Predictors for Success in Calculus I

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Work in Progress: Predictors for Success in Calculus 1

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ABSTRACT:

"What are the factors for success for students in calculus 1?" Because calculus 1 is considered a gateway course in most STEM majors, this is a common question among universities as attrition rates of students in these majors is considered high.

This paper explores the use of different statistical approaches to analyzing data on students who have taken calculus 1 at a large research extensive university. Hierarchal Linear Modeling (HLM) analysis will be used in determining if the class attributes of different sections determine the success of students in calculus 1 in college. HLM analyses will be used because students are nested within each class, or section. Another model involving the mediation variable of anxiety level will be explored. Lastly, a multivariate model will be used to examine relationships between all factors.

For all models, the dependent variable is student grade in calculus 1. In the HLM model, the level one factors are student academic level, gender, whether the student attended on campus tutoring, and whether the student is a STEM major (or more specifically an Engineering major). The level two factors come from the students' class section and include whether the section is online or face-to-face (mode of instruction), the semester (Fall vs. Spring), the size of the class, and the instructor's gender.

Keywords: calculus, student success, statistical analysis

Introduction

Many universities have seen a decrease in calculus success rates over the last decade. The success of students in calculus is an indicator of student success in future courses. Moreover, failure to pass calculus (with a C or better) negatively impacts the student retention rates and delays degree completion. These observations bring up several questions regarding what contributes to student success that need to be addressed. If the answers to these questions indicate definitive variables that contribute to success, we should then move to modify placement exams, curriculum, faculty attitudes, or pedagogy training as needed.

Math Placement

Although many universities use SAT, ACT, or some other standardized placement exam to determine if students will be successful in certain math courses, it has been reported that these scores under-predict the grades for female students [1]. In their article, Kessel and Linn [1] claim that college admissions officers may be missing talented females if they are relying heavily on ACT and SAT scores. Their article points out that "females report spending more time reflecting on similarities among problems, organizing and linking their ideas, and reviewing material" and because of this, they tend to be less able to demonstrate speed and recall of classroom taught algorithms – which is what most standardized exams test over [2]. Although females have similar scores to their male counterparts in math classes, Kessel and Linn [1] cited a national survey of over 800,000 students in which 72% of females and 62% of males who originally declared a major in math or a math related field switched majors during their career. Influences such as unrelatable instructors and discouraging advisors were factors that were cited as influencing the decision to switch; however, many females also cited that the competitive culture of these math and math related fields contributed considerably toward their decision to switch.

If a student does not place into calculus or decides that they are not prepared, they typically will start in precalculus. McGowen [3] analyzed enrollment data from 1980 to 2000 in an effort to gather information on the student-body makeup for precalculus in college. This study also analyzed data on the various math pathways students take after precalculus (i.e., subsequent math courses). Many students take 3-4 years of math in high school, so courses such as college algebra and precalculus are usually thought of as a repeat of information for these students. According to their grades, this does not appear to be the case [3]. Regardless of whether students did not retain the material, they had an insufficient foundation, or the students were misplaced in their math class, the data shows that college-level precalculus classes act as a "filter and not a pump" [3]; i.e., many students do not go on to calculus as they do not feel prepared to do so.

Research has reported that 23% of students who complete precalculus with a C or better are able to successfully complete calculus [4]. One reason that has been cited for this low rate is that the curriculum for precalculus is non-standardized across the nation [4]. To help improve this rate, Carlson et al. [4] developed an exam – and thus, a set of guidelines – containing topics that should be taught prior to calculus (similar to a common final exam). Their research focused on how precalculus should prepare students for calculus and discussed the development of a tool called the Pre-Calculus Concept Assessment (PCA). The PCA would then be used to assess students' reasoning abilities and understanding of precalculus. This test is a 25-item multiple choice exam that includes topics such as, e.g., rate of change of a function, interpreting the meaning of a graph, computational reasoning, and inverses. Carlson et al. [4] found that students with high scores on this particular assessment were almost twice as likely to pass calculus than those with low scores. They also found that scores on this assessment had a higher correlation with final course score than the math SAT score did.

