Practical Design of PID-type Controllers with Constraints

Dr. Vassilios Tzouanas, University of Houston, Downtown

Vassilios Tzouanas is an Associate Professor of Computer Science and Engineering Technology at the University of Houston – Downtown, in Houston, Texas. He also serves as assistant department chairman. He received all his degrees in chemical engineering and obtained his Ph.D. from Lehigh University. His area of specialization is process modeling, simulation and control. He has worked in the industry for 19 years where he held technical and management positions with major operating companies as well as process control technology development companies. Since 2010, he has been with UHD where he teaches university courses in process control, modeling and simulation, process design and operation, applied thermodynamics and heat transfer, and numerical methods. Dr. Tzouanas' research interests include process modeling, simulation and design, process control, and renewable energy systems. Dr. Tzouanas is an ABET Program Evaluator (PEV) for Engineering and Engineering Technology programs. He is also member of AIChE and ASEE.

Steely Earl Varon Falar, University of Houston, Downtown

Mr. Steely Falar is a senior student in the Controls and Instrumentation Engineering Technology Program at the University of Houston-Downtown. He holds an AAS degree in Controls and Instrumentation from Houston Community College. His interests include process control, programmable logic controllers, and design of effective human-machine interfaces.

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Abstract
The proportional-integral-derivative (PID) controller has extensively been used in the process industries and is taught in most undergraduate engineering and engineering technology programs. Various PID design/tuning methods have been proposed over the years such as the famous Ziegler-Nichols method, the Internal Model Control (IMC) method, and the many variations of it. Given a process model, these methods estimate values for the PID tuning parameters: proportional gain, integral time, and derivative time. Many of the tuning methods include a tunable parameter, for instance the filter time constant in the case of the IMC method that the user must "tune". Furthermore, none of these techniques considers process constraints in the PID design. However, from a practical viewpoint, process and final control element constraints must be accounted for.

Recently, a methodology based on has been developed to design PID controllers subject to controlled variable as well as manipulated variable (size and rate) constraints while a performance criterion is optimized. This paper extends this methodology to determine the "tunable" parameters of other PID design methods while process and equipment constraints are satisfied. Estimation of the IMC filter time constant is considered. Simulation and experimental results demonstrate the practicality of the new PID design method.

1. Introduction
Over the years, a great deal of research has been devoted to the design of proportional-integral-derivative (PID) controllers which are widely used in the process industries. [1]-[3]. The famous Ziegler-Nichols tuning method [1] was developed more than 70 years ago and is still widely used. However, over the years, new methods have been proposed which result in better control performance and improved robustness. One of the most widely used methods is the Internal Model Control (IMC) method for designing PID controllers [4] and its many variations of it. Most of these techniques are analytic in nature and given a linear process model, they estimate the tuning parameters of the PID controller as long as the user provides an estimate of the filter time constant. Although simple guidelines are provided on how to select this filter time constant, it is in essence a "tunable" parameter, however.

In addition, a number of methods have been proposed to tune a PID controller such that a performance criterion is optimized, e.g. [5]-[7]. Almost exclusively, such methods are concerned with optimizing a performance criterion and the efficiency of the optimization methodology. However, from a practical point of view, it is also desirable to optimize the tuning of PID controllers subject to operating constraints.

Recently, a methodology has been developed to design PID controllers subject to controlled variable as well as manipulated variable (size and rate) constraints while a performance criterion is optimized. This method has been applied to linear and nonlinear processes, various types of PID algorithms and single or cascade feedback control structures [8]-[10]. Its usefulness has been demonstrated using simulation studies as well as experimental runs.

This paper extends this methodology to determine the "tunable" parameters of other PID design methods while process and equipment constraints are satisfied. Estimation of the IMC filter time constant is considered. Simulation and experimental results demonstrate the practicality of the new PID design method.

The remaining of the paper is organized as follows: Section 2 describes the methodology and its implementation in Microsoft Excel. Section 3 presents simulation results on the performance of the proposed method for a number of processes. Section 4 presents experimental results while Section 5 discusses the impact on undergraduate education. Finally, Section 6 summarizes the main results.

2. The Proposed Tuning Method

2.1 Block Diagram Representation

Consider a process under feedback control as shown in Figure 1.

