AC 2011-81: MODELING ROCKETS IN INSTRUMENTATION LAB

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Abstract

A final project for an instrumentation laboratory course was developed involving the prediction of the maximum altitude of a model rocket. The course is part of a mechanical engineering core curriculum. The final rocket project is intended to integrate the theoretical and experimental methods used in the preceding eight experiments included in the course. The rocket lab is conducted over the final five weeks of the semester, with one three-hour laboratory meeting per week. In the first week, the students assemble the rockets from commercial kits and find the drag coefficient for each rocket using a wind tunnel. The speed of the wind tunnel is varied from 10 MPH to 100 MPH in ten steps and the drag force is plotted as a function of the square of the velocity. The drag coefficient is then calculated from the slope of this plot.

On the second week of the rocket sequence, the engine thrust is measured using a strain gage mounted on a ¼” x ¼” aluminum cantilever beam fitted with a machined cylinder sized for the rocket motor. Using a data acquisition system, the strain on the beam is measured as a function of time during the rocket engine ignition sequence while the rocket motor discharge is directed to a metal drum. From the data file generated from this experiment, the students calculate the thrust of the rocket motor as a function of time, employing their skills learned in their solid mechanics course.

On the third week of the laboratory, the instructor assists the students with the development of a numerical solution, incorporating the data measured during the previous two weeks. The students are required to incorporate the curve generated for thrust as a function of time, the weight of the rocket and the drag coefficient, which is used to calculate the drag force as a function of velocity. Using this information, the students develop a computer code to numerically integrate the net force on the rocket twice as a function of time. This allows them to calculate the expected maximum height of the rocket, based on the experimental data obtained in the laboratory.

The final experiment in the fourth week of the project involves launching the rocket and measuring the angle of the rocket with respect to the horizon at a known distance from the launch pad when the rocket reaches its highest point. This allows the actual maximum height to be calculated and compared with the prediction from the numerical model.

Introduction

The instrumentation laboratory course is designed to give students a hands-on opportunity to gain familiarity with measurement tools in the mechanical engineering field. Some of these measurement tools include a micrometer, caliper, pitot tube, barometer, pressure transducer, turbine flow meter, oscilloscope, multimeter, strain gage and a data acquisition system. The lab course includes one hour of lecture immediately followed by two hours of lab time. The course meets one time per week, which allows 14 meetings per semester. The portion of the course involving the model rocketry work is the "project" segment of the course, encompassing the final five weeks of the semester. By this time, the students have had previous exposure to all of the measurement tools needed to perform the model rocketry experiment and, at this point, are asked to design their own experiments to accomplish the goal of predicting the maximum height of a rocket.

Previous work in this area has been done by Boyer et al. [1] which dealt primarily with an introduction to aerospace engineering, using model rocketry as part of this introductory course, including the concept of impulse. Suchora and Pierson [2] use model rocketry as part of a freshman introductory course, primarily in order to generate interest among first-year engineering students. Newberry [3] addresses many of the concepts used in the present paper, including the measurement of rocket motor thrust in the prediction of rocket maximum altitude. Morris and Zietlow [4] describe the use of model rocketry in design competition for senior-level students. In this setting, students are required to develop their own methods for analyzing rocket parameters. Self et al. [5] compared the launching of a model rocket to a projectile launched from a catapult. In this work, the prediction of horizontal travel was the primary focus as opposed to computing the ultimate rocket height as an objective.

The present research differs in two respects with the previously-mentioned works. One of the initial aspects of the student project involves a student laboratory experiment where the rocket drag coefficient is determined using pitot tube measurements for speed in a wind tunnel and the resulting force exerted on the rocket via a load cell (Sting balance). This feature of the present research is absent in previous work published. An additional feature of the experiment described here is that the rocket motor impulse was measured using a strain gage, mounted by the students on a cantilever arm. The strain gage was connected to a data acquisition system and a program was created by the students for acquiring the thrust data. Using the information from these experiments, rocket height was predicted using a numerical scheme accounting for time-variable thrust and time-variable rocket mass. This height prediction was then compared against a measurement taken at an actual launch, including the calculation of uncertainties for both the calculated and measured launch heights.

