AC 2007-2901: PROMOTING HOLISTIC PROBLEM-SOLVING IN MECHANICS PEDAGOGY

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Promoting Holistic Problem-Solving in Mechanics Pedagogy

Abstract

The authors propose three strategies that are designed to enhance students' understanding and problem-solving ability in introductory mechanics courses: (1) employing multiple-method problem-solving, in which students solve a given problem using more than one method; (2) organizing systems of linear equations into a standard "tabular" format which resembles matrix format; and (3) emphasizing the discussion and use of assumptions in problem-solving activities. The authors give a rationale for each strategy, present a review of several mechanics textbooks to determine the prevalence of these strategies, and provide local student performance data that, while as yet inconclusive, suggests a possible method for assessment of the strategies' efficacy.

Introduction

Mechanics provides the scientific foundation for nearly all branches of engineering and constitutes an essential component of the education of nearly all engineering students. Through mechanics, students learn not only fundamental principles that govern the behavior of structures and machines, but they also develop the rigorous habits of mind of establishing and critiquing assumptions, translating physical problems into well-posed mathematical equations, and assessing the meaning and validity of their solutions (possibly leading to reformulation and new solutions). It is this broader understanding of mechanics that informs our holistic approach to teaching.

In a previous work[6] we studied the ability of mechanics students to think critically on the basis of their ability to use of free body diagrams, use vectors, coordinates and sign conventions, and address of units and physical dimension. We discovered that about three quarters of the time, students committed some error in at least one of these areas, even if they arrived at the correct answer. We also surveyed textbooks to determine how these matters are presented, and discovered several inconsistencies and inadequacies in their treatment.

Here we present three issues – referred to as the "targeted issues" – that we believe are important to promote problem-solving skills and broader understanding of mechanics. These issues are (1) multiple-method problem-solving, in which a given problem is solved in more than one way, (2) writing equations in a standard form that is amenable to computation, and (3) careful address of assumptions. Strategies to address these issues are referred to as the "targeted strategies".

Considering the first issue, can material be developed in a general manner such that the choice of method is presented as a fundamental part of the problem-solving process? Or must certain problems be "pigeon-holed" such that their solutions are hard-wired to only a certain approach? We probe these questions using the example of solving problems with both polar coordinates and Cartesian coordinates.

Regarding the second issue, we provide a rationale for encouraging students to solve equations systematically, rather than in an ad hoc manner. In particular, we advocate using a standard tabulation of the equations, which mimics matrix form, so that students can better appreciate the meaning of the equations and solve them reliably and clearly. This will also prepare students for computational methods that often require standard forms of equations.

Regarding the third issue, we stress the importance of adequately discussing and using assumptions in solving problems, even when a cursory treatment might simplify problem solutions. In the long run, neglecting the meaning and proper use of assumptions sets a standard in which students do not critically examine all elements of a problem.

To provide insight into how consistently these issues are addressed in mechanics education, we review several common mechanics textbooks to provide measures of how these issues are addressed in specific, standardized instances. To this end, we selected problems in textbooks that (1) are used widely across several books allowing for comparisons to be made, and that (2) sharply illustrate the address (or lack of address) one of the targeted issues.

Finally, we outline an assessment model that attempts to measure the effectiveness of the targeted strategies. Despite shortcomings in the methodology and inconclusiveness of the data, we suggest that the underlying concept – designing a procedure that can assess pedagogical effectiveness by measuring outcomes in future courses – is sound and potentially useful.