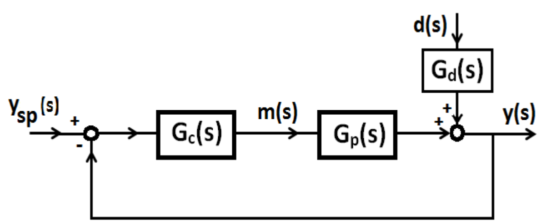

Figure 1. Schematic of a feedback control loop.

where (in the Laplace domain):

$G_p(s)$	is the process model
$G_c(s)$	is the controller transfer function
$G_d(s)$	is the model of the disturbance
$m(s)$	is the manipulated variable
$y(s)$	is the controlled variable
$y_{sp}(s)$	is the controlled variable setpoint
$d(s)$	is the disturbance (or load)

At the heart of the proposed methodology lies the process model. Most industrial processes, at least in the petrochemicals and refining sectors, can be described by a first or second order plus time delay model. On the other hand, a number of model reduction techniques, such as the "half rule", could be used to reduce the process model to a first order plus dead time model (FOPDT) [11]. So, without loss of generality, it is assumed that the process is represented by a FOPDT model of the form given by Equation (1):

$$G_p(s) = \frac{K_p \cdot e^{-\theta \cdot s}}{\tau_p \cdot s + 1} \qquad (1)$$

where

K_p is the process gain

τ_p is the time constant

θ is the dead time (or time delay)

2.2 PID Algorithms

PID controllers can take different forms. Some of the most common ones are: ideal, cascade, or parallel form. The proposed methodology is independent of the controller form and for the sake of brevity will be demonstrated for the case of the ideal form.

The transfer functions of the ideal PID controller is as follows:

$$\frac{m(s)}{e(s)} = K_c \cdot \left[1 + \frac{1}{\tau_i \cdot s} + \tau_d \cdot s\right] \qquad (2)$$

where

K_c is the proportional gain

τ_i is the integral time (time per repeat)

τ_d is the derivative time

K_i is the integral gain

K_d is the derivative gain

For computer-based control, the controllers are discretized and programmed in the velocity form. For example, in the case of the ideal PID controller, when P and D act on the control error, the following equation is used.

$$m(k) = m(k-1) + K_c \cdot [(e(k) - e(k-1) + \frac{\tau_s}{\tau_i} \cdot e(k) + \; + \frac{\tau_d}{\tau_s} \cdot (e(k) - 2e(k-1) + e(k-2)] \qquad (3)$$

When P and D act on the process variable, the following equation is used:

$$m(k) = m(k-1) + K_c \cdot [(y(k-1) - y(k) + \frac{\tau_s}{\tau_i} \cdot e(k) + \; \frac{\tau_d}{\tau_s} \cdot (2y(k-1) - y(k) - y(k-2)] \qquad (4)$$

In equations (3) and (4), τ_s is the controller's execution period.

2.3 The IMC Design Method for PID Controllers

The original method was first presented in [4]. A good summary of the tuning equations for various types of process models is included in [11]. For a model as shown in equation (1), the tuning equations are:

For PI only controllers:

$$K_c = (\frac{1}{K_p}) \cdot (\frac{\tau_p}{\tau_c + \theta}) \qquad (5)$$

$$t_i = t_p \qquad (6)$$

For PID controllers:

$$K_c = (\frac{1}{K_p}) \cdot (\frac{\tau_p}{\tau_c + \theta}) \qquad (7)$$

$$t_i = t_p \qquad (8)$$

$$td = ti / 4 \qquad (9)$$

It can be seen from equations (5) and (7) that tuning parameters will be calculated as long as the user provides a value for the filter time constant, τ_c. Depending on the assigned value, the controller tuning will be affected. This work will estimate a value for the filter time constant while a performance criterion is optimized subject to process constraints. The approach is based on the concept of co-simulation.

2.3 A Co-Simulation Approach

Considering a single loop, evaluation of control performance and tuning of the PID controller are done simultaneously over a desired time horizon while a performance criterion is optimized and process constraints, meaningful to the practicing engineer, are satisfied. In simple terms, the co-simulation approach follows the steps shown in Figure 2.