Experimental Setup

The rocketry project for the Instrumentation Lab course takes place over a period of five weeks, which happens to be the last five weeks of the course. By then, students are familiar with most of the measurement techniques to be used in the rocketry experiment. The "project" nature of the rocket experiment gives them the opportunity to see an application for the measurement methods they have learned in class and provides an integrated series of experiments, culminating in one final objective. Thirty-five percent of the course grade derives from the project.

Table 1 summarizes the sequence of the experiments in the lab and the written work to be submitted by the students. The first class meeting involving the rocket project involves building the rockets from a kit. Each section of the lab course enrolls a maximum of 12 students. The students are broken up into four groups, with a maximum of three students per group. Each group builds one rocket model. The "Baby Bertha" model by the Estes model rocket company was chosen because of its large diameter, making wind tunnel measurements more robust.

Page 22.1076.3

Table 1. Milestones of Rocketry Project	
Week 10	**During Lab:** Build rocket model and write LABVIEW program to sample data from a strain gage module.
Week 11	**Hand In:** (1) Free body diagram of rocket in flight. (2) Definition of drag coefficient, identifying each term in the equation. (3) Procedures for measuring engine impulse. (4) Certify that you have watched the video on Mounting Strain Gage.
	During Lab: (1) Perform drag test in wind tunnel. (2) Set up for rocket motor thrust testing.
Week 12	**Hand In:** Drag curve and calculation of drag coefficient.
	During Lab: Measure rocket motor thrust.
Week 13	**Hand In:** Results of numerical solution for rocket height.
	During Lab: Launch rockets.
Week 14	**Hand In:** Written Project
	During Lab: Presentations

Besides building the models in the first lab meeting, fundamentals of fluid drag are discussed, since most of the students have not taken fluid dynamics at this point in the curriculum.

The second week in the project sequence involves performing a drag test in the wind tunnel. The wind tunnel used for this experiment is equipped with a built-in force measurement system, allowing the user to read lift force, drag force and wind velocity via a computer interface. A custom-made cylindrical aluminum bushing was made by the lab technician, the outer dimensions of which are identical to those of a rocket motor. The bushing is drilled out to match the size of the wind tunnel bracket used for measuring lift and drag. This arrangement allows the rocket to be held securely in a horizontal position while performing the wind tunnel tests. The test was conducted by running the wind tunnel at ten even increments of speed up to 100 miles per hour. The students manually recorded the drag force at each point as displayed on the computer interface. Each group of students was asked to choose the unit system for the air velocity and force, since the wind tunnel allowed the capability to read wind velocity in meters per second or miles per hour. The drag force could also be recorded in either pounds or Newtons. The students then plotted the drag force vs. the air velocity, fitting the following parabolic curve through the data

$$F_d = kv^2 \tag{1}$$

where F_d is the drag force, v is the velocity and k is a constant of proportionality. By minimizing the error between the experimental data and the mathematical model described by Equation (1), the constant k is found using a software package of the students' choice, including Microsoft Excel, Matlab, EES or MathCad. Next the drag coefficient is calculated for the rocket by the equation

$$F_d = \frac{1}{2}\rho A c_D v^2 \tag{2}$$

In this equation, ρ is the air density, A is the cross-sectional area of the rocket and c_D is the dimensionless drag coefficient. The students measure the room temperature and barometric pressure and then calculate the density of the air using the ideal gas law. The cross sectional area of the rocket is obtained from physical measurements of the diameter of the body and the fin dimensions. The resulting drag coefficient is then used in the calculation phase of the project, incorporating the density of the outside air on the day of the launch. Since the launch is normally performed in December, the outside air on the day of the launch is normally more dense than during the wind tunnel test. In the past, smaller diameter rockets were used which generated less drag in the wind tunnel. The Estes "Baby Bertha" rockets used in the latest edition of the class created 50% more drag, making the effect of the measurement errors less significant.

While each of the four lab groups rotates through the wind tunnel testing experiment, the remainder of the students is involved in mounting strain gages on aluminum cantilever beams to be used in the rocket motor thrust testing phase of the experiment. Each aluminum beam is of square cross section with a thickness of 0.25 inches and is approximately 10 inches in length. Although each group builds only one rocket, each student mounts a strain gage to an aluminum beam. In this way, each student obtains first-hand experience with the details involved in strain gage mounting. This also affords additional redundancy to the supply of cantilever beams available for measuring rocket motor thrust. The techniques required for mounting strain gages can be quite exacting and it is not unusual to have the strain gage bonding fail in a student-applied strain gage. Another common problem is the overheating of strain gages during the soldering phase of the mounting procedure. Once each strain gage is mounted, the students connect their beams to the data acquisition system. The students then test their data acquisition programs and the adequacy of their gage mount by manually flexing the beam to ensure test data is being properly recorded in response to the beam deformation.

