

**AC 2008-2720: OFFERING HONORS COURSE OPTION WITHIN AN ORDINARY MATHEMATICAL COURSE FOR UNDERGRADUATE STUDENTS IN ENGINEERING MAJORS**

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# Offering Honors Course Option within an Ordinary Mathematics Course for Undergraduate students in Engineering Majors

## Abstract

In order to attract talented students, many selective small universities have established honors programs in recent years<sup>10,13</sup>. However, because it is difficult for those small universities to schedule enough honors program students to fill all the seats of a regular-sized class, it is often too costly to offer sufficient honors courses for students in an honors program alone. A cost effective solution to this problem is to allow students who are not in the honors program to take any seats that may remain after the honors students have registered. An alternative is to offer an honors course option in a regular non-honors course by asking the honors program students who take that course to learn some extra content and do some extra work. With either solution the act of mixing a significant number of honors students with non-honors students usually results in a bi-modal distribution based on the students' level of academic preparation. To teach such a class, instructors often face more challenges balancing the content so that the strong honors students can be adequately motivated without intimidating the other relatively weaker students. This paper reports on the first year's experience offering an honors course option within an ordinary mathematics course that had a mix of both honors and non-honors engineering students.

## 1. Introduction

Typical calculus or differential equation courses teach students to work on well-defined and oversimplified problems. Average college students believe that the solution to all problems simply implies finding the right formulas and plugging data into those formulas. Consequently, the learning of mathematics comes down to remembering formulas. Given application problems where the solutions are not based on formula association, most students do not know how to start their work. With the increasing complexity of postmodern technology, bridging the gap between real-world problems and problems in textbooks becomes an increasingly critical pedagogical issue. Berkey and Vernescu<sup>1</sup> presented an extensive survey about the curriculum reform effort of project-oriented education in 30 years. Many articles, projects, and books addressed the issue, but no magic remedy existed to solve the problem<sup>9,2,7,3</sup>. In 2003-2004, the author participated in three summer workshops of the National Computational Science Institute<sup>8</sup> (NCSI) on computational science education and appreciated the pedagogical value of teaching an application oriented mathematics module on mathematical modeling. Such an approach can help students understand that mathematics is a language for accurately describing complex natural and artificial phenomena in science and engineering.

While teaching courses in ordinary differential equations over the past three years to engineering students, the author introduced modifications to these courses by adding a Mathematical Modeling Module (MMM) of 8 hours. The feedback from student participants was mixed and bipolarized. The reason behind the disparate feedback became clear after a couple of years. The ability to reach the appropriate level of abstract thinking was critical for correctly modeling complicated problems<sup>11,4</sup>. The author observed that there were two main groups of students,

those who could handle the required level of abstraction and those who could not. Hence, the author proposed to test this module on a group of talented students in the honors program at the author's affiliated university. The proposal to teach the mathematics module to honors students got the support of the director of the honors program. This paper reports the first year's experience offering an honors course option to honors program students in engineering majors (MA345HON) within an ordinary math course (MA345) that was taken by a majority of other students. The major difference between the ordinary course MA345 and the honors course MA345HON was that the honors program students who took the honors course option needed to learn the mandatory MMM and complete a final team project. The major goal of the honors options was to teach students how to conceptualize problems, build mathematics equations from the conceptualized invariants, solve them numerically and/or analytically, and finally validate and verify their solutions.

The layout of the rest of this paper is as follows: Section two presents the course objectives and discusses their pedagogical rationale. Section three describes the content of the MMM, tool support, tutorial, and team project requirements as well as addressing some implemental issues. Section four summarizes formative assessments, student evaluations, and opportunities to improve the honors course option. Future work is presented in the conclusion of this report.

## **2. Objectives and Pedagogical Rationale**

Mathematical modeling bridges the gap between mathematical theories and applications. Modeling methodology and process are systematic approaches to conquering complexity. The author believes that the current mathematics curricula of most engineering programs have not given adequate emphasis on mathematical modeling<sup>9,2,8,7,6</sup>. The increasing gap between problems in the real world and problems in textbooks leads many engineers in the technology industry into believing that they can rarely use the college mathematics in their work places. The negative opinion of practicing engineers in turn misguides college students. It is clear that many colleges have not adequately taught students how to formulate applications and model their problems by using the mathematics as a scientific language. Many problems in the real world are too complicated to be described by existing models and solved by existing formulas. It is desirable to introduce some basic modeling methodology to college students so that they know how to divide large problems into small problems solvable by their college mathematics.

