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Mechanics of Reinforced Concrete Beams – The Whole is Greater than the Sum of its Parts

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Mechanics of Reinforced Concrete Beams – The Whole is Greater than the Sum of its Parts

Abstract

This “Best in 5-Minutes” presentation will detail a lecture given in a senior level reinforced concrete design class titled “Is The Whole Greater Than the Sum of Its Parts? – Aristotle’s Insight into the Mechanics of Reinforced Concrete”. The purpose of this lecture is to allow the students to understand how the composite nature of a reinforce concrete beam allows for its flexural strength (the whole) to be much greater than just the sum of the flexural strengths of the unreinforced concrete and steel reinforcement (its parts).

This lecture begins by the instructor bringing a portion of a reinforced concrete beam into the classroom, having the students physically measure the relevant dimensions, and posing the question: “Is the whole (flexural strength) greater than the sum of its parts?”. The class then works in groups of 4-5 students to decide how they are going to answer the question. The instructor circulates around the room and guides the students through the calculations necessary to prove that the strength of the whole is indeed greater than the sum of the parts. The instructor them concludes the lecture by discussing the engineering mechanics behind why the flexural strength of a reinforced concrete beam is significantly higher than the sum of the flexural strengths of the unreinforced concrete beam and the steel reinforcement.

Later in the semester is it pointed out to the students that this is not true for other types of reinforced concrete structural members, such as axially loaded columns and in the analysis of reinforced concrete beams for shear, where the strength of the reinforced concrete is calculated as merely the sum of the strength of the unreinforced concrete and steel reinforcement.

Introduction

In the department of civil and environmental engineering at Manhattan College, all senior civil engineering students are enrolled in Reinforced Concrete Design. This course meets for 4 lecture hours a week and presents the design of reinforced concrete structural members. Recently, the author has introduced an exciting and engaging lecture into the course, titled “Is The Whole Greater Than the Sum of Its Parts? – Aristotle’s Insight into the Mechanics of Reinforced Concrete”.

Reinforced Concrete Design Course

As part of their structural design course sequence, all civil engineering students at Manhattan College are required to take Reinforced Concrete Design. The purpose of this course is to expose the students to the design of reinforced concrete structural members, including beams, slabs, and columns. This course is generally taken during the first semester of senior year and is comprised of four lecture hours per week. These students have already taken a steel design course as their first class in structural design and also a civil engineering materials course that has exposed them to the material properties of concrete.

The first week of the semester in the reinforced concrete course is used to review both the material properties of unreinforced concrete and relevant laboratory tests, including compression, split tension, and flexure. The second week of the course is used to introduce the students to the mechanical response of reinforced concrete beams, which includes a discussion of the different types of failure modes and an overview of the internal couple method. During the third week of the course, the students learn how to calculate the flexural strength of reinforced concrete beams through application of the internal couple method. The lecture titled "Is The Whole Greater Than the Sum of Its Parts? – Aristotle's Insight into the Mechanics of Reinforced Concrete" is presented during the fourth week of the semester.

Is the Whole Greater than the Sum of its Parts?

This lesson begins with the instructor preparing the classroom before any students arrive by writing on the board the question "Is the Whole Greater than the Sum of its Parts?" and displaying a portion of a reinforced concrete beam that has previously been loaded to failure (**Figure 1**). Once all of the students all arrive, the instructor divides the class of approximately 20 students into 5 groups. He then askes each of the groups to apply the statement to the flexural strength of the beam and to either prove the statement to be true or false.