Multiple-approach Problem-solving vs. Pigeon-Holing

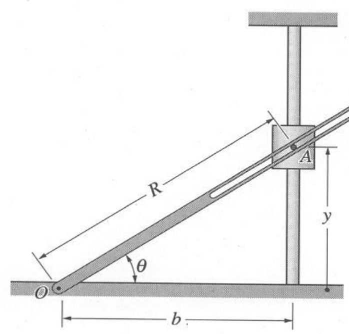
We believe that one strategy that fosters problem-solving ability and long-term retention of concepts is the regular use of "multiple-approach problem-solving", in which a *given* problem is solved by multiple methods. We distinguish this idea from the mediation of multiple methods through many problems, in which each problem is solved by only one method. We have developed several exercises that make use of multiple-method problem-solving in the sense that we mean, some of which are described below:

- Solving a problem using both Cartesian and polar coordinates;
- Determining components of vectors by direct trigonometry and use of the dot product;
- Determining cross products by inspection, geometric reasoning, and determinants using Cartesian components;
- Determining the moment of a force about an axis using different base points on the axis;
- Deriving different sets of equivalent equilibrium equations, such as by using different moment equations or different combinations of substructures.

In the absence of multiple-method problem-solving, a tendency arises to "pigeon-hole" problems and problem-solving approaches; that is, certain types of problems become associated with only one solution method. For example, consider the presentation of the basic equation of particle dynamics, $\sum \mathbf{F} = m\mathbf{a}$, assuming that the kinematic description of acceleration in the standard coordinate systems (Cartesian, polar, normal tangential) has already been covered. One option is to present the equation in *unified* vector form, and leave the issue of choosing an appropriate system of coordinates as a central element of the problem-solving process. The other option is to present the equation *separately*, i.e. separate discussions are provided corresponding to each coordinate system. It is this second option that fosters pigeon-holing of problems.

To illustrate how pigeon-holing might occur, consider the typical problem of analyzing the dynamics of a particle constrained to a straight track, but driven by a rotating arm. An example from Pytel & Kiusalaas[7] is provided in Figure 1 (henceforth referred to as "Problem A").

Figure 1. Problem A: A Particle Constrained to a Straight Track, Driven by a Rotating Arm.



Source: Pytel & Kiusalaas[7]

Problem A is reasonably approached using either polar coordinates (considering that the arm is rotating) or Cartesian coordinates (considering that the particle moves along a single Cartesian axis). Pigeon-holing this problem to only one of these methods limits the student's perspective, and possibly sets the student on a path of attempting to think "which method *must* I use for that problem?" If the problem is completed using only polar coordinates – including reporting the acceleration of the particle in polar coordinates – does the student absorb the fact that the acceleration of the particle itself is directed along the y-axis? And if the problem is completed using only Cartesian coordinates, does the student understand that the trigonometric relations used in this approach are embedded in the polar relations?

Exposing students to *both* the polar and Cartesian approaches for Problem A enables them to capture these and other insights, and in this way, the student learns more than simply the details of two different methods. Through the *comparison* of the two methods and their results, the student builds confidence in working with the fundamental principles and develops a deeper insight into the underlying physics. The student also begins to form assessments regarding the benefits and drawbacks of each method; ultimately, this will help the student to cultivate good judgment in deciding how to approach other problems. We do acknowledge the need to carefully engage students in multiple-method problem-solving, in order to avoid overwhelming them with too much information.

To understand the degree to which multiple-method problem solving occurs in mechanics pedagogy, we reviewed several textbooks to record whether the equation $\sum \mathbf{F} = m\mathbf{a}$ is presented in a unified or separated manner, and whether problems equivalent to Problem A (either sample problems or unsolved exercises) are presented with Cartesian, polar, or both coordinate systems. The data are reported in Table 1.

Table 1. Textbook Approaches to Introducing and Using Basic Coordinate Systems in Particle Dynamics.

Textbook by Author	Treatment of F = ma Separated by Coordinate Systems	Problem A Presented in Polar Coordinates	Problem A Presented in Cartesian Coordinates
Bedford & Fowler[1]	YES	N/A	N/A
Beer, Johnston & Clausen[2]	YES	Presented as a Review Problem	
Boresi & Schmidt[3]	YES		X
Hibbeler[4]	YES	X	
Meriam & Kraige[5]	YES	X	X
Pytel & Kiusalaas[7]	YES	X	X
Ruina & Pratap[9]	NO	N/A	N/A
Tongue & Sheppard[11]	YES	N/A	N/A

Problem A described in Figure 1. N/A = no problem comparable to Problem A found in text.

