ASEE's 123rd Annual • Conference & Exposition • New Orleans, LA • June 26-29, 2016

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Paper ID #16911

# Pedagogic Mediation of Dynamic Geometry in Teachers' Mathematical Activities

**Muteb M. Alqahtani, Rutgers University**

I am a doctoral candidate in mathematics education in the Ph.D. program at the Graduate School of Education, Rutgers University-New Brunswick, and I teach in the Department of Urban Education at Rutgers University-Newark.

**Dr. Arthur Belford Powell, Rutgers University**

# Pedagogic Mediation of Dynamic Geometry in Teachers' Mathematical Activities*

Muteb M. Alqahtani
*Rutgers University*
*muteb.alqahtani@gse.rutgers.edu*

Arthur B. Powell
*Rutgers University*
*powellab@andromeda.rutgers.edu*

*Dynamic geometry environments can support learning of geometry through meditating learners' activity. To understand how dynamic geometry environment mediate the activity of mathematics teachers, we used Rabardel's categories of instrument mediations in an instrument-mediated activity [1, 2]. We analyzed the discursive and inscriptive interactions of 4 mathematics teachers who worked for 15 weeks in a team to construct geometric figures and solve open-ended geometrical problems in a collaborative, dynamic geometry environment. In addition to Rabardel's epistemic and pragmatic mediations, we found and coined a third mediation–pedagogic mediation–by which teachers use the environment to help other team members understand particular geometric objects and relations among them.*

**Keywords:** Dynamic geometry, Instrumental genesis, Mediated activities, Pedagogic mediation

Technologies developed for teaching and learning mathematics are powerful and potentially useful. One significant factor of successful integration of technology in mathematics classrooms is understanding how technology influences teachers and students' social interactions and shapes their mathematical knowledge. Vygotsky [3] emphasized the role of tools and signs for cognitive development. He argued that intellectual development occurs through engagement in tool-mediated activities that allow for social interactions. Several studies use Vygotsky's notion of mediation to explain learners' interactions with technological tools in mathematical activities [for example, 4, 5-7]. Technological tools mediate learners' activity and provide additional tools and signs that can support students' mathematical discourse and building of meaning. Helping students construct mathematical meaning while interacting with mathematical tools requires teachers to carefully plan and implement how their students engage in mathematical activities. However, for mathematics teachers' to use technological tools in their classrooms effectively, they need to learn how to use the tools. This creates a need for investigating how teachers learn to use mathematical tools and the mediations of these tools in teachers' activities.

Rabardel and Beguin [1] theorized how technological tools mediate learners' activity. They describe two mediation roles of tools in users' activity: epistemic (focuses on understanding the properties of the object) and pragmatic (concerned with transferring the object to a desirable form). In dynamic geometry environments (DGEs), the mediations of learners' mathematical activities can take different forms. This type of environments allows users to create geometric objects (points, lines, line segments, circles, etc.) with certain relations between them.

* This paper is based upon work supported by the National Science Foundation, DRK-12 program, under award DRL-1118888. The findings and opinions reported are those of the authors and do not necessarily reflect the views of the funding agency.

For example, a line segment can be used as a radius to create a circle. These objects can be dragged around the screen, which allows the users to observe the consequences of their dragging to understand the relations among the different objects. Users act on geometric objects and DGEs react to their actions in a manner that corresponds to engineered infrastructure that responds to the theory of geometry [8, 9]. This co-active relationship between the environment and users allows users to monitor and reflect on their activity. In an instrument-mediated activity in DGE, the environment provides instantaneous feedback to users' actions. This feedback can inform the users' actions and shape their thinking. In this paper, we report on a third mediation role that mathematics teachers used while working on geometrical tasks in a collaborative dynamic geometry environment. We analyzed the interactions of four middle and high school teachers and identified the third mediation role of the environment.