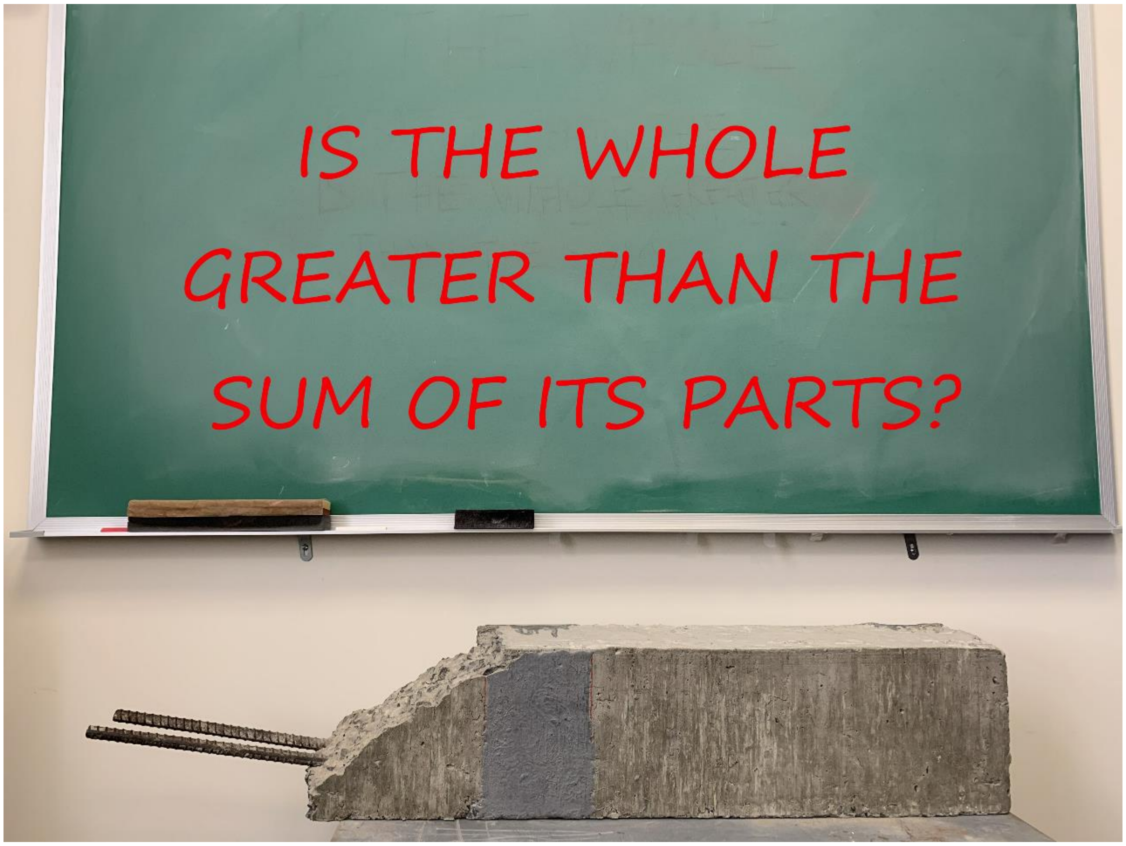

Figure 1: Reinforced Concrete Beam Displayed During the Lecture

After posing the question, the instructor gives each group approximately 5 minutes to discuss how they will approach the problem before he begins to circulate around the classroom and interact with the students. While the question that has been posed is a general yes/no type question, the steps necessary to arrive at the answer are initially unclear to the students. The purpose of posing the question this way is to promote group and classroom discussion on how to best answer the question given the knowledge that they have already acquired. While there is not a clear right or wrong method to solve the problem, the instructor guides them to a particular approached based on knowledge that they have previously received in their current course and previous courses.

The different groups determine very quickly that *the whole* refers to the flexural strength of the reinforced concrete beam; however, the groups generally cannot initially agree on what *the parts* refers to. Through group and class-wide discussions, the decision is made to consider the parts to be the unreinforced concrete beam and the steel reinforcing.

Before the students can calculate the different values of the flexural strength, they must first obtain information about the reinforced concrete beam, including the width, depth, and effective depth of the cross-section and the number, size, and grade of the steel reinforcement. The students use a measuring tape which is provided by the instructor to determine the width, depth, and effective depth. The markings on the exposed rebar are used to determine the rebar size and its grade (yield strength). Since it is not possible for the students to determine the compressive strength of the concrete in the classroom, the instructor provides them with a value of 4 ksi. The cross-section of the reinforced concrete beam is shown in Figure 2 and the values of the relevant parameters are displayed in Table 1.

