NOVEL VISUAL ALGORITHM TO TEACH BENEFIT-COST RATIO ANALYSIS

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NOVEL VISUAL ALGORITHM TO TEACH BENEFIT-COST RATIO ANALYSIS TO FIRST-YEAR ENGINEERING STUDENTS

It is well known that the benefit-cost ratio (BCR) incremental analysis is the most used technique for economic analysis and decision making in the public sector. That is precisely one of the reasons why it is usually taught within engineering economic analysis courses at the undergraduate level. However, often times, freshmen and sophomore students find traditional incremental-analysis algorithms long and tedious, mostly when dealing with four or more alternatives. Moreover, even those who appear to grasp the steps more easily tend to show low levels of retention. Visual pedagogical techniques could facilitate the teaching/learning experience of BCR incremental analysis, while improving the level of retention over time. It has been established that visual approaches can help accomplish both of the aforementioned goals. With this intention in mind, we have derived a novel, simple, and purely-visual algorithm to select the best of many alternatives using the BCR analysis, with the advantage of avoiding the almost-purely-verbal route of traditional incremental analysis. In our approach, BCR values for all alternatives are plotted on a 2-dimensional, 3-axis diagram: two parallel and one perpendicular coordinates. The best alternative is selected by the maximum vertical or perpendicular distance from the points to the incline BCR=1. Results and analysis of experiments carried out to compare students' preference and performance using the traditional verbal approach versus our novel visual algorithm are presented. The proposed algorithm has been preferred by a cohort of engineering economic analysis freshmen students. Further experiments are currently being carried out to assess retention and ratify present results.

Introduction

The impetus of this study is at least two fold: (1) visual pedagogical materials are more effective among engineering students, and (2) the benefit-to-cost ratio analysis is widely accepted as the preferred method to evaluate and fund public sector projects. This introduction is meant to provide a brief context of both facts.

Visual Learning Preference among Engineering Students

It has been widely accepted that engineering students prefer visual methods to perceive and, then, better process information[1,2,3]. In fact, it has been reported that visual aids can improve learning by up to 400%.[4] Also, published literature reports that as much as 65% of the general population of the world are visual learners[5]. By visual, it is not only meant actual graphics, but also descriptions or analogies that can be easily pictured or imagined. According to Jonassen and Grabowski[6], visual learners prefer graphs, diagrams, or pictures added to text-based material. There are numerous instruments developed with the intension of measuring verbal-visual preferences. Particularly, on a highly-cited paper that included five dimensions of

learning/teaching styles, Felder and Silverman developed a self-scoring instrument called the Index of Learning Styles (ILS) consisting of 44 simple questions[7]. This instrument has been used by various researches to measure verbal-visual preferences among engineering students. A short collection of some of those results follow.

In a sophomore-level chemical engineering class of 143 students, it was found that 69% were visual and 30% verbal (1% none)[8]. From a cohort of 858 engineering students at the University of Ontario, 80% were found visual[9]. Eighty-five percent of the students in a particular engineering class at the University of Washington were reported to be visual, as determined also by ILS. A simpler instrument called the Verbal-Visual Learning Style Rating (VVLSR), which was developed and validated by Mayer and Masa[10], has also been used. (By validation, it is meant that it was compared with 13 other instruments and found to have a strong correlation with them. The authors concluded that "in some cases, a simple self-rating of spatial ability or of learning style can be an effective substitute for longer, more time-consuming instruments". By "simple self-rating" instrument, they were directly referring to the VVLSR.) . It consists of only one self-rated question that measure the learner's verbal-visual preference on a 7-point scale. Using the foregoing instrument, it was found that out of a cohort of 59 mechanical engineering students from two Midwestern universities only 6.8% preferred verbal learning versus 44.1% visual and 49.2% either.

Therefore, instructors could make the teaching-learning transaction more effective by designing new highly-visual teaching material (or converting existing one). Due to the nature of engineering sciences—i.e. its foundation on physics—this endeavor might not seem too difficult for the most part. However, for subjects such as *engineering economic analysis* (EEA) the development of visual material might be a little more challenging. Some important exceptions to the foregoing are decision trees, cash flow diagrams (CFD), graphs, etc. Particularly, CFD comes in handy for most of the EEA techniques due, in part, to the mathematical underlying nature of those techniques. (In fact, CFDs have inspired the development of a technique for money equivalence that makes use of the physical concept *moment of a force[11]*.) In contrast, the benefit-to-cost ratio (BCR) technique is mathematically different and therefore does not benefit sufficiently from the visual aid provided by CFD's.