A process model is controlled using a PID controller. Assuming an initial value for the filter time constant, tuning parameters are estimated using the IMC method and the closed loop system performance is evaluated over a desired time horizon for expected set point and/or load changes. For instance, the Integral Absolute Error (IAE) could be used to measure system performance. Then, using an optimization algorithm and process constraints, the filter time constant is updated and the process is repeated until the performance criterion reaches a minimum value. Thus, by following this co-simulation based approach, tuning parameters are obtained which help ensure that process constraints will be satisfied prior to implementing them on the real process.

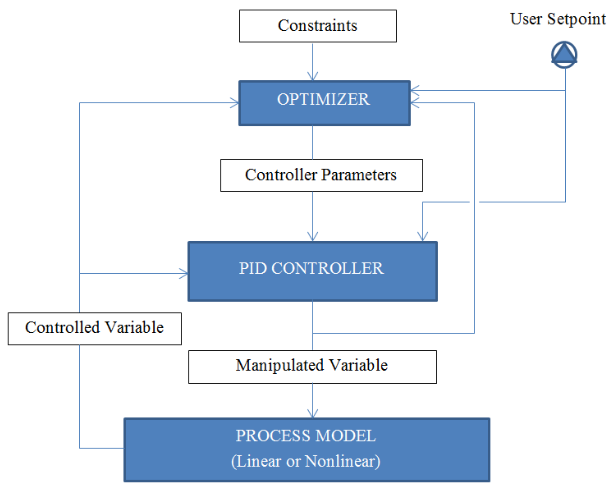

Figure 2. PID Controller Design Using Co-simulation

This co-simulation approach has been used for the tuning of single loop PID controllers subject to constraints [8], [9]. The same approach was also used to simultaneously tune PID controllers in a cascade control structure [10].

2.4 Optimal Estimation of the IMC Filter Time Constant

The objective is not to replace but rather complement previously proposed analytic tuning approaches by directly considering important process constraints such as manipulated variable size and rate of change constraints as well as constraints on the controlled variable and the tuning parameters. Manipulated variable constraints are meant to reflect the inherent capacity of the process to cause and/or reject change in the case of setpoint response or load disturbances, respectively. Controlled variable constraints are meant to meet desired objectives such as product quality constraints. Constraints on the tuning parameters are meant to limit the search space by utilizing experiential knowledge or analytical knowledge.

In this study, the IMC filter time constant will be estimated. Its estimation is done by optimizing either the integral absolute error (IAE) or the integral square error (ISE) over a desired time horizon, t_f. Other performance measures could also be considered. The performance criteria and the various constraints are mentioned next.

The performance criteria used in the present work, integral absolute error (IAE) and integral square error (ISE), are given by Equations (10) and (11), respectively.

$$IAE = \int_0^{t_f} |e(t)|\, dt \qquad (10)$$

$$ISE = \int_0^{t_f} e(t)^2\, dt \qquad (11)$$

The controlled variable constraints are positional only and are shown in Equation (12).

$$y_{LL} \le y(t) \le y_{UL} \qquad (12)$$

Positional and rate of change constraints for the manipulated variables are considered. They are shown in Equations (13) and (14).

$$m_{LL} \le m(t) \le m_{UL} \qquad (13)$$
$$\Delta m_{LL} \le \Delta m(t) \le \Delta m_{UL} \qquad (14)$$

To limit the search space for appropriate tuning parameters and speed up convergence of the optimization algorithm, tuning parameter constraints are considered as shown in Equations (16) through (17).

$$K_{C,LL} \le K_C \le K_{C,UL} \qquad (15)$$
$$\tau_{I,LL} \le \tau_I \le \tau_{I,UL} \qquad (16)$$
$$\tau_{D,LL} \le \tau_D \le \tau_{D,UL} \qquad (17)$$

To account for robustness against modeling errors, maximum sensitivity function constraints [18] are considered as shown in Equation (18).

$$M_S \le M_{S,UL} \qquad (18)$$

The subscripts LL and UL stand for lower limit and upper limit constraints.

As in [8], the process models are assumed to be first order plus time delay (FOPTD). The Euler integration method is used to solve the resulting ordinary differential equations. For computer implementation, the discrete version of a velocity type PID controller is used. The computer platform is Microsoft Excel and the Solver function with the GRG (gradient) optimization algorithm is used. The upper/lower limits for controlled and manipulated variables are process specific. The upper/lower limits for tuning parameters are set as multiple/fraction of the tuning parameters obtained using the IMC method for a value of the filter time constant which is equal to the process time constant.