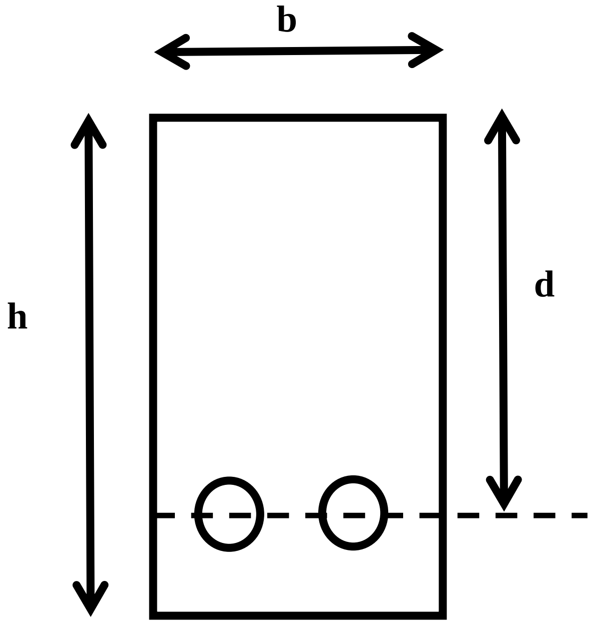

Figure 2: Reinforced Concrete Cross-Section

Table 1: Reinforced Concrete Beam Parameters

Parameter	Value
Depth, h [inch]	8
Effective Depth, d [inch]	6
Width, b [inch]	6
Rebar Number and Size	(2) No. 5
Rebar Diameter, d_b [inch]	0.625
Total Rebar Cross-Sectional Area, A_s [in^2]	0.62
Rebar Yield Strength, f_y [ksi]	60
Concrete Compressive Strength, f'_c [ksi]	4

The Whole

In order to calculate the flexural strength of the reinforced concrete beam, the students use the internal couple method [1] and assume an equivalent rectangular concrete stress distribution as described in Section 22.2.2.4 of ACI 318-14 [2]. This calculation has been taught to the students during the previous week of classes, several example problems have been solved, and homework problems have been assigned. As a result, this step in the problem is a review and serves to reinforced material that they have been taught, but necessarily mastered yet.

The aspect which is new is that in all previous problems, the students were only given a two dimensional sketch of the cross-section and the values for all of the relevant parameters were directly provided. In this lesson, the students are provided with an actual reinforced concrete beam and they must physically measure the relevant dimensions. This method of instruction may appeal more to sensory, visual, and active learners who would benefit by physically seeing the three dimensional beam and actively making measurements. Additionally, the students must recall information that they learned during the first week of the course relevant to reading the markings on steel rebar. The exposed portion of the rebar contains a grade line that indicates that the rebar is grade 60 (yield strength of 60 ksi).

Assuming that the steel reinforcement yields at failure, the tensile force in the rebar is given by:

$$T = A_s f_y = (0.62\ in^2)(60\ ksi) = 37.2\ kips$$

Since the compressive force in the concrete at failure must equal the tensile force in the rebar:

$$C = T = 37.2\ kips$$

Utilizing the equivalent rectangular concrete stress distribution as described in Section 22.2.2.4 of ACI 318-14 and shown in **Figure 3**:

$$C = 0.85 f_c' ab$$

Where *a* is the depth of the equivalent rectangular stress block.

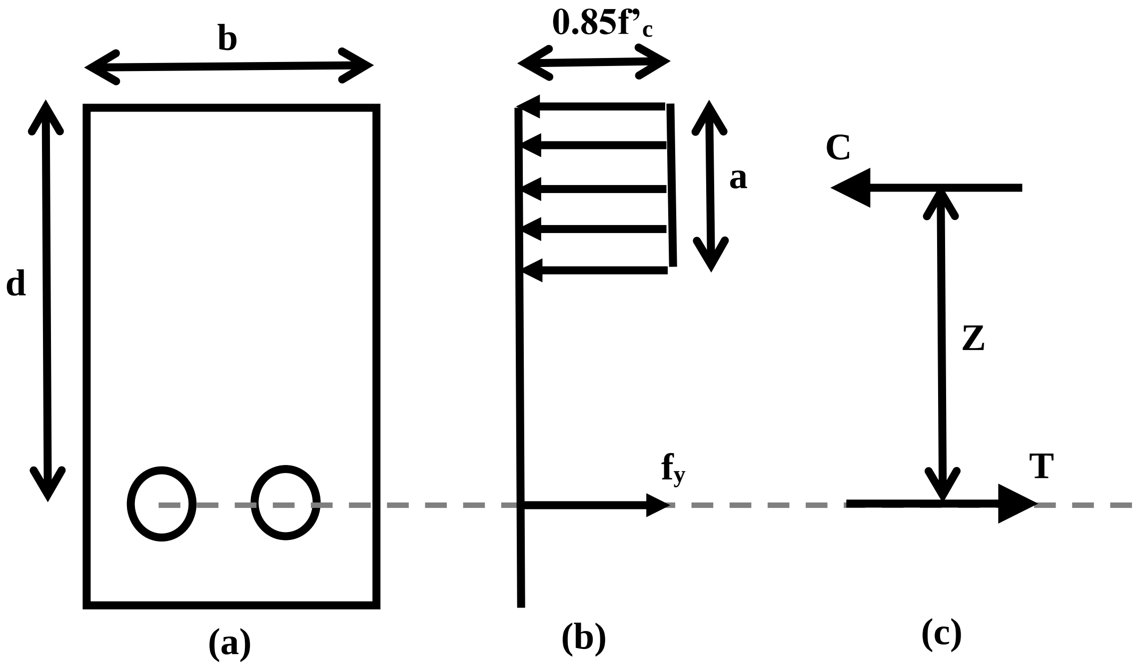

Figure 3: (a) Cross-Section, (b) Stress Distribution, and (c) Internal Couple

By solving the previous equation for a:

$$a = \frac{C}{0.85 f_c' b} = \frac{37.2\ kips}{0.85(4\ ksi)(6")} = 1.824\ inch$$