Visual Learning and the Benefit-to-Cost Ratio Analysis

The motivation for teaching BCR analysis to engineering students is, in great part, due to its wide use in the public sector, and hence the colossal amount of money that is invested on project-alternatives decided through this method. Just to put it in perspective, the Federal Highway Administration (FHWA) fiscal year (FY) 2014 budget requested $41 billion "to improve the condition and performance of the Nation's highway and bridge infrastructure"[12]. Moreover, the current administration signed into law the Moving Ahead for Progress in the 21st Century Act (MAP-21)[13], which is translated into, most likely, stable funding for the growth and development of the country's transportation system. In fact, it was a congress bill, namely the Flood Control Act (FCA) of 1936, which gave birth to the current use of BCR analysis[14]. Its mathematical form might be based on a famous text portion from the FCA: "…the Federal Government should improve or participate in the improvement of navigable waters or their tributaries, including watersheds thereof, for flood-control purposes if *the benefits to*

whomsoever they may accrue are in excess of the estimated costs..." (Italics added). Hence, the figure-of-merit in the BCR analysis is the ratio of equivalent worth (either present, annual or future) of benefits to that of costs. A proposed project with BCR greater than unity indicates that it can be considered for investment, provided that financial resources are available. For the cases where a set of mutually exclusive alternatives are assumed, BCR incremental analysis is carried out.

Traditional Algorithm for BCR Incremental Analysis

Most textbooks are reasonably consistent with the manner they present BCR incremental analysis. Following is an algorithm to carry out incremental BCR analysis as taught in traditional textbooks:
1. Identify all relevant alternatives
2. Calculate the B/C ratio of each alternative
3. Rank-order the projects
4. Identify the increment under consideration
5. Calculate the B/C ratio for the incremental cash flows
6. Use the incremental B/C ratio to decide which alternative is better
7. Iterate step 4 until all increments have been considered
8. Select the best alternative from the set of mutually exclusive competing projects.

Of course, in textbooks those steps are explained in detail and examples are provided to illustrate the algorithm. In an attempt to present the steps in a more visual format, some other authors use a flowchart diagram approach[15], which appears to be a more visually-effective method than the purely-verbal series of steps shown above; however, it is still verbal. The following example demonstrate the use of the verbal-algorithm above:

Example: *The local government of Lynchburg, VA is studying alternatives to solve the issue of highly-frequent rear-end collisions at the intersection of Wards Road and Atlanta avenue. After a lengthy preliminary study, there are eight alternatives under present consideration. The present worth (PW) and Costs (C) associated with those alternatives are provided in Table 1. Alternatives are labeled A, B, C, D, E, F, G, and H, for simplicity. Using incremental BCR analysis, determine the best alternative to be recommended.*

Solution (Note: All cash flows presented from now on are to be understood as to be multiplied by 1000. So for example, in table 1, the cost of alternative A is to be understood as $4 million ($4,000 *1000), the PW Benefit of alternative H as $20 million (20,000 *1000), and so forth.):

Step 1) Since the potential alternatives are already provided, step 1 is already taken care of. Step 2) we calculate the BCR of each alternative, which is shown in Table 2; only those alternatives with BCR ≥ 1 are further considered. Therefore, since alternative G has a BCR=0.83 < 1 is immediately rejected. Step 3) Now, the alternatives left are ranked in increasing order of the costs. If the do-nothing alternative is under consideration, it should be the first on the list. For the case in question, the order should be D, B, A, C, E, F, and H, as shown in Table 3. Step 4) There is no need to compare the do-nothing alternative with alternative D (the first on the ordered list shown in Table 3) since D has a BCR > 1. In the incremental analysis to be carried out,

alternative D will be the first defender and B the first challenger; the analysis will progress depending on the winner at each comparison. Step 4) Table 4 shows the incremental $\Delta B/\Delta C$ ratios for each challenge. Note that the winner of each challenge is underlined and in bold font. Thus, alternative B is selected over alternative D; however, then, alternative A is selected over alternative B; alternative remains selected when compared against alternative C; then, E defeats A; in the following two iterations alternative E is selected over both F and H. Therefore, Alternative E is economically justified and selected among the eight mutually exclusive competing alternatives originally considered as the solution to be implemented. In other words, project E stands out as the one whose benefits to "whomsoever it accrues are in excess of the estimated costs".