3. Simulation Results

In this section, a number of simulation examples illustrate the proposed method. Its performance is tested for setpoint changes. Tuning parameters, subject to performance criteria, are estimated for PI and PID controllers. Control performance is compared to that obtained using the default IMC method where the filter time constant is set equal to the process time constant. In all cases, the controller execution period is set to 0.5 units of time and equals the integration step of the Euler method.

Example 1(Lag Dominant Process)

This process is lag dominant. Its model is given by the following equation:

$$G_{p2}(s) = \frac{1 \cdot e^{-1s}}{20s+1}$$

The relative time delay, is:

$$\tau = \frac{\theta}{\theta + \tau_p} = \frac{1}{1+20} = 0.0476$$

Controller tuning using the proposed optimization method is done subject to the constraints summarized in Table 1.

Table1. Tuning Constraints for Example 1

	min	max
Max/Min MV Change (%)	-3	3
Max/Min MV Rate of Change (%/min)	-3	3
CV Setpoint Change	1.5	1.5
Max/Min CV Change	10% under shoot	10% over shoot
Proportional Gain, K_C	0	$5K_{C\_IMC}$
Integral Time, τ_i	0.001	2 max(τ_p, 8 θ)
Derivative Time, τ_d	0	τ_i /5

The tuning parameters and corresponding IAE values are shown in Table 2 for the traditional IMC method and the IMC method with an optimal filter time constant. For the IMC tuning method, the filter time constant has been set equal to the process time constant.

Table2. Tuning Results for Example 1

	IMC	IMC with Optimal Filter
τ_c=	20	10
K_c =	0.95	1.82
τ_i =	20.00	20.00
IAE	63.00	33.00

Figure 3 shows the closed loop performance of a PI controller when it is tuned using the traditional IMC method (see IMC lines) and the IMC method using an optimal filter time constant (see IMC_OPT lines). The top part shows the controlled variable response to a set point change while the bottom part shows the required movement of the manipulated variable.

From a performance viewpoint, the classical IMC method yields the slowest response. The proposed method brings a balance between speed of response and excessive manipulated variable movement. In the case of the new method, a more aggressive tuning was limited because the maximum manipulated variable constraint became active.

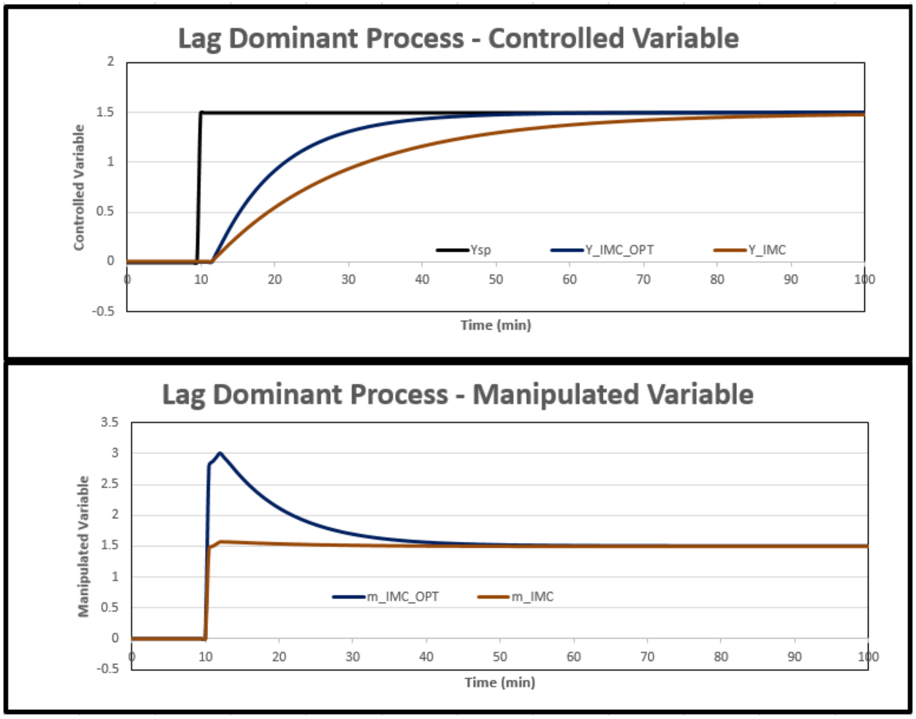

Figure 3. Setpoint response for a lag dominant process

Example 2 (Time Constant/Time Delay Balanced Process)