NCSI is one of the leading organizations that advocate the reform of mathematics and computing education with more emphasis on application oriented modules and mathematical modeling<sup>8,6</sup>. After participating in three NSCI workshops in computational science education and experimenting with teaching application-oriented modules in existing courses for three years, the author is convinced that one of the best practices for motivating engineering students to learn mathematics is to expose them to many mathematical applications related to their engineering work. While a systematic education reform takes too long to effectuate, a pragmatic approach for motivating engineering students to learn mathematics is to insert a small dose of application oriented modules into a traditional course.

The performance objectives of MMM and the team projects are as follows:

1. Knowledge of using modeling methodology and processes to divide and conquer complicated problems
2. Skills to use mathematical modeling tools to model an application incrementally
3. Techniques to use graphics tools to present information intuitively
4. Capability to combine mathematical analysis with numerical solutions to gain insight and justify answers to a problem
5. Framework to write a mathematical paper
6. Experience of working in teams and learning from each other
7. Understanding that mathematics is the scientific language to accurately express complex natural and artificial phenomena

### 3. Content, Tool Support and Project Requirements

The MMM is originally a module of 8 class hours designed to insert into MA345, a course of 4 credit hours entitled Ordinary Differential Equations and Matrix Methods. Students of MA345 are typically sophomores or juniors in engineering majors who have taken 12 credit hours of the calculus series. By that time, almost all of them are familiar with at least one programming language (C or Java) and know how to use mathematical software such as Maple or MATLAB. The prerequisite of MMM is the same as that of MA345. The author has offered the MMM to students in MA345 for three years in a row. Since fall 2006, the author started to offer the MMM (10 hours) as an honors course option (MA345HON) mainly to students in the honors program. There is not much change in the content, but there are different implementation strategies and different expectations. Five hours of the content in MMM, such as modeling methodology and process together with case studies are still taught to all students. But two hours of tutorial on Stella<sup>12</sup> and one hour on project requirements are only taught to students who register for the MA345HON in a computer lab. All students taking the MMM are required to complete a team project and attend a final presentation of 2 hours. After finishing the project, students need to submit the project report and related programs, and give an oral presentation of 20 minutes. Another difference between MMM for the previous MA345 and MMM for MA345HON is the different expectations and different difficulty levels of the projects. The author used to give three application problems of different levels of challenge to students of MA345 before. Students in MA345HON in fall 2006 only got the most challenging ones that the author used to give. The content of the MMM are summarized in Table 1.

Topics	Hours	Offering time	Notes	Asso. Objectives
Errors vs. mistakes, model validations vs. verifications.	1	The third week	All	1.
Modeling Process and Methodology	2	The third week	All	1. 7.
Project requirements and assignment	1	The ninth week	Honors	5, 6
Stella tutorial	2	The 10 <sup>th</sup> -11 <sup>th</sup> week	Honors	2, 3
Modeling systems and higher order equations	2	The 11 <sup>th</sup> week	All	1, 4. 7.
Final Presentation	2	End of the course	Honors	All objectives

Table 1

In the first lecture on MMM, the deviations between models and application problems, and the errors of numerical solutions are introduced. Students learn the basic concepts of validation and verification. Validation checks whether the model reflects the essence of the application problem and verification checks whether the solution is correct. The terms error and mistake mean differently in the validation and verification lectures. Errors can be controlled, but mistakes can be beyond human control and are sometimes catastrophic when some critical factors have been neglected in the assumption of the modeling. Students learn that the validity of a model is based on sound modeling assumptions and the correctness of a numerical solution is based on proper error control. During the next two hours, we use case studies to learn compartmental analysis and incremental and iterative processes in mathematical modeling. Stella (created by High Performance System Inc) is a software tool for mathematical modeling and simulation. It uses the simplest description of fluid flows to build dynamic models and simulate the numerical solutions. There are two hours of tutorial for students of MA345HON. However, students learn mostly by themselves by running 5 to 10 tutorial examples. The next two hours on MMM, all students learn the strategies about how to model systems and high order equations from case studies. In these MMM modules, applications include fluid mixing problems, population models, logistic models, Newtonian mechanics models, electrical circuits, coupled spring mass systems, and many nonlinear systems such as predator-prey models and disease spread models. Most linear models can be found in the textbook and are taught in regular classes. But the difference is that systematic modeling methodology and tool support were introduced to make much more complicated nonlinear problems accessible to and comprehensible by undergraduates. Since all students learned the modeling methodology and processes, they were tested on how to formulate application problems and build corresponding differential equations.