STEM Attrition

Chen [5] compiled a statistical analysis report on attrition of STEM college students by using data from a cohort of students who started their post-secondary education in the 2003-2004 academic school year. In this report, it was found:

> About 28 percent of bachelor's degree students and 20 percent of associate's degree students entered a STEM field (i.e., chose a STEM major) at some point within 6 years of entering postsecondary education in 2003-04. At the bachelor's degree level, biological/life sciences was the most popular field, attracting 11 percent of students, and mathematics and physical sciences were the two least popular fields, attracting 2-3 percent of students. At the associate's degree level, a higher percentage of students chose computer/information sciences (9 percent) than other STEM fields (1-6 percent). Many of these STEM entrants left STEM several years later by either changing majors or leaving college without completing a degree or certificate. A total of 48 percent of bachelor's degree students and 69 percent of associate's degree students who entered STEM fields between 2003 and 2009 had left these fields by Spring 2009. Roughly one-half of these leavers switched their major to a non-

> STEM field, and the rest of them left STEM fields by exiting college before earning a degree or certificate.

The study used a multinomial probit (MNP) model with three outcome variables: persistence in STEM major; switching to a non-STEM major; or leaving school without a degree. One interesting finding was that among those that switched to a non-STEM major, the most important variables were the intensity of STEM coursework as well as the type of math courses taken during the student's first year. Chen [5] discovered that taking a lighter course load with less challenging math courses while having a poor performance in those courses lead to a high chance of switching out of STEM. Another noteworthy result from this study:

> All other factors being equal, bachelor's degree STEM entrants who first attended public 4-year institutions had a higher probability of leaving STEM by switching majors than those who started at private nonprofit 4-year institutions. Bachelor's degree STEM entrants who were male or who came from low-income backgrounds had a higher probability of leaving STEM by dropping out of college than their peers who were female or came from low-income backgrounds, net of other factors. Similarly, bachelor's degree STEM entrants who first attended institutions that were among the least selective had a higher probability of leaving STEM due to dropping out than students who first attended highly selective institutions.

When Chen [6] examined whether STEM persisters and leavers differ in their specific courses performance, they found that the "STEM persisters and leavers were also distinguished by their first-year math coursetaking: proportionally more STEM persisters than STEM leavers took advanced math courses such as calculus in the first year." In fact, it was determined that over 70% of STEM persisters had successfully completed calculus 1 (or a higher math class) during their first year of college work [6].

Daempfle [7] examined the attrition rates of students in STEM areas – in particular, the notion that freshmen classes should "weed out" those not prepared for the field. It is known that the highest rate of students switching out of a STEM major occurs at the end of the freshman year. Many students who switched indicated that most faculty in the STEM areas valued their research over teaching [7]. Daempfle explored responses from students regarding their classes and instructors and found that some of the reasons for leaving STEM included faculty expectations, epistemological expectations, gender, and minority status. Many students described STEM courses as having a "chilly climate", lack of discussion in class, competitiveness, and "one-way" lectures. Some students reported valuing their STEM experience in high school more than in college. Also, those students that stayed in their STEM major reported mentoring experiences and personal contact with their instructors. Daempfle [7] also reported that most switchers had a mean GPA of about 3.0 prior to switching (which is similar to that of the non-switchers). As such, cognitive ability, course rigor, and amount of student reading were not strongly correlated with retention in STEM majors. Daempfle [7] then states that one obstruction for first year college students is the "incongruity between secondary student preparation and post-secondary faculty requirements." Daempfle continues by citing several studies which show the statistically significant differences of perceived importance of certain courses and of certain traits of students by secondary and post-secondary instructors. Lastly Daempfle [7] analyzed studies which examined epistemological factors that contributed to attrition − the main factor being that non-persisters held an absolute view on science and math topics (right or wrong answers only).

Both Chen's and Daempfle's reports on attrition of STEM majors raise some interesting points. If we are concerned about attrition rates, we must look beyond standardized tests like the SAT and ACT. We need to look at our methods of instruction in important classes such as calculus and precalculus. We also need to find ways to change students' attitudes toward STEM courses and majors. Only time will tell if these changes can increase success in these important math courses and thus increase retention in STEM majors.