This is balanced process from a lag and time delay viewpoint. The process transfer function model is given below. Its relative time delay, τ, is 0.33.

$$G_{p2}(s) = \frac{1 \cdot e^{-5s}}{10s+1}$$

The tuning results are given in Table 3. The closed loop system performance for the two tuning methods is shown in Figure 4.

Table 3. Tuning Results for Example 2

	IMC	IMC with Optimal Filter
τ_c=	10.00	5.06
K_c =	0.67	0.99
τ_i =	10.00	10.00
IAE	45.00	35.51

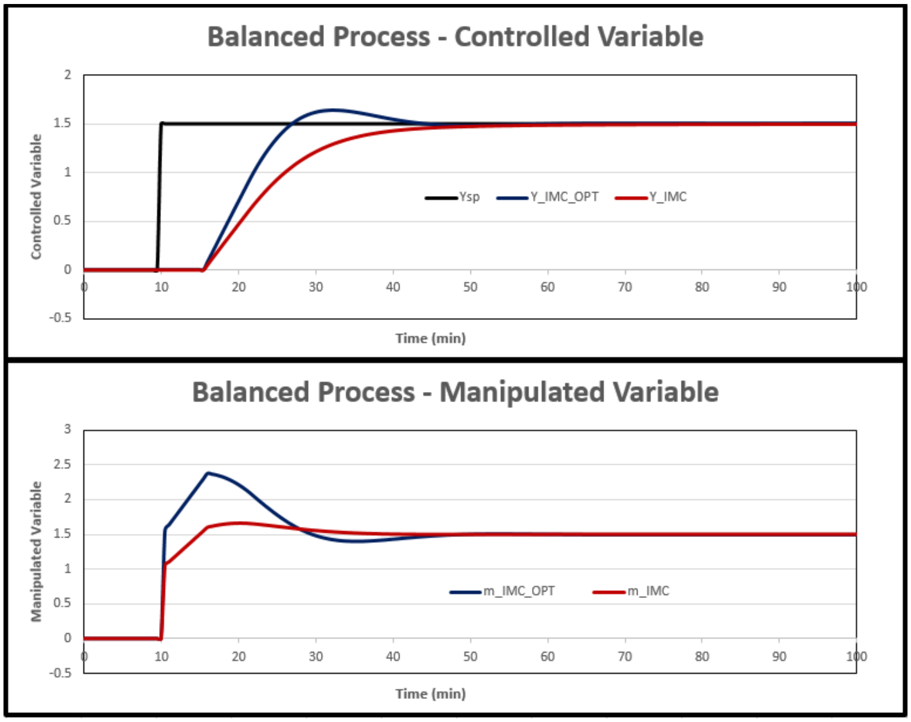

Figure 4. Setpoint response for a balanced process

From a performance viewpoint, and for a balanced process, it is demostrated that new method with an optimal filter time constant yields a faster response than the classical IMC method while process constraints are respected.

<u>Example 3 (Time Delay Dominant Process)</u>

. The process transfer function model is given below. Its relative time delay, τ, is 0.91.

$$G_{p3}(s) = \frac{1 \cdot e^{-10s}}{s+1}$$

The tuning results are given in Table 4. The closed loop system performance for the two tuning methods is shown in Figure 5.

