on opposite sides of the equation, and changing the variables from known to unknown requires reprogramming. The required algebraic manipulation is undesirable from a labor and accuracy standpoint. At present, however, many modern engineering tools include equation solvers that do not require isolation of the dependent variables. This greatly increases the flexibility of the tool resulting in simplicity and much less labor in repetitive analyses.

Conclusion

The authors believe that the first course in mechanics of materials should present not only the basic theory, but also an approach to problem solving which encourages the student to: (1) describe the problem model with assumptions and limitations, (2) preface equations with a clear statement of the principle involved, (3) solve the equations with appropriate modern engineering tools and (4) conduct a critique of any answer. In addition, the student should learn that the mathematical model providing an analysis solution of a problem can almost always be converted into a design tool for a similar physical system.

Teaching the student to model a general physical problem with the fundamental equations written in symbolic form, with no variable values specified, helps the student to more fully concentrate on the fundamental principles taught in the course. Introducing the modern engineering tool to solve the equations removes the necessary manipulation of the equations to isolate the dependent variables. Training the student to examine and test the answer becomes an important goal in our course. The proposed approach can also be used in follow up design and nondesign courses that includes advanced mechanics of materials, machine design, structural analysis, structural design, etc. Students should be prepared to solve the more complex problems, and use of the currently available modern engineering tools makes that possible.

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