RESEARCH PROPOSAL

The Issue

The College of Natural Sciences and Mathematics at the University of Houston took on a project in the Summer of 2018 to help correct what they believed was a major issue for student success in calculus: placement. In their solution, they assigned a team of instructional professors in the math department to re-write the placement exams for both precalculus and calculus 1. As part of this project, this team also developed video lessons and quiz modules for remediation needs to help students overcome areas where the students tend to show weakness in before taking the placement exams. The initial results of this effort indicate that fewer students enrolled in calculus their first semester (with more starting off in precalculus) and the withdrawal rate for both calculus 1 and precalculus have dropped significantly. Shockingly, this project was taken on without analyzing student data beforehand (other than the examination of success rates).

Research Questions

1) Does placement exam score, academic level, gender, whether the student is a STEM (Science Technology Engineering or Math) major, and the number of times the student attended tutoring during the semester affect student scores in calculus in college?
2) Does a student's anxiety of math correlate with their exam scores in calculus?
3) If a student took precalculus, does their grade correlate with their calculus grade and are these students more or less successful than a student who enrolled FTIC in calculus?

METHOD

This research explores the use of Multiple Regression, Hierarchal Linear Modeling (HLM) analysis and Mediation Models in examining attributes that contribute to student success in calculus. These methods give different views of the student data.

HLM is used to determine if the class attributes of different sections determine the success of students in calculus in college. HLM analysis is used because students are nested within each class or section. The dependent variable is student grade in calculus, the level one factors are student academic level, gender, whether the student is a STEM major, and if the student attended tutoring. The level two factors come from the students' class section and include whether the section is online or face to face (mode of instruction), the semester (fall vs. spring), the time of day the class section is offered and the instructor's gender.

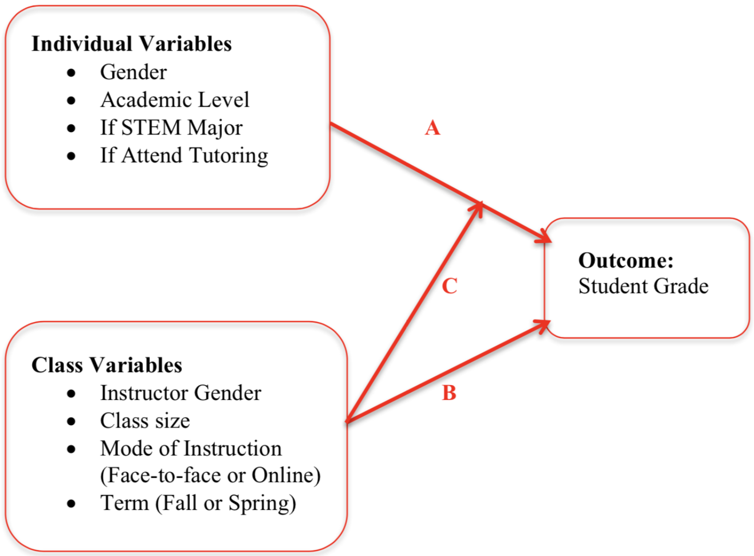

Figure 1: Conceptual model for HLM analysis.

Data

Student data from a large, diverse, research extensive university has been gathered from Spring of 2016 to Fall of 2018. The data includes over 7500 students within over 25 different course sections. Student information will include gender, admission term, academic level, major, term course is taken, placement exam score, previous college math scores (if applicable), and final grade in calculus 1 (including withdrawals and dates of withdrawal). Instructor information includes the gender of the instructor, whether the instructor is full or part-time and whether the instructor taught the course online or face-to-face. The data also contains survey results from 265 calculus 1 students from a math anxiety survey.

Measures - Hierarchal Linear Model

Student (Level 1). The student level (level-1) data used in this analysis will include information on the student's academic level, gender, if the student is a STEM major, and if the student attended tutoring. The academic level information will be coded as 1 for freshman, 2 for sophomore, 3 for junior, 4 for senior, 5 for post baccalaureate, and 6 for other. Student gender will be coded 0 for female and 1 for male. The information for whether the student is a STEM major will be coded as 1 if the student is a STEM major at the time of taking calculus and 0 if not. The tutoring variable is how many times the student attended the free math tutoring center on campus.

Class Section (Level 2). The classroom level (level-2) variables included the instructor's gender, the mode of instruction, the size of the class, and the term the class was taught in. Instructor gender will be coded as 0 for female and 1 for male. The mode of instruction will be coded with 1 for face-to-face instruction and 0 for online instruction. The term will be coded with 1 if the term was the Spring semester and 0 if it was the Fall semester. Summer and Mini semester information will not be used.