Table 4. Tuning Results for Example 3

	IMC	IMC with Optimal Filter
τ_c=	1.00	7.69
K_c =	0.09	0.06
τ_i =	1.00	1.00
IAE	82.21	64.72

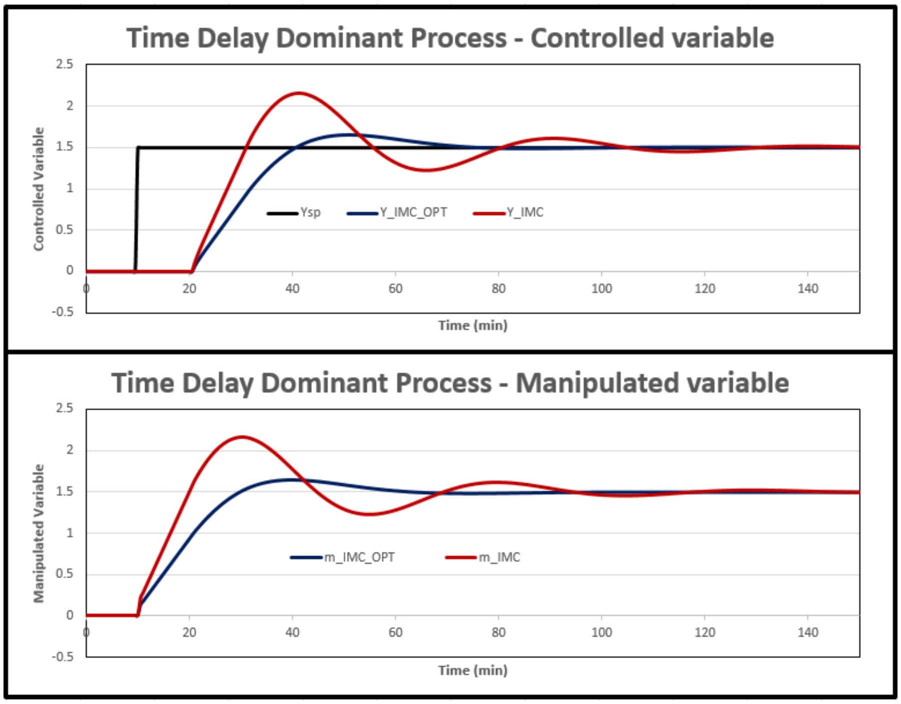

Figure 5. Setpoint response for a balanced process

From a performance viewpoint, and for a time delay dominant process, the classical IMC method yields a faster response than the proposed method. However, the proposed method results in less IAE while it respects the imposed process constraints. The classical method violates the imposed max limit for the manipulated variable.

4. Experimental Results

The proposed tuning method was applied to the twin water tank experimental system reported by [9] and [10]. Figure 6 shows the water twin tank system. In this twin tank process, water is pumped into the first tank (Tank 1) using a variable speed pump. From tank 1, water flows into the second tank (Tank 2) because of hydraulic pressure difference. From tank 2, water flows back into the reservoir. The control objective is to maintain the level of tank 2 at a desired setpoint using a PI controller. The manipulated variable is the speed of the pump or more precisely the DC voltage to the pump.

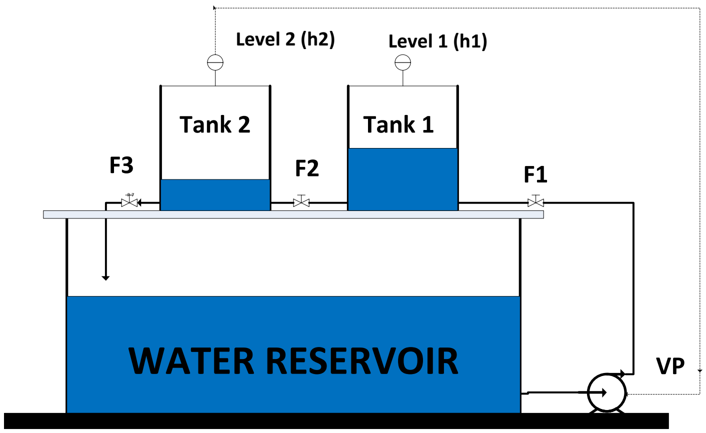

Figure 6. Experimental Twin Water Tank Process

Using experimental step test data, the following transfer function model was developed:

$$\frac{h_2(s)}{V_P(s)} = G_p(s) = \frac{21.25 \cdot e^{-10 \cdot s}}{66 \cdot s + 1}$$

Where h_2 is the tank 2 level in cm and V_p is the pump voltage in VDC.

The previous model was used to tune a PI controller using the classical IMC method and the proposed method. Furthermore, simulation runs were performed to determine the expected closed loop perfornace using these two tuning methods. The tuning results are given in Table 5. The closed loop system performance for the two tuning methods is shown in Figure 7.