The third week of the project involves actually testing the rocket motor thrust. To accomplish this, a machined aluminum bracket to hold the rocket motor is fastened to one end of the beam with a number 8 screw, which passes through a hole drilled in the beam and the bracket. Figure 1 shows the beam with the strain gage mounted and the motor bracket connected to the end. The beam is then clamped to the table with a C-clamp. The rocket motor is placed into the motor bracket and an uncovered 55 gallon drum is placed below the rocket motor to minimize fire danger. The data acquisition program is set to run for 10 seconds, which is more than an adequate amount of time to capture the rocket motor burn phase. The data acquisition rate is set

Figure 1. Photograph of the underside of the ¼" beam with the mounted strain gage and the rocket motor bracket used for measuring engine thrust.

to 100 Hz. Figure 2 shows the rocket motor during the testing procedure. The data from this experiment provides the last piece of information needed for the students to calculate the predicted rocket height. Figure 3 shows a plot of the data from the rocket motor thrust experiment. A force of 10 N is obtained for a very short time, but the thrust is less than this for a majority of the motor burn period.

The fourth week of the project involves an actual launch of the rocket, after the students have submitted their numerical solution for the prediction of the maximum rocket height. The actual rocket height is measured as part of the launch exercise by recording the angle from the horizon to the rocket at maximum altitude. The students place an observer a known distance from the rocket launch pad, as measured with a distance-measuring wheel, and use a locking angle recording device to visually sight and record the angle of the rocket with respect to the horizon at maximum altitude. Using the measured angle and the known distance from the launch pad, the students use trigonometry to calculate the actual rocket altitude attained.

Figure 2. Rocket motor thrust test underway.

Comparing Measured Altitude to Predicted Altitude

As noted in Table 1, the students submit a free body diagram of the rocket in flight during the second week of the rocket project segment of the course. This free body diagram is quite simple with only three forces. One force is applied in the upward direction (thrust) and two forces are applied in the downward direction (weight and drag). The net imbalance in these forces determines the acceleration of the rocket. Equation (3) shows this basic equation.

$$\sum F_y = ma = T - \frac{1}{2}\rho A c_D v^2 - mg \qquad (3)$$

This equation is discretized by writing acceleration in a finite difference form, specifically