Measures - Mediation and Moderated Mediation Models

The model for mediation focused on using the students' test scores from class (test) as the independent variable with final semester average (grade) as the dependent variable. The students' score on an anxiety survey (anxiety) was used as the mediator in the model.

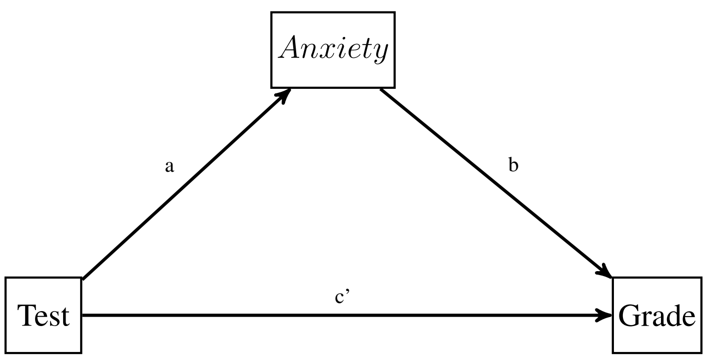

Figure 2: Single mediator model.

Results

Hierarchical linear modeling was used to statistical analyze a data structure where students (level-1) were nested within class sections (level-2). Of specific interest was the relation between student's grade in calculus 1 (level-1 criterion variable) and the student-level (level-1) predictor variables of gender, academic level, if STEM major and if the student attended tutoring and class section variables of the teacher's gender, class size, mode of instruction and term offered (level-2 variables). The average class size is 261 with a minimum size of 101 and maximum size of 534. Model testing proceeded in four phases: intercept-only model, 'within class model, means-as-outcome model, and the intercepts- and slopes-as-outcomes model.

The representation of the level 1 model with outcome for case *i* within unit *j* is given by:

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{1ij} + \beta_{2j}X_{2ij} + \beta_{3j}X_{3ij} + \beta_{4j}X_{4ij} + \epsilon_{ij}$$

where β_{0j} is the intercept for the level 1 equation and $\beta_{1j}, \beta_{2j}, \beta_{3j}, \beta_{4j}$ are the level 1 coefficients of each X. Also, ϵ_{ij} represents the level 1 random effect. The level 1 coefficients and intercept become the outcome variables for the level 2 variables. The level 2 model is

$$\beta_{qj} = \gamma_{q0} + \gamma_{q1}W_{1j} + \gamma_{q2}W_{2j} + \gamma_{q3}W_{3j} + \gamma_{q4}W_{4j} + u_{qj}$$

where $\gamma_{qj}, q = 0..4$, are the level 2 coefficients of each W and u_{ij} represents the level 2 random effect.

The first step in the analysis is to run the empty or unconditional model (Model 1) and compute the intraclass correlation coefficient (ICC) in order to compute the proportion of the total variance in each class. For this data, I found

$$ICC = \frac{59.49}{59.49 + 735.81} = 0.0748$$

which means that 7.5% of the variation in grades is due to factors at the class section level. This value meets the criterion of at least 5%, so I proceeded with the HLM analysis.

Model 2 is the "within class section" model, or the random intercept and random slope model. In this model we can see if normal within class environments affect student grades. All level 1 variables are significant in the model ($p < 0.001$ for gender, stem, and academic level, $p < 0.01$ for tutoring). It is interesting to note that the slope for tutoring is negative. One reason for this may be because students mostly go to tutoring when they are struggling with the material.

Model 3 represents a "means as outcomes" model with the level 2 variables of mode of instruction, semester taken, and class size included (faculty gender was not significant at any level). Six sections of the sample are online with 849 students total, the mean of this group is 59.08 with a standard deviation of 31.91. Twenty-three sections are face to face classes with enrollment of 6730 students with a class grade mean of 72.62 and standard deviation of 27.26. These results along with the significant γ coefficient ($\gamma = 6.839$, $p < 0.01$) indicate that the delivery style of the online sections may need further examination. The fall semester averages ($\mu = 74.70$, $\sigma = 27.20$) were significantly ($t = 15.44$, $p < 0.001$) higher than the spring semester averages ($\mu = 64.40$, $\sigma = 28.64$) for calculus 1 (see table 2). One last interesting fact from the data is that the classes with the larger class sizes did better then those with smaller class sizes (see table 3). This could be due to the fact that the night and online classes are the ones which are smaller in size.