By utilizing the internal couple method and summing the moments about the centroid of the rebar, the nominal flexural strength of the reinforced concrete bean is calculated by:

$$M_n = CZ = C\left(d - \frac{a}{2}\right) = (37.2\ kips)\left(6" - \frac{1.824"}{2}\right) = 15.77\ k - ft$$

Where Z is the moment arm.

Thus, the strength of the whole is 15.77 k-ft.

The Parts

Unreinforced Concrete

During the students' junior year, they take a required course named Civil Engineering Materials. In this course the students are exposed to various laboratory tests involving unreinforced concrete, including compression, split tension, and flexure tests. During the course Reinforced Concrete Design, the flexural strength of unreinforced concrete beams is reviewed during the first week of the semester and the students are taught how to calculate the cracking moment of unreinforced concrete beams using the flexure formula. The cracking moment, which is the largest internal bending moment that an unreinforced concrete beam can support, is given by equation 24.2.3.5b of ACI 318-14:

$$M_{cr} = \frac{f_r I_g}{y_t}$$

Where f_r is the modulus of rupture, I_g is the gross moment of inertia of the cross-section, and y_t is the distance from the neutral axis to the bottom of the cross-section.

According to ACI 318-14, the modulus of rupture for normal weight concrete is given by:

$$f_r = 7.5\sqrt{f_c'} = 7.5\sqrt{4000\ psi} = 474.3\ psi$$

The cracking moment of the unreinforced concrete beam is calculated by:

$$M_{cr} = \frac{(474.3\ psi)\dfrac{(6")(8")^3}{12}}{(4")} = 2.53\ k - ft$$

Thus, the strength of the unreinforced concrete is 2.53 k-ft.

Steel Reinforcement

It is the calculation of the strength of the steel reinforcement that generally is the most difficult for the students. Initially, they want to quantify the strength of the steel rebar in terms of its tensile strength, so they must be reminded by the instructor that in this context we are concerned with its flexural strength.

In order to guide the students in the right direction, the instructor asks the students to recall information that they learned in their Structural Steel Design course (taken the previous semester) and to identify the limit states that apply to the flexural strength of steel shapes. Once the students provide the correct answers (lateral torsional buckling and yielding), the instructor then asks them to identify which of the two limit states would apply in this situation. A class-wide discussion is then held regarding lateral torsional buckling of beams. It is concluded that this limit state does not apply to circular cross-sections since they have infinite axes of symmetry and do not have a major and minor axis. As a result, the flexural strength of each individual piece of steel reinforcement is given by the plastic moment:

$$M_p = F_y Z_x = \frac{F_y d_b^3}{6} = \frac{(60\ ksi)(0.625")^3}{6} = 0.20\ k - ft$$

Thus, the strength of the steel reinforcement is 0.40 k-ft.

The Whole is Greater than the Sum of its Parts

Based on the results of their previous calculations (strength of the whole = 15.77 k-ft and sum of the strength of the parts = 2.93 k-ft), the students are able to prove that the whole flexural strength is indeed greater than sum of the flexural strength of its parts. This conclusion is intuitive for most of the students; however, the students are generally surprised that the whole is approximately 5.5 times stronger than the sum of its parts. Additionally, they tend to be surprised that the flexural strength of the steel rebar is approximately 1/5th of the flexural strength of the unreinforced concrete beam.

The conclusion that the whole it greater than the sum of its parts is then used start a larger discussion into the behavior of composite materials. The instructor explains that when two or more materials are combined into a composite material the mechanical strength, along with other desirable engineering properties, are generally improved. It is shown to the students that each of the two materials (unreinforced concrete and steel reinforcement) are helping to increase the strength of the other material. This is done to dispel a common oversimplification /misconception that students typical have about reinforced concrete, which is that “concrete is weak and steel reinforcement is used to strengthen the concrete”.

It is explained to the students that the steel reinforcement increases the flexural strength of the concrete by carrying the tensile stress which develops below the neutral axis so that the compressive stress in the concrete can increase until the concrete ultimately fails in compression above the neutral axis. If the steel reinforcement was not present, the concrete would fail once the internal moment is equal to the cracking moment of the cross-section.