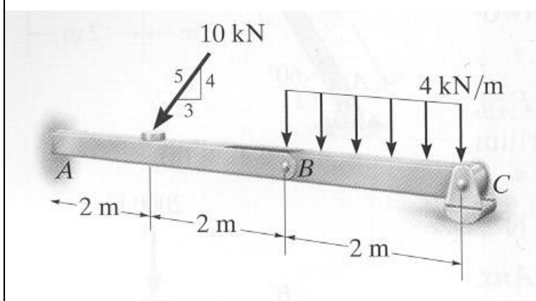
The data reveal that the vast majority of textbooks separate the discussion of $\sum \mathbf{F} = m\mathbf{a}$ by coordinate system; only one text (Ruina & Pratap[9]) presented a unified approach. With respect to presentation of problems comparable to Problem A, two books present the problem in only one system, three using both (including one as a review problem), and three others did not contain problem of sufficient similarity. While we advocate presenting $\sum \mathbf{F} = m\mathbf{a}$ in a more unified approach, we are encouraged that several texts do promote multiple-method problem solving in specific instances. While we did not collect data to exhaustively account for how often texts use multiple-method problem-solving throughout, our general experience and perusal suggests that the data presented in Table 1 is reasonably demonstrative of the general situation.

Organization of Systems of Equations: Newton Form vs. Tabular Form

A second issue that we believe is important in developing student problem-solving skills is the systematic organization of equations. In traditional problems in undergraduate Statics and Dynamics, the basic governing equations ($\sum \mathbf{F} = m\mathbf{a}_{CM}$ and $\sum \mathbf{M}_{/O} = \dot{\mathbf{H}}_{/O}$) generally yield a system of linear equations. State variables such as reactions, internal forces, and accelerations at particular instants can appear in various combinations as given or unknown quantities. Depending on what is given and what is unknown, direct transcription of the governing equations in the standard "Newton" form does not necessarily yield the standard linear form $Ax = b$, in which all of the unknown variables are on the left-hand side of the equation (x), and the known quantities ("forcing terms", though not necessarily physical forces) appear on the right-hand side (b).

To illustrate this, consider a typical problem of the static analysis of a simple frame, taken from Hibbeler[4] and outlined in Figure 2 (henceforth referred to as "Problem B"). In this problem, the equations are written directly from Newton's equations, rendering all terms – both known and unknown – to the left-hand side. The right-hand side of each equation is zero.

Figure 2. Problem B: Static Analysis of a Simple Frame, with Governing Equations Written in Direct Newton Form.



$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad B_x = 0$$
$$\circlearrowleft + \Sigma M_B = 0; \qquad -8\text{ kN}(1\text{ m}) + C_y(2\text{ m}) = 0$$
$$+\uparrow \Sigma F_y = 0; \qquad B_y - 8\text{ kN} + C_y = 0$$

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x - (10\text{ kN})\left(\tfrac{3}{5}\right) + B_x = 0$$
$$\circlearrowleft + \Sigma M_A = 0; \qquad M_A - (10\text{ kN})\left(\tfrac{4}{5}\right)(2\text{ m}) - B_y(4\text{ m}) = 0$$
$$+\uparrow \Sigma F_y = 0; \qquad A_y - (10\text{ kN})\left(\tfrac{4}{5}\right) - B_y = 0$$

Source: Hibbeler

We affirm that the presentation of equations in the standard Newton form (as in Figure 2) is necessary and useful, for it clearly displays the physical source of each term. But beyond this, we strongly advocate introducing an intermediate step prior to solving the equations. In this step the equations are re-written in a "tabular" form, in which the equations are re-arranged to write all unknown terms on the left-hand side, and all known quantities on the right-hand side. In addition, the equations are written such that like terms on the left-hand side are vertically aligned. The tabular form of the equations from Figure 2 is provided in Figure 3.