The four mathematics teachers (three middle school teachers and one high school teacher) participated in a 15-week professional development course. They interacted in an online, collaborative, dynamic geometry environment called Virtual Math Teams with GeoGebra (VMTwG), which allows for synchronous interactions among users through a multiuser interface of GeoGebra and a chat feature (see Figure 1). The environment is available publicly at <http://vmt.mathforum.org> for any teacher or educator to use. GeoGebra is also a free dynamic mathematics software that can be used in many different platforms (see http://www.geogebra.org). VMTwG records users' actions in GeoGebra as well as their chat massages. The teachers met synchronously in VMTwG for two hours every week to work on open-ended geometrical problems that were designed to engage teachers in productive mathematical discourse about geometric ideas [10, 11]. We analyzed teachers' weekly sessions (14 sessions) by reading their discussions and viewing their GeoGebra actions to understand how the environment mediated their activities. The produced data included about 2000 pages of chat postings and descriptive interactions in VMTwG. The 14 sessions can be viewed using VMT replayer, which shows teachers' chat postings and GeoGebra actions at the same time. Our analysis of their interactions in these 14 sessions allowed us to categorize how VMTwG mediated teachers' activities as they discussed and solved geometrical tasks. For each session, we examined all team members' chat postings and highlighted the mathematical moments of the session. Here to code these discursive mathematical moments, we used Gattegno's [12] dialogic notion of what constitutes mathematics: teachers discussing ideas about geometric objects, relations among objects, and relations among relations. After highlighting those discursive moments, we examined how teachers used the environment in each of those moments and for what purpose. Using Rabardel and Beguin's [1] mediation roles, we categorized teachers' use of VMTwG into epistemic and pragmatic use as well as pedogogic.

We were able to identify the pedagogic mediation, in which teachers concerned themselves with how to teach with this environment. They used it to teach each other and convey their ideas in a didactic manner as well as to discuss how VMTwG could be used to teach certain school topics. In these pedagogic meditative moments, the environment is used as a mediator not particularly to make transformations on the objects or explore objects' properties and relations, but to help others understand these transformations, properties, and relations. We chose the following example to show how VMTwG mediated pedagogically teachers' mathematical activity.

## Example: Task 5.01

After working individually on different visualizations of Thales and Pythagoras' theorems, in Task 5.01, the teachers were encouraged to discuss their resulting insights and wonderings about the theorems. In Task 4.01, the teachers worked individually to construct a triangle inscribed in a semicircle and to notice the measures of the inscribed angles. Then they were invited to develop a proof for Thales' theorem. In Task 5.01, teachers discussed what they noticed about Thales theorem. One teacher said that she never heard of Thales' theorem and others teachers said they were not familiar with its proof. One teacher, ceder, said she was able to prove the theorem using a hint that was given in Task 4.01 about constructing a radius to form isosceles triangles. Another teacher, sunny blaze, said: "so as the points were dragged, there were 2 isosceles triangles inside the right triangle... did anyone see this?" She then said: "i wish we had the tab in front of us to confirm". This comments encouraged ceder to offer to do the construction. The others agreed and asked her to make the construction (see Figure 1).