	A	B	C	D	E	F	G	H
Cost ($)	4000	2000	6000	1000	8500	9000	7500	12000
PW Benefits ($)	7500	4200	9000	2500	17500	9000	6200	20000

Table 1. Costs and benefits for alternatives corresponding to problem example 1

	A	B	C	D	E	F	G	H
Cost ($)	4000	2000	6000	1000	8500	9000	7500	12000
PW Benefits ($)	7500	4200	9000	2500	17500	9000	6200	20000
B/C = (Benefits)/(Cost)	1.88	2.1	1.5	2.5	2.06	1	0.83	1.67

Table 2. BCR ratio calculated for each alternative. Notice that alternative F has a BCR=1. Since Alternative G has a BCR < 1 is immediately rejected.

	D	B	A	C	E	F	H
Cost ($)	1000	2000	4000	6000	8500	9000	12000
PW Benefits ($)	2500	4200	7500	9000	17500	9000	20000
B/C = (Benefits)/(Cost)	2.5	2.1	1.88	1.5	2.06	1	1.67

Table 3. Alternatives ranked in increasing order according their costs.

	B - D Increment	**A** - B Increment	C - **A** Increment	**E** - A Increment	F - **E** Increment	H - **E** Increment
Δ Cost ($)	1000	2000	2000	4500	500	3500
Δ Benefits ($)	1700	3300	1500	10000	-8500	2500
Δ B/C	1.7	1.65	0.75	2.22	-17	0.71

Table 4. Set of iterations of BCR incremental analysis for example 1. Note that the winner of each challenge is emphasized in bold font and underlined.

Novel Visual-Algorithm for BCR Incremental Analysis

We will now introduce our visual algorithm and compare it to the traditional verbal format presented in the previous section. For the sake of consistency, the same example solved before will be used here as a way of comparison. Our entire algorithm is presented in Fig. 1. In short, the best alternative is the one whose vertical distance to the incline is largest. In this case, the largest distance correspond to the point labeled as "E", which refers to alternative E. We will now explain this in detail and guide through the simple construction of this visual algorithm.
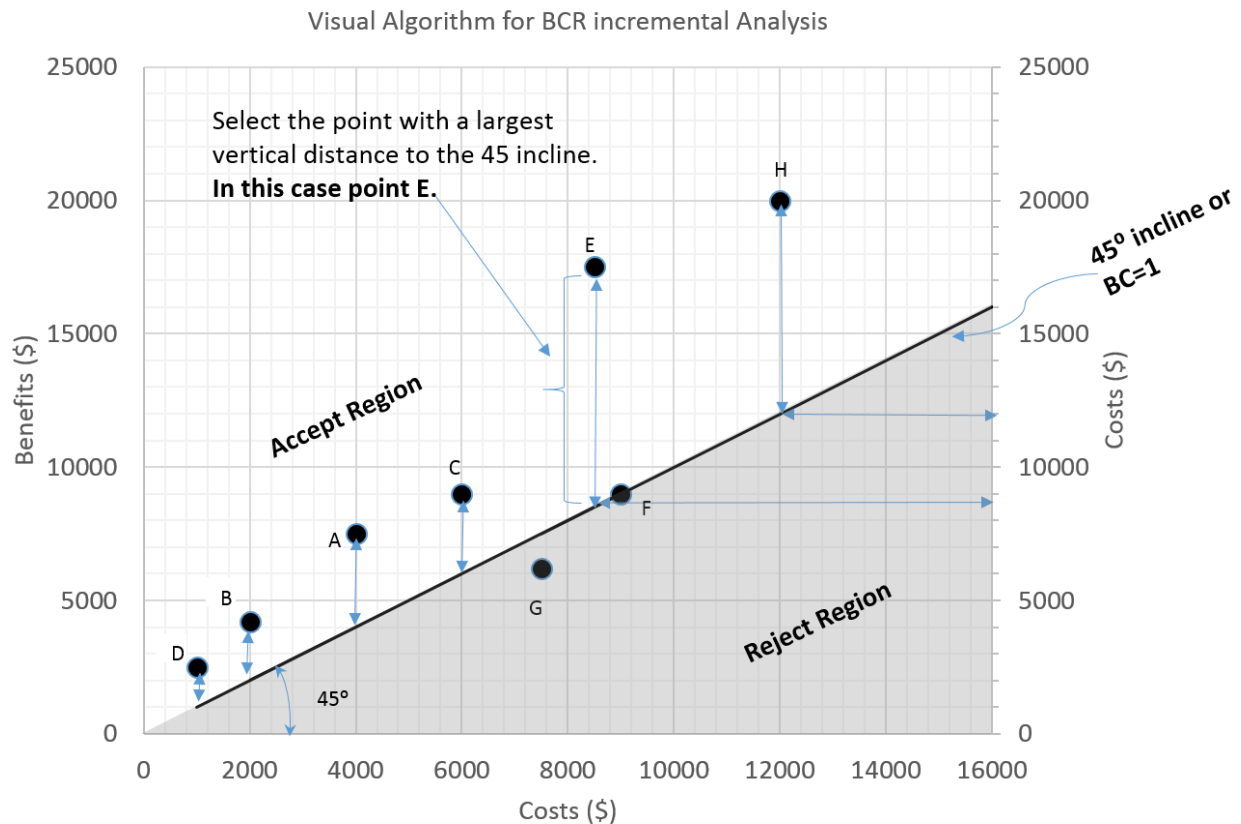