Figure 3. Equations from Problem B (Figure 2) Written in Tabular Form.

		B_x				$=$	0
				$(2\text{m})C_y$		$=$	8 kNm
			B_y	$+\ C_y$		$=$	8 kN
A_x		$+\ B_x$				$=$	6 kN
			$-(4\text{m})B_y$		$+\ M_A$	$=$	16 kNm
	A_y		$-\ B_y$			$=$	8 kN

The rewriting of equations into tabular form (as in Figure 3) might appear unnecessary, especially in cases in which the equations are highly decoupled and their solution is nearly trivial, as with the equations presented in Figure 2. It is also reasonable to question whether techniques to setup equations belong in mechanics courses or should be left to engineering mathematics courses.

We see many reasons to regularly include the tabular form as part of the overall problem-solving process. First, regardless of whether the solution of the equations is trivial, writing equations in tabular form calls attention to the distinction between *writing* and *solving* equations. We have found repeatedly that students race ahead to "solve" problems before properly formulating them. Writing equations in tabular form is a clear and tractable approach that will add structure to student reasoning, such as steering them to consider whether the system is properly posed and has an equal number of equations and unknowns. It will further promote general habits of organizing both thought and exposition, ultimately fostering better problem solving and communications skills.

Second, learning how to write equations in tabular form – a close cousin to matrix-vector form – immediately provides a format that is ready for input into standard equation solvers, such as in computer software or programmable calculators. Thus, using tabular form prepares the student for real problems that *require* computational solutions (unlike textbook problems that are usually amenable to hand calculation). In addition, use of tabular form provides flexibility to encourage capable students to use matrix computation instead of or in addition to hand calculation, while not placing other students at a disadvantage. Many students of the first author, including some in fields other than mechanics, have remarked that the habit they learned in Statics, to write equations in tabular form, has proved useful in more advanced courses, such as linear circuit analysis.

Third, the establishment, manipulation, and solution of linear systems of equations lie at the heart of countless engineering computations. Despite the fact that these computations are often hidden from the user in many software applications, and many practicing engineers seldom perform linear algebra calculations in their day-to-day work, familiarity with linear systems is useful to provide a context to help the engineer understand "what's under the hood" in many applications. Moreover, facility with linear equations helps the engineer to conceptualize an entire system of equations as a single entity, helping him or her to make qualitative judgments about computed solutions. Learning to write equations in tabular form is a good early step that will prepare students for these situations.

We surveyed several Statics texts to determine whether equations were re-written in tabular form. The survey was standardized by selecting examples from each text in which equations were generated from the static analysis of a simple frame (i.e. comparable to Problem B).

Table 2. Textbook Approaches to Organizing Linear Systems of Equations, using Problem B.

Textbook by Author	Tabular/Matrix Form Used	Tabular Form Never Used
Bedford & Fowler[1]		X
Beer, Johnston & Clausen[2]		X
Boresi & Schmidt[3]		X
Hibbeler[4]		X
Meriam & Kraige[5]		X
Pytel & Kiusalaas[7]		X
Riley, Sturges & Morris[8]		X
Ruina & Pratap[9]	X	
Soutas-Little & Inman[10]	X	
Tongue & Sheppard[11]		X

Problem B described in Figure 2.

Table 2 provides a summary of how the textbooks surveyed organize equations after initial deriving them in Newton form. We found only two examples (Ruina & Pratap[9] and Soutas-Little & Inman[10]) in which equations were re-arranged into a standard tabular or matrix form. We found that otherwise, textbooks solved equations using a variety of ad hoc substitution methods or simply provided solutions without any method.