Table 5. Tuning Results for Twin Tank Process

	IMC	IMC with Optimal Filter
τ_c=	66.00	17.61
K_c =	0.04	1.10
τ_i =	66.00	66.00
IAE	222.18	82.80

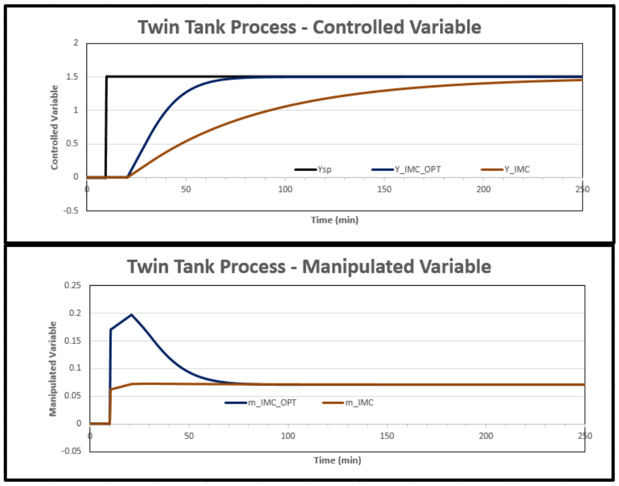

Figure 7. Setpoint response for the Twin Tank process (Simulated response)

Based on the data in Table 4 and the reponses shown in Figure 7, it is apparent that the new method estimates a filter time constant which optimizes the IAE while a maximum rate of change for the manipulated variable is observed (0.17 VDC/s).

The calculated tuning parameters were applied to the actual experimental system. The response of the water tank process when the PI controller is tuned using the classical IMC and the proposed IMC tuning methods is shown in Figure 8.

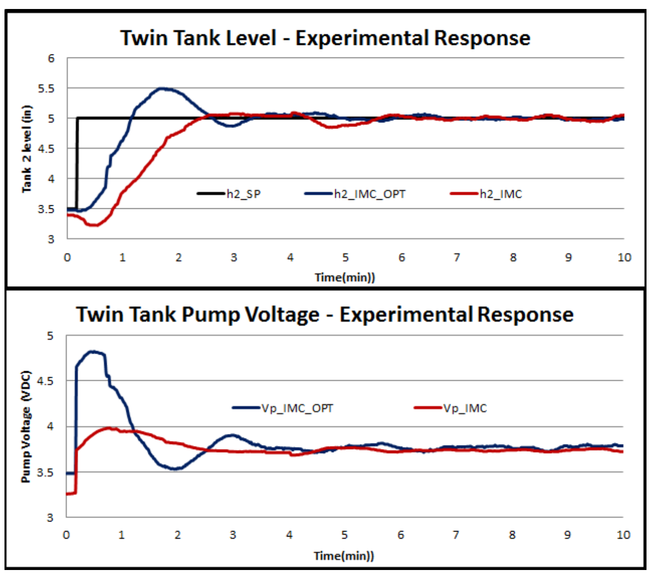

Figure 8. Setpoint response for the Twin Tank process (Experimental response)

The experimental data show that the new tuning method yields a faster response (blue line) than the classical IMC method (red line) and a smaller IAE.

5. Impact on Undergraduate Education

The importance of undegraduate research has been emphasized by many. Undegraduate reaseach is one of the different ways to provide high impact learning experiences to students.

Undegraduate engineering tecnology students are encouraged to participate in research from an applied engineering viewpoint. Reseach opportunities are provided through capstone design projects or senior level courses. This work has been included in our senior level Process Control Systems course when the discussion focuses on tuning methods for PID controllers. Furthermore, a few students have been able to participate in the development and experimental evaluation of the proposed methodology. As a result, they have a better understanding of PID contol, tuning methods, the importance of process/physical constraints on the design of a control system, and had the oportunity to experimentally evaulate the new method. Last but not least, they have been able to co-author a paper which is an important accomplishment for undergraduate students.

6. Conclusions

The paper was concerned with the design of PID controllers. A new method was proposed to estimate an optimal value for the filter time constant of the classical IMC tuning method for PID controllers. The new method is based on the concept of co-simulation and accounts for process constraints. Simulation and experimental results demonstrate the advantages of the proposed method.

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