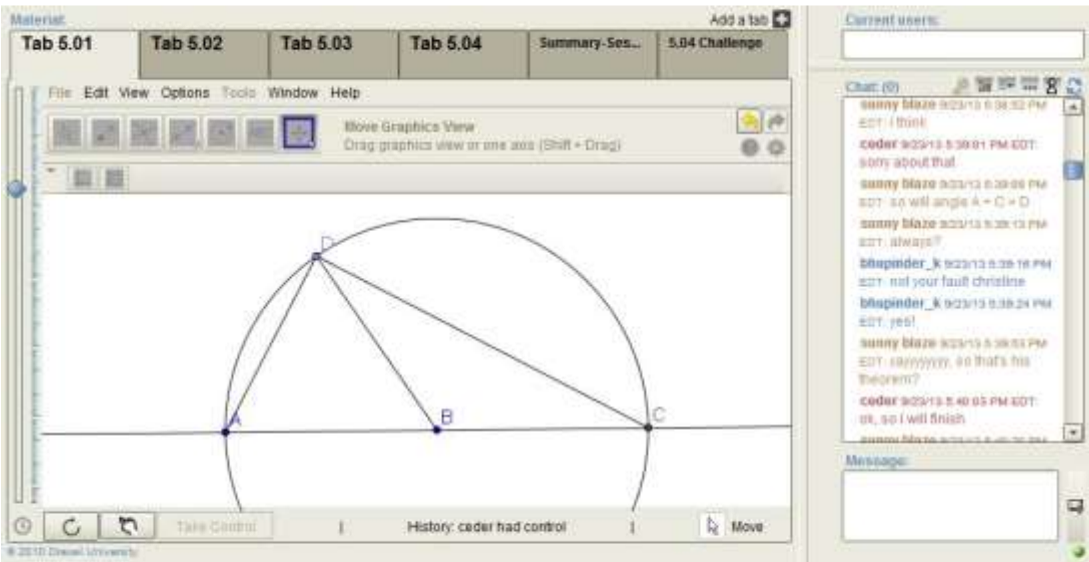


**Figure 1.** For Task 5.01, the construction of a member of Team 1.

After ceder constructed the figure in Figure 1, sunny blaze stated that she was not sure why there are two isosceles triangles. Part of their discussion follows:

| # | User | Chat Post |
|---|---|---|
| 59 | sunny blaze | (besides the fact that I'm not 100% sure what the theorem actually is), if D and B were not connected by a line segment, then how would you be able to see that there are two isosceles triangles inside the right triangle? Is the idea to see that there are 2 isosceles triangles no matter what vertex of the right triangle is dragged or is the idea to see that you can always form a right triangle inside a circle |
| 60 | ceder | the point is that there is always a right angle at D |
| 61 | ceder | and he used the fact that isosceles triangles have two angles that are equal and that the sum of interior angles of a triangle are 180 since he had previously proven that |

| | | |
|---|---|---|
| 62 | ceder | so A=ADB |
| 63 | ceder | and C=CDB |
| 64 | ceder | because of the isosceles triangles he say |
| 65 | ceder | saw* |
| 66 | sunny blaze | see i was way off, i wasn't sure what to look for because I didn't read Topic 4, i just went in and viewed the tabs. i really wish i would have spent more time on this last night |
| 67 | ceder | yeah, I find that document to be a big help |
| . | . | . |
| . | . | . |
| 79 | ceder | so we know those angles are equal because of the two isosceles triangles |
| 80 | ceder | then... |
| 81 | ceder | A+ADB+C+CDB=180 |
| 82 | ceder | is that ok so far? |
| 83 | bhupinder_k | right |
| 84 | ceder | I know its hard with all the letters |

In line 59, sunny blaze inquires about the theorem and her team member's, ceder's, construction. She wonders about the purpose of the construction of line segment BD in relation to the theorem: is it to show that no matter which vertex is dragged the formation of two isosceles triangles is invariant or is it to show that an inscribed triangle in a circle is always right? In response, in lines 60 and 61, ceder explains that the theorem says that angle D is always a right angle and that to prove his theorem Thales used the isosceles triangles and the fact that the sum of the interior angles of a triangle is 180. She lists the congruent angles that she will use to prove the theorem later. In lines 79 to 84, ceder states that the sum of the interior angles of triangle ABC is 180 then she asks if everyone is following her argument. The session continues after these lines and ceder shows the team her proof of Thales' theorem. During her demonstration, she was concerned with whether her team members were following her argument (see lines 82 and 84). She also dragged point D to show that the right angle was invariant. Later in the session, other team members took control and dragged different points vigorously to explore the invariance of the right angle and the isosceles triangles.

## Discussion

Analyzing the mediational interactions of a team of four middle and high school teachers working with digital collaborative and dynamic visualization technologies, VMTwG, allowed us to extend Rabardel's mediational roles of instruments [1, 2]. The mediational role of VMTwG in mathematics teachers' activity can take three forms: epistemic, pragmatic, and pedagogic. In epistemic mediation, teachers interacted with VMTwG to investigate objects properties and their relations. Pragmatic mediation occurred when teachers used the environment to transform objects into certain forms. Pedagogic mediation evidenced when teachers engaged the environment to help others learn about mathematical objects and their transformations and relations among objects. This mediational role was visible in the teachers' activity but it could also be visible in students' mathematical activity. Understanding the mediational of VMTwG in learning mathematics can inform our understanding of how learners interact with other technological tools for doing mathematics and how learners build their knowledge using the tools. This understanding can be extended to engineering students. Their learning' activities can be mediated epistemically, pragmatically, or pedagogically by technological tools. The different mediations of tools in students' learning activity allow us to see clearly how students engage in an activity, which helps educators to support students' learning.

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