$$a = \frac{\Delta v}{\Delta t} = \frac{v_{i+1} - v_i}{\Delta t} \qquad (4)$$

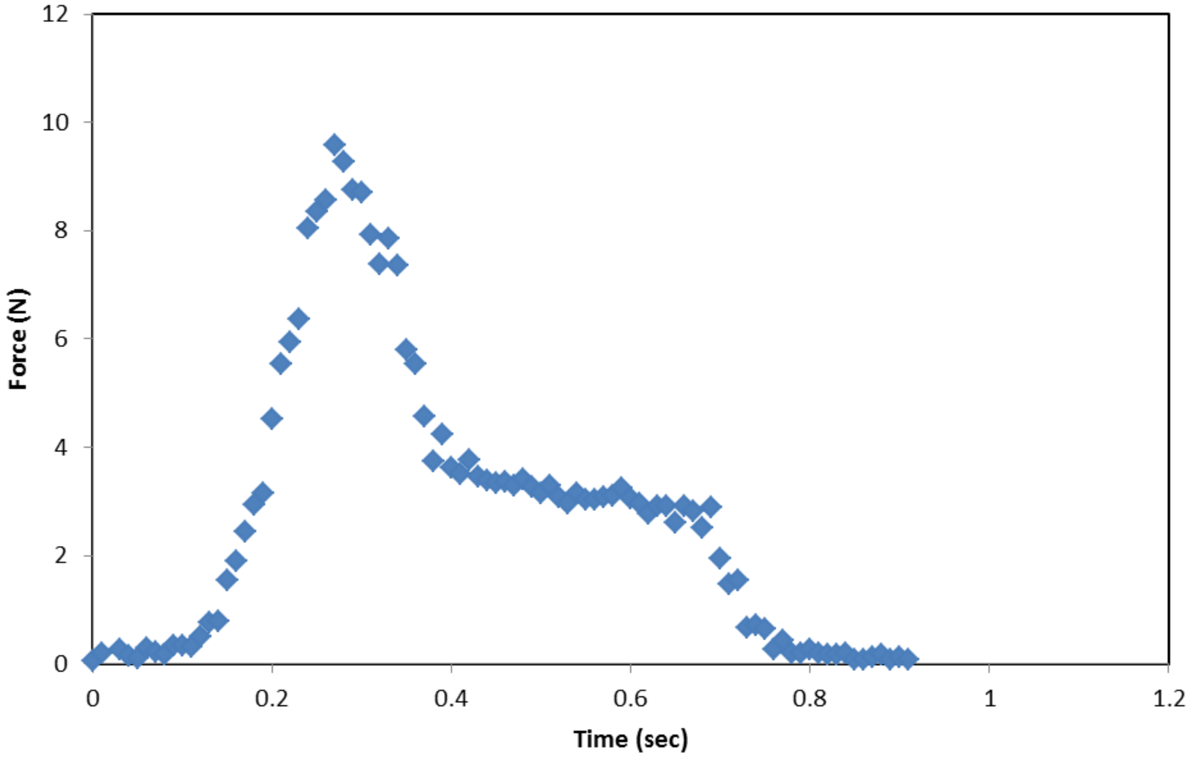

Figure 3. Rocket motor thrust as a function of time, recorded at 0.01 sec intervals.

Solving for the velocity at the end of a general time step, the discretized form of Equation (3) becomes

$$v_{i+1} = \Delta t\left(\frac{T_i}{m_i} - \frac{\rho A c_D v_i^2}{2m_i} - g\right) + v_i \tag{5}$$

Note the subscripts on the thrust, mass and velocity terms. Each of these variables is subscripted because they vary with respect to time. The thrust values (T_i) are recorded as shown in Figure 3. The mass values are computed proportionally to the amount of fuel burned. If the rocket motor impulse at any given time during the firing of the rocket motor is computed as

$$I_i = \sum T_i \Delta t \tag{6}$$

And the total impulse from the entire burn duration of the rocket motor is

$$I_n = \sum_{i=1}^{n} T_i \Delta t \tag{7}$$

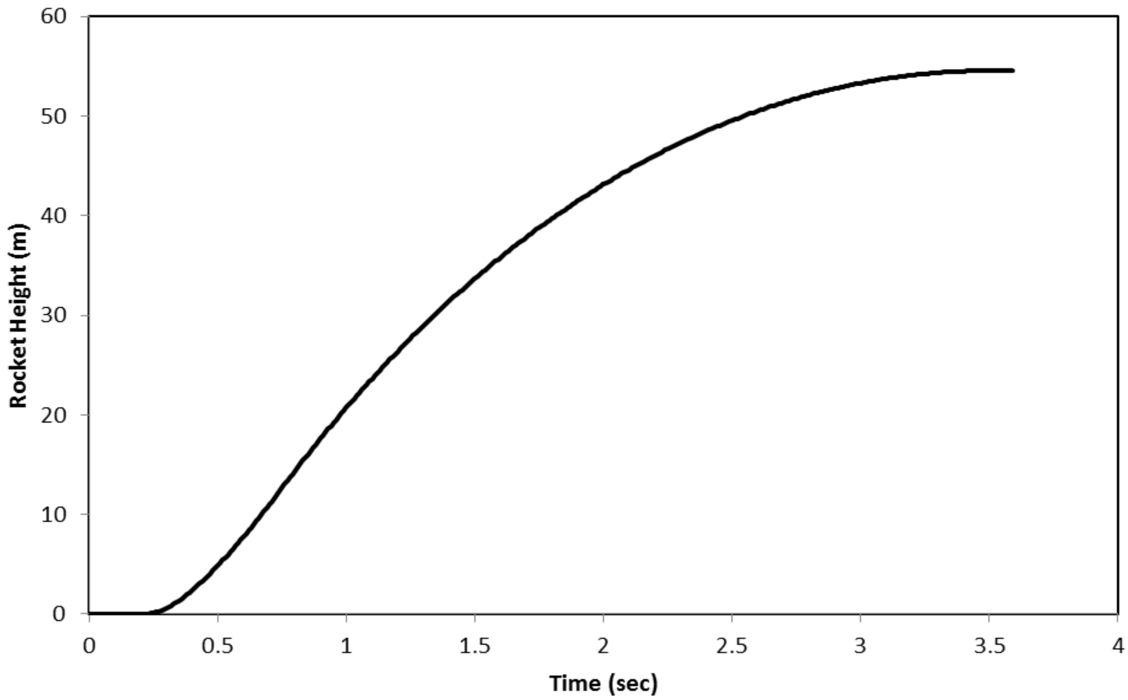

Figure 4. Calculated rocket height as a function of time.