There were a total of sixteen students in the honors program. Ten of them chose the honors course option MA345HON. When mixing the honors course with an ordinary course, there was an issue of a fair grading system. It is common in many universities to have a senior undergraduate course and a graduate course offered together. Instructors typically give graduate students an extra project or some extra problems on exams, or both. As an ad hoc solution, the author used the same strategy to grade the students in the honors course option as well as the regular students. While the students in MA345 had 40% of their score for the final exam, the students in MA345HON had both a team project and a final exam, with each counting for 20%. Another difference was that many bonus problems were optional for students in MA345, but mandatory for students in MA345HON.

The team projects were due about 5 weeks after assigned. A team was comprised of three or four students. Ten honors students for MA345HON made three teams. There were two more teams of non-honors students who were talented in mathematics and volunteered to take the project option. Hence, there were five teams in fall 2006 from two sections of the differential equations courses that the author taught. The evaluation of the projects was based on the objectives of the MMM and the project requirements. They were itemized as follows with each item worth 20%. The items were:

1. Oral presentation with Power Point slides,
2. Written project report,

3. Models in Stella and/or Maple as well as the validation of models,
4. Analysis and verification to the solutions,
5. Intellectual merits and special research efforts.

In addition to the associated objectives, the quality of every item was based on the 3-C principle: Correctness, Completeness, and Conciseness. On average, each team member was expected to spend about 24 hours (6 hours per week for 4 weeks) working on the project. Intellectual merits included good ideas for verification such as using available analytic solutions in special cases to check numerical solutions, or using the solutions of phase plane equations to verify their numerical solutions to a two variable system of equations. Special research efforts included relaxing unrealistic model assumptions, generalizing formulas for more realistic applications, finding data from real-world applications to support their results, etc.

#### **4. Formative Assessment and Opportunity for Improvement**

To evaluate the teaching materials and the course delivery, we will follow the three principles suggested by the NRC for mathematics assessment<sup>5</sup>:

1. Content Assessment reflects what is most important for students to learn
2. Learning Assessment enhances learning and supports instructional practice
3. Equity Assessment supports every student's opportunity to learn

The relative importance of the MMM and content assessment has been addressed in section two and it is supported by numerous references<sup>9,2,8,6</sup>. The learning assessment consists of two components - the quality of the projects and the feedback from the students who took MA345HON. Since the author has taught MMM for more than three years, similar project problems have been assigned many times. When comparing the projects completed by students in MA345HON with similar projects done in the previous semesters, the author found that the overall quality of the former was significantly better. The assessment from the feedback of participating students was summarized in a table. Each question in the table assessed one of the objectives set for the MMM and projects. Students were asked for their opinions about whether those objectives have been achieved, and if so, to what degree.

17 students worked on 5 projects and 14 students participated in the presentation and submitted the survey (see Table 2). Except for the fifth objective, the survey indicates that all other six objectives have been achieved. The answers to the last question indicate that almost all students enjoyed the projects. But, about half of students answered, "Maybe yes". One student commented: "good project, but the time and complexity proved overwhelming". This comment may well explain why half of students enjoyed it with some reservations. Comparing the mixed and bi-polarized feedback and evaluations from the MA345 course that the author offered in previous years, this survey clearly demonstrates that offering such a challenging level of MMM to the honors students was the right choice for both instructor and students. An ongoing team project is to make the approach effective to groups of students with a wider range of aptitude in abstract thinking.