It is further explained to the students that the concrete increases the strength of the steel reinforcement by allowing it to be placed further from the neutral axis. If the steel reinforcement was simply to act as a beam on its own, the neutral axis would pass through its own centroid and the moment arm between the internal compressive and tensile forces (using the internal couple method) would be equal to $4d_b/3\pi \approx 0.424d_b$ (**Figure 4**). When the concrete and steel reinforcement act as a composite material, the neutral axis is shifted upward, and the moment arm is equal to the distance between the centroid of the reinforcement and the centroid of the equivalent rectangular stress block in the concrete, which is equal to $d - a/2$. Using the values from the lesson, the value of the moment arm is increased from 0.265" to 5.09".

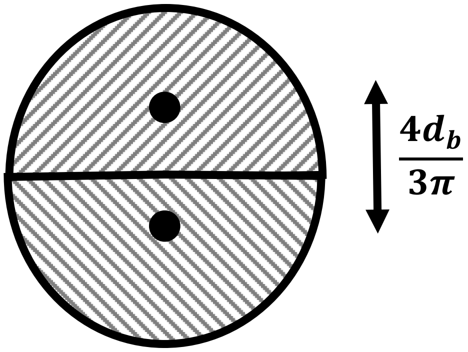

Figure 4: Moment Arm of a Circular Cross-Section (Internal Couple Method)

The Whole is Equal than the Sum of its Parts

The question "Is the whole greater than the sum of its parts?" is revisited during two additional topics during the course. During the topic of shear design of reinforced concrete beams, the students are provided with equation 22.5.1.1 from ACI 318-14:

$$V_n = V_c + V_s$$

Where V_n is the nominal shear strength of a reinforced concrete beam, V_c is the shear strength of the concrete, and V_s is the shear strength of the shear reinforcement (stirrups).

During the topic of design of short columns, the students are provided with equation 22.4.2.2 from ACI 318-14:

$$P_o = 0.85f_c'(A_g - A_{st}) + f_yA_g$$

Where P_o is the nominal axial compressive strength with no eccentricity, f'_c is the compressive strength of the concrete, A_g is the gross cross-sectional area of the column, A_{st} is the cross-sectional area of the longitudinal steel, and f_y is the yield strength of the longitudinal steel.

For both of these applications, it is emphasized that the strength is merely equal to the sum of its parts. For shear, the nominal shear strength is the sum of the shear strength of the concrete and the shear strength of the stirrups. Whereas for compression with no eccentricity, the nominal axial compressive strength is equal to the compressive strength of the concrete plus the compressive strength of the steel reinforcement.

Discussion and Conclusions

This “Best in 5-Minutes” paper outlined an interactive lecture used in a senior level Reinforced Concrete Design course at Manhattan College. This lesson utilizes a popular quote attributed to the Greek philosopher Aristotle to illustrate the mechanical behavior of reinforced concrete beams.

It is common for some students to be initially intimidated by the technical nature of senior level design courses. One of the purposes of this activity is to increase the interpersonal rapport between the students and instructor so that the students feel more comfortable working with each other and also feel more confident to participate in class. This lesson is approached as an interactive group activity in order to promote discussion. The same conclusions could be drawn if it was approached as merely an example problem dictated by the instructor. This would require less class time; however, the interactive group activity is more memorable to the students and allows the significance of the conclusions to be more lasting. This activity uses discussion to teach the process of learning to the students. In order to prove the statement to be true they must connect various concepts that they have previously been exposed to (some of them in different courses). The discussions had during the lesson use active learning to motivate the students to think independently and enhances student involvement.

This lesson appeals to different types of learning styles. Sensory and visual learners can be stimulated by being able to see the reinforced concrete beam and to physically make measurements. This is in comparison to typical example problems in the course where two dimensional illustrations are used and all values are directly provided. Active learners also benefit from this lesson by being able to interact with their classmates during the activity. Additionally, students who fall into the intuitive, verbal, or reflective learning styles benefit from this lecture by being able to think through the question, come up with ideas, and reason out how to prove that the statement is true.

The instructor has used this innovative and exciting lecture during the fall 2018 and fall 2019 semesters and at this time he has not performed any formal assessment; however, there has been positive feedback from the students. The instructor is planning on performing an assessment during the fall 2020 semester to evaluate if this method of presenting the material enhances the ability of students to retain and understand the mechanics behind the behavior of reinforced concrete beams.

References

[1] A. Aghayere, Reinforced Concrete Design, Upper Saddle River, NJ: Pearson, 2019.

[2] ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)," American Concrete Institute, Farmington Hills, MI, 2014.