Then the mass at any given time is

$$m_i = m_o - m_f \frac{I_i}{I_n} \tag{8}$$

where m_i is the mass of the rocket at any given time during the rocket motor burn, m_o is the initial mass of the rocket with the motor installed prior to launch, and m_f is the mass of the fuel obtained from the manufacturer. Combining this information, the velocity of the rocket as a function of time is computed using Equation (5). By numerically integrating the velocity, the height of the rocket can be computed as a function of time. The maximum height is found when the velocity goes to zero and begins to become negative as the rocket starts to fall. Figure 4 shows a plot of this computed height as a function of time. As can be seen in this figure, the final calculated rocket height is approximately 55 meters. There was some variability in this calculated value between students of ten percent or so due to laboratory measurement uncertainties. The calculation of uncertainty was a required component of the lab report for the rocket project and was calculated using the standard root sum error expression

$$\omega_R = \sqrt{\left(\frac{\partial R}{\partial a}\omega_a\right)^2 + \left(\frac{\partial R}{\partial b}\omega_b\right)^2 + \ldots + \left(\frac{\partial R}{\partial z}\omega_z\right)^2} \tag{9}$$

where R is the calculated rocket height. The variables a, b, and z are some of the measurements used in computing the rocket height, such as the beam dimensions, the strain gage factor and drag force measurement. The partial derivatives used in this expression were found by perturbing the measurements in the calculation scheme and recording the effect on the calculated height. Dividing the change in calculated rocket height by the amount of the corresponding parameter perturbation provides a numerical approximation of the derivative.

The students were given a choice as to which type of computational software they would use in computing the rocket height. Almost all of the students used Microsoft Excel with only one student out of 20 using EES [6]. The advantage of using EES is its automated package for computing the uncertainty of the calculated rocket height; Microsoft Excel does not provide an automatic feature for calculating this parameter. The actual measured rocket height was normally within 20 percent of the calculated rocket height for most of the students. The most common error was that the density of the actual outside air was not used in the calculation, which made the measured rocket height lower than the calculated height. This problem was exacerbated on two of the launch dates because of light rain. Due to the rigidity of the class schedule, postponement of the launch was not an option. It is suspected that the impact of raindrops made a large contribution toward suppressing the height attained by the rockets on those particular days.

Since the number of credit hours associated with the Instrumentation Lab class is only two, the only assessment included as part of the course is the grading of the lab reports and the pre-lab calculations. With no final examination applicable to the class, the assessment of student learning for the rocket project is somewhat limited to a subjective analysis of the project calculations and written reports submitted by the students. As noted previously, most of the laboratory techniques used in the rocket experiment are covered in prior experiments in the course. As such, the rocket project allows students to reinforce their knowledge of these laboratory techniques as part of a larger project using multiple experimental steps. More importantly, the motivation of the students for participating in the rocket project is noticeably higher than in the experiments involving simple calibration procedures for instruments. As such, the students have a stronger appetite for the experiments during the rocket project because they see it as important in accomplishing a larger overall goal. Therefore, it is believed that the reinforcement offered through the repetition of the experiments offered by the rocket project, in combination with the improved motivational level of the students, greatly enhances student learning in the Instrumentation Lab class.

Summary

The rocket project was extremely well received by the students. The experiments in the Instrumentation Lab course have historically consisted of calibrations of various instruments and student interest has been very low. With the addition of the rocket project to the course, evaluations completed by the students were very positive in response to the project. Eighteen student evaluations out of the 38 returned at the end of the course specifically mentioned the rocket project as being the component of the course that was the most enjoyable and the most intellectually stimulating. The rocket project will be retained and perhaps even expanded in future years in the Instrumentation Lab course.

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