Figure 1. Entire novel visual algorithm to perform incremental BCR analysis applied to example 1 with eight mutually exclusive alternatives, labeled as points A, B, C, D, E, F, G, and H. The best alternative is selected based on the largest vertical distance (LVD) from the point to the 45° incline. In this case, point "E" has the LVD and hence alternative E is justified as best.

Consider again Fig. 1, which shows an edited 3-axis plot of benefits and costs for the eight alternatives considered in example 1. In general, the number of alternatives can be as high as they can be in practice. As a matter of fact, it will become evident that our visual algorithm gains even more advantages as the number of alternatives increases. As mentioned before, the plot has 3 axes: two vertical axes and one horizontal axis. (In reality, the third axis is intended more for clarity and can in fact be omitted.) While the left vertical axis corresponds to the benefits, both

the right vertical and horizontal axes are assigned to the costs. A simple construction of this visual follows:

1. **Plot the benefits versus costs** for each alternative and label them, as shown in Fig. 2A

2. **Draw a 45° incline from the x-axis**. This line corresponds to BCR=1. Label the region below the 45° degree incline as "reject region". Any alternative below the "reject region" is considered rejected immediately. Likewise, label the region above the incline as "accept region. Only those alternatives above this line will have further consideration (See Fig. 2B).
3. **Add an extra vertical axis** to the right of the plot corresponding to the costs. See Fig. 2C.