The last model, model 4, is an “intercept- and slopes-as- outcomes” model. This model included the level 1 variables of gender, STEM, and academic level, and the level 2 variable indicating the semester the course was offered. All coefficients for this model are significant (see Table 4). These results confirm the model 3 results of semester offered being significant predictor of student grades. All of the results are summarized in Table 4.

Because semester offered appeared to have a strong significance on the class average in calculus 1, an examination of students who took precalculus before calculus was warranted. Of the 7579 students in the sample, 1917 had grades for precalculus at the same university (more may have taken precalculus elsewhere). A paired samples t-test was performed and it was found that although the correlation was significant ($r = 0.508$), the difference in the averages was also significant. In fact, the 95% confidence interval for the difference in grades in precalculus versus calculus 1 is from 23.675 to 25.896 ($\mu_D = 24.785$, $\sigma = 24.798$, $t = 43.762$, $p < 0.001$). In other words, students going from precalculus to calculus 1 should expect, on average, a 24 point drop in their class grade.

Anxiety was measured by a survey given online to students in a calculus 1 class. A mediation model was used to assess if anxiety has any effect on test scores. In the model the mediated effect of anxiety is not significantly different from zero, $ab = -0.012 (SE = .010), z = -1.239, p = 0.215$. Although the effect of test scores does drop slightly when anxiety score is higher.

Using multiple regression analysis to predict a student’s calculus 1 grade, seven independent variables were used: placement exam score, mode of instruction, number of visits to the tutoring center, student gender, semester class taken, whether or not the student is a STEM major, and academic level. The overall variance explained by these predictors is 30.6%. The

ANOVA test showed an F test statistic of 105.288 with $p < .001$. Each predictor, except student gender ($\beta = -2.556, p < 0.05$) and academic level ($\beta = -1.945, p < .0.01$), was positively related to the student's grade (see table 5). The model equation for prediction using unstandardized coefficients is $\hat{y} = -9.738 + 0.721(avgprec) + 6.357(mode) + 6.058(sem) + 4.528(STEM) + 0.389(tutor) + 0.066(placecal) - 1.945(acadlevel) - 2.556(gender)$.

Discussion

When introductory courses, such as precalculus and college algebra, were created at the college level, the intention was to prepare the underprepared students to go on to advanced classes, specifically calculus. The idea was to develop "those algebraic skills that were deemed necessary for success in mainstream calculus" [8]. It appears though that the curriculum in these preparatory courses may need to be examined thoroughly.

The academic level of students at the time of taking calculus at the University of Houston appears to be a very strong indicator of success in the course. As students wait to take this course, averages decline, with the exception of post baccalaureate students (see table 6). Freshman students account for 51.8% of the total students taking calculus, however, freshmen only account for 27.7% of the W grades given (withdrawal from the class between the fourth and tenth week of the semester) and they account for more than half (59.7%) of all passing grades.

Many of the success factors discussed in this paper can be addressed when advising students. College advisors need to be aware that if a student waits to take calculus (or possibly any gateway math course), their chance of success decreases. Proper placement of students is also a key issue. Since we have revised our placement exam and policy we have seen an increase in the passing rate among freshmen (from 79.88% in fall of 2017 to 82.62% in fall of 2018). Only time will tell if this trend continues. Future analysis of this data is essential to determine if our efforts in revising our placement exam is effective in the long run.

Table 1. *Unweighted Descriptive Statistics*

Variable	Mean	Std Dev
Class Level Variables:		
Facgen	0.31	0.47
Size	261.34	142.75
Mode	0.79	0.41
Sem	0.533	0.505
Student Level Variables:		
Gender	0.61	0.49
Acad Level	1.79	0.99
STEM	0.69	0.46
Tutoring	1.41	5.17