The degree of certainty that you feel the particular objective has been achieved.	Yes, Sure	Yes, in some degree	I am not sure	Surely have not
1. Knowledge of using modeling methodology and processes to divide and conquer some complicated problems	10	4	0	0
2. Skills to use mathematical modeling tools to model an application incrementally	10	4	0	0
3. Techniques to use graphical tools to present information initiatively	10	4	0	0
4. Capability to combine mathematical analysis to numerical solutions to gain insight and justify answers to a problem	10	4	0	0
5. Framework to write a mathematical paper	4	6	3	1
6. Experience to work in teams and learn from each other	10	3	0	1
7. Understanding that mathematics is the scientific language to accurately express complicated natural and artificial phenomena	9	4	0	0
Over all, comparing the time that you spent on the module/project and the knowledge and skills that you gained from them, if you have the choice again, do you still choose the project option?	6	7	1	0

Table 2

As a follow-up to the MMM and project of previous years, the author selected a couple of teams who did the best projects to give presentations in some undergraduate mathematics conferences. Several students of MMM actually took an independent study course under the mentorship of the author. One major effort was to help students to generalize their knowledge in mathematics to multi-variable and vector space settings so that they were ready to apply their mathematics to engineering problems. One interesting consequence of having students take the honors option is that those students make ideal candidates to take independent study courses and do research under the mentorship of the affiliated instructor. After grading the project reports of those students, the instructor knows their aptitudes and attitudes quite well and, therefore, is able to more effectively direct the selected students to do research work.

There exist several problems with the current implementation of the MMM. Firstly, instructors need to renew and/or modify the problems for projects each semester so as to eliminate potential plagiarism. Problem generation or modification is quite a burden for an instructor to do alone. Secondly, students in the honors program have to fight two battles to succeed in this course. One is a final exam and the other is the final project. The author wonders whether the instructor should waive the final for those who take MA345HON and give the final project 40% weight. Thirdly, some negative psychological effects seemed to be noticeable in the non-honors students when they were surrounded by other students who had the privilege of learning more. This problem does not seem to exist when graduate and undergraduate seniors are mixed in the same class and taught differently.

## 5. Conclusion and Future Work

There are many quality educational modules in computational science posted on open source websites<sup>8,6</sup>. Instructors may freely customize and adapt these modules to serve their targeted students. Recently, the author teamed up with three other colleagues with interdisciplinary expertise from the fields of mathematics, computer science, and teacher education. Our goal is to adapt and develop some easy-to-share computational science modules for enhancing the conventional classroom education and promoting problem solving experiences of undergraduate students. We are working on a preliminary framework concerning how to classify and package the modules into module sets according to their content relationships and difficulty levels. Easy-to-share modules<sup>8,6,14</sup> from other colleagues can help instructors obtain many alternative projects in the future thereby solving the first problem mentioned above. To avoid the negative psychological effect to other students in the same class, instructors may consider moving the extra course content for the students in the honors option out of the regular class. As an advisor to a newly established SIAM student chapter, the author would like to use the every second week's evening activities to serve both the SIAM student chapter and the students in the honors option. This would give the author the flexibility of moving the content that is required only for honors program students out of the regular class hours. Because the membership of the SIAM student chapter is open to all committed students, the other non-honors students are free to join in if they really want learn more about mathematical modeling.

This paper reports the first year's experience of offering an honors course option to the honors program students in engineering majors within an ordinary mathematics course that was also taken by a majority of other students. The paper also presents the objectives of the honors course option along with its pedagogical rationale and addresses some implementation problems as well as related future work. The director of our honors program found that mandatory honors course loads vary from 18 to 36 credit hours in different honors programs across the country. Our school offers 23 credit hours of honors courses to students in engineering majors. This honors course load is at the low end. The honors program committee agreed to increase the honors course load in mathematics by 3-4 credits. Nevertheless, like many small universities, we have difficulty offering or scheduling sufficient courses for students in the honors programs alone. This paper has shown that it is possible to provide a cost effective alternative for students in honors programs to earn sufficient honors credit hours. Most honors designated courses in our school are upper-level major courses for juniors and seniors. With MMM, honors course options can be flexibly implemented at the freshman and sophomore levels. As a result of such offerings talented freshmen and sophomores could be identified as possible candidates for early research opportunities. Offering honors course options and providing research opportunities to freshmen and sophomores in honors programs can help universities reduce the attrition rate of those talented students.

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