4. **Measure the vertical distance from each** point (or alternative) to the incline. The best alternative corresponds to the largest distance associated. In our particular example, this corresponds to alternative E.
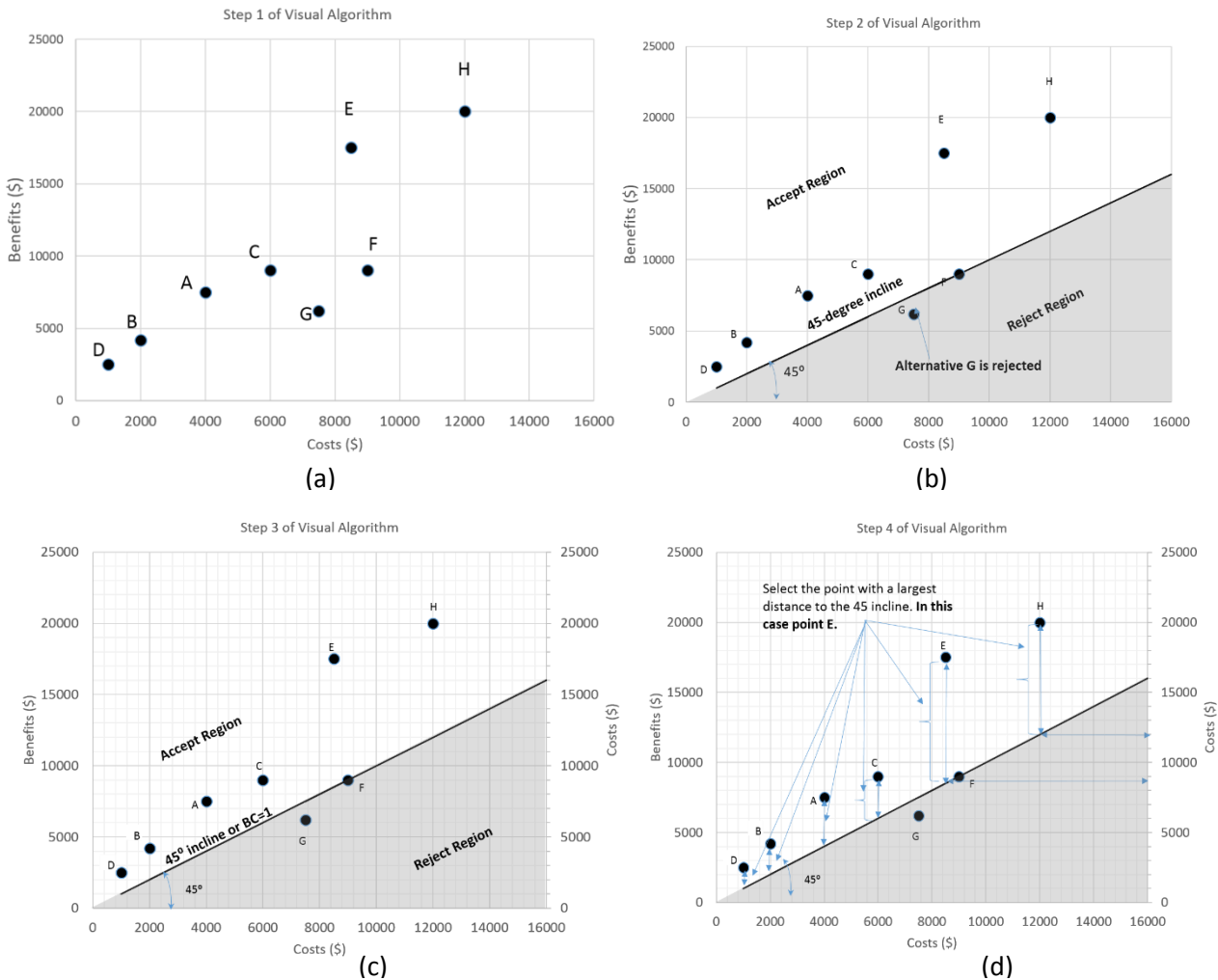

Figure 2. Four steps to build the visual algorithm shown in Fig. 1. (a) Plot the benefits versus costs for each alternative and label them; (b) Draw a 45⁰ incline from the x-axis and demark the accept and reject regions; (c) add an extra (cost) vertical axis on the right; (d) Measure the vertical distances from each point to the incline. The largest vertical distance correspond to the best alternative.

Application of Method to a Cohort of Freshmen Students

The verbal benefit-to-cost ratio incremental analysis was introduced to a group of 20 freshmen engineering students, during a 50-minute lecture. In the following class period, the novel visual algorithm was introduced; however, while the verbal approach used up the entire 50-minute lecture, the visual was only presented for about 20 minutes. A survey was performed to assess the preference of the students and the results are shown in table 5. There were only two questions in the survey: "In class we learned two methods to carry out benefit-to-cost ratio incremental analysis, (1) state which method you prefer; and (2) explain why you selected that method.

A total of 18 students responded the survey, half of which preferred unconditionally the visual method. Also, five students unconditionally preferred the verbal approach. Additionally, four students stated that for problems containing three or more alternatives they would definitely use the visual approach, otherwise (for problems with two alternatives) the verbal seemed more reasonable.