Table 2. *Class Semester Comparison*

Semester	Mean	Std Dev
Fall	74.70	27.19
Spring	64.40	28.64

Table 3. *Class Size Comparison*

Average size	Mean	Std Dev
161	62.23	5.28
409	74.67	5.44

Table 4. *Multilevel Results for Predicting Student Success in Calculus*

	Model 1: Unconditional Model		Model 2: Within Class Model		Model 3: Means as Outcomes		Model 4: Intercept and Slopes as Outcomes	
Independent Variable	Coefficient	SE	Coefficient	SE	Coefficient	SE	Coefficient	SE
Intercept	67.940***	1.450	68.395***	1.149	67.753***	0.773	68.305***	0.799
FacGender								
Size					0.030***	0.007		
Mode					6.839**	2.275		
Sem					−3.372	1.853	−4.420*	2.021
Fixed Effects:								
Tutoring			−1.767***	0.453				
Gender			−4.851***	0.813			−4.879***	0.824
Acadlevel			−5.065***	0.606			−5.131***	0.627
STEM			7.859***	0.950			8.00 ***	0.978

Chi-square table:

Model	SD	Variance	*df*	χ^2
Model 1: Intercept	7.713	59.491	28	704.865***
Model 2: Intercept	5.986	35.832	20	354.700***
Model 3: Intercept	3.688	13.602	25	130.490***
Model 4: Intercept	5.229	27.348	27	259.029***

a. Dependent Variable: Avgcal

*p < .05. **p<.01. ***p<.001

Table 5. *Coefficients*[a]

Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	-9.738	3.527		-2.761	0.006
	avgprec	0.721	0.029	0.483	24.765	0.000
	mode	6.357	1.546	0.080	4.111	0.000
	sem	6.058	1.129	0.106	5.366	0.000
	STEM	4.528	1.184	0.076	3.826	0.000
	tutor	0.389	0.086	.088	4.543	0.000
	placecal	0.066	0.018	.072	3.562	0.000
	acadlevel	-1.945	0.597	-0.068	-3.258	0.001
	gender	-2.556	1.157	-0.044	-2.208	0.027

a. Dependent variable: avgcal

Table 6. *Averages in Calculus 1 by Academic Level*

AcadLevel	Mean	N	Std Dev
Freshman	77.871	3925	24.181
Sophomore	67.179	19838	29.464
Junior	61.057	1131	29.658
Senior	52.968	429	31.543
Post Bacc	74.819	111	29.402
Total	71.113	7579	28.114

Table 7. *Averages in Calculus 1 by Colleges*

College	Mean	N	Std Dev
Natural Sciences & Math	81.258	2374	22.980
Engineering	85.401	918	19.646
Technology	59.491	1965	29.123
Non-STEM	64.659	2319	28.998
Total	71.113	7579	28.114

CODEBOOK

Variable	Mean	Std Dev	Min	Max	N	Description
Avgcal	71.11	28.13	0	122.70	7579	Numeric grade for calculus 1
Avgprec	84.75	18.95	0	109.50	1917	Numeric grade for precalculus 1f taken at UH
PFGrade	0.67	0.47	0	1	7579	Pass-Fail (0=fail, 1=pass)
Placecal	48.27	38.12	0	100	7579	Placement exam score for calc
Placeprecal	28.67	38.42	0	100	7579	Placement exam score for precalc
Gender	0.61	0.49	0	1	7579	Gender of Student (0=female, 1 = male)
AcadLevel	1.79	0.99	1	6	7579	Academic level of student (1=frsh)
Fresh	0.52	0.50	0	1	7579	If freshman (0=upperclassman, 1 = freshman)
STEM	0.69	0.46	0	1	7579	If STEM major (0=not, 1 = is STEM)
ENG	0.12	0.33	0	1	7579	If ENG major (0=not, 1 = is Engineering)
NSM	0.31	0.46	0	1	7579	If NSM major (0=not, 1 = is NSM)
Tech	0.26	0.44	0	1	7579	If Tech major (0=not, 1 = is Technology)
Tutoring	1.41	5.17	0	105	7579	Number of times student attended tutoring center during the semester
Mode	0.79	0.41	0	1	29	Mode of instr (0=online, 1=f2f)
Facgen	0.31	0.47	0	1	29	Instr Gender (0=female, 1=male)
Day	0.38	0.49	0	1	29	Two or Three days a week (0=two, 1=three)
Time	0.59	0.50	0	1	29	Time of day taught (0=evening, 1=day)
Sem	0.48	0.51	0	1	29	Term class was taken (1=Spring, 0 = Fall)
Size	261.34	142.75	101	534	29	Size of class/section

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