Preference	Purely Visual	Purely Verbal	Conditional	
Number	9	5	4	
Percent	50%	28%	22%	
Rationale	Easier; quicker; more representative of overall picture; more understandable; student is visual; more effective; easier for long problems.	More accurate; student is sequential; not a fan of graphs	If problem is long (3 or more alternatives) students prefer the visual method; If problem is short (2 or 3 alternatives) students prefer verbal method.	

Table 5. Results of survey applied on a cohort of 20 first-year engineering students on the preference between the verbal versus our novel visual algorithm to teach/learn BCR incremental analysis. A total of 18 students responded the survey. The "rationale" row contains a summary of the responses to the question: "Explain why you selected that method".

In order to corroborate the above results, a three-question quiz was applied to the aforementioned cohort. The first question explicitly asked the students to solve an eight-alternative problem using the verbal approach; the second question, asked them to solve the first question using the visual approach; and the third question gave the students the option to freely select which method to use in order to solve a seven-alternative problem. Sufficient time was allocated for this assessment in order to not constrain students to resort to the "fastest approach".

The results show that there was no statistically difference between the level of performance using one approach or the other. In fact, the majority of the students developed an ability to carry out

incremental BCR analysis using both techniques. However, near 75% of the students preferred to freely use the novel visual algorithm in question three of the quiz.

Discussions

On the simplicity of the algorithm

At its root, the visual algorithm presented is grounded on the fact that the best alternative can be selected based on the following simple numerical algorithm:

$Best\ Alternative\ \stackrel{m}{=} Max[C_i(BCR_i - 1)]$ For all alternatives i=1…N   (1)
where N is the number of mutually exclusive alternatives; C_i is the cost of alternative i; and, BCR_i is the benefit-to-cost ratio of alternative i. The symbol $\stackrel{m}{=}$ means "measured by".

So Equation (1) means that the best alternative is measured by maximizing the quantity in square brackets. Notice that Eq. 1 takes care of alternatives whose initial BCR are less than 1, since in those cases the quantity in brackets becomes negative.

An implication of Eq. (1) is that in reality the BCR incremental analysis can be reduced to a regular non-incremental analysis. The fact that mutually alternatives can be compared as a group, instead of incrementally verifies the foregoing. Another important fundamental observation is that Eq. (1) is simply a maximization of the profits, which is after all the goal of all engineering economic techniques.

On the Visualness of the Algorithm

As shown in Fig. 1, our proposed algorithm can be summarized in a single graphic. Likewise, the steps to construct it can be represented through graphics (see Fig 2). Furthermore, the tools needed to explain each step consist of simple geometrical and algebraic concepts. Additionally, a monochromatic representation of the algorithm could be easily used without incurring in confusion—although it might be useful to use various colors at least the first time it is introduced.

On the Performance and Preference of Students

The results of the preliminary experiments carried out—since more are currently being performed—show that freshmen students perform similarly well using either method. However, as the length (number of alternatives) of the problem increases, students tend to prefer the easier and quicker visual algorithm. Perhaps a perfect analogy to explain this was that provided in the survey by one of the students: *"…One could compare these two methods to monetary investments: the step-by-step computation approach* (a.k.a. verbal method) *has a lower initial cost, but a higher marginal cost per alternative. On the other hand, the graphical approach* (a.k.a. visual method) *has a relatively higher initial cost, but a lower marginal cost per alternative."*

Summary and conclusions

We have developed and proposed a novel, simple, visual algorithm to perform benefit-to-cost ratio incremental analysis. The simplicity of our algorithm stems from the fact that its root BCR analysis seeks to maximize profits. Mathematically, this translate into simplifying the problem to a regular analysis where mutually exclusive alternatives are analyzed in any direction rather than just incrementally.

Our visual algorithm has been preferred by our engineering students at Liberty University. Some experiments have been carried out with regards to comparing student's preference and performance after being taught using the traditional verbal algorithm versus our novel visual algorithm. The results show our first-engineering students can perform as well using either method but by find our visual algorithm easier, quicker, and easier to understand, and prefer to use it when dealing with problems containing three or more alternatives. Further experiments are being carried out in order to assess retention and ratify the present results.

We recommend this algorithm to be further explored and implemented in other programs across the nation and the world. This will allow to have a larger cohort to compare results of acceptance as well as performance.

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