Use and Discussion of Assumptions

Another key element that we believe is central to developing problem-solving skills is the ability to understand, use, and develop reasonable assumptions. Our experience suggests that students in introductory mechanics courses regularly neglect to use essential assumptions, even when given. Similarly, they appear to have difficulty formulating necessary assumptions that are not supplied, often introducing insufficient or irrelevant information. The authors' teaching pays close attention to formulating assumptions and illuminating their implications when used.

We inquired to what degree texts state assumptions, discuss their origin and importance, and incorporate them into the text or sample problems. Even when some assumptions appear to be trivial, we believe that the act of *paying attention to an assumption* serves to establish healthy mental habits that foster rigor and stimulate critical thinking.

As an example, we examined to what degree the assumption of a massless cable is presented in elementary dynamics texts, and how these assumptions are highlighted in the context of solving problems. Such an assumption is common and nearly indispensable in elementary mechanics. We surveyed several texts to determine whether the massless cable assumption is validated by analyzing the "massless dynamics" of a segment of cable (i.e. as if in static equilibrium), or present a segment of cable as a two-force body.

Table 3 provides data that summarize the treatment of the "massless cable assumption" in elementary dynamics texts. As shown, most texts mention cables (alternatively wires, cords, tethers, etc.) in the statement of problems in which they appear. About half of the books make the point that the cables are to be assumed massless (or light, etc.). None appeared to present the cable in the context of massless dynamics, or remind the reader that a segment of cable can be treated as a two-force body.

Table 3. Treatment of the Assumption of Massless Cables in Dynamics Texts.

Text	Mention cable, string, wire, tether, hawser, or cord	Specify cable as massless	Mention massless dynamics or cable as 2-force body
Bedford & Fowler[1]	Most times	Never	Never
Beer, Johnston, & Clausen[2]	Always	Never	Never
Boresi & Schmidt[3]	Always	Sometimes	Never
Hibbeler[4]	Always	Sometimes	Never
Meriam & Kraige[5]	Always	Sometimes	Never
Pytel & Kiusalaas[7]	Most times	Never	Never
Ruina & Pratap[9]	Most times	Most times	Never*
Tongue & Sheppard[11]	Always	Most times	Never

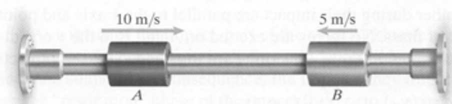
*An example of this type was included in an earlier edition.

While dwelling on the massless cable assumption might appear to be of questionable utility, we believe that paying attention to such assumptions is instrumental in communicating to the student that *every* element or component of a mechanical system is subject to analysis; hopefully this will foster their free thinking in trying to analyze components of various systems that they will encounter.

Other instances in which assumptions are often made without full discussion or development involve conservation of energy and conservation of momentum. These important principles are often approached with the narrow objective of providing an “easy” calculation. For example, consider the excerpt from a problem about the collision of two masses on a rail, taken from Bedford & Fowler[1] and presented in Figure 4. The problem solution correctly assumes conservation of linear momentum, but no commentary is given the validity of the assumption. The adjective “smooth” provided in the problem description does imply that no horizontal external force will act on the masses, but this is not mentioned as part of the reasoning to justify using conservation of linear momentum. We recommend citing details such as this and drawing impulse Free Body Diagrams which explicitly illustrate the absence of external impulses. Then, the conservation of momentum can be deduced rather than merely asserted.

Figure 4. Problem C: Excerpt from Sample Problem in which Conservation of Linear Momentum is Assumed.

The 4-kg masses A and B in Fig. 16.14 slide on the smooth horizontal bar. Determine their velocities after they collide if (a) they are coated with Velcro® and stick together and (b) their coefficient of restitution is $e = 0.8$.



Strategy

(a) If the masses stick together, they have the same velocity after their collision. We can determine the velocity from conservation of linear momentum.
(b) Knowing the coefficient of restitution, we can determine the velocity of each mass after the collision by using conservation of linear momentum together with the definition of the coefficient of restitution, Eq. (16.16).

Solution

(a) The velocities of the masses before the impact are $v_A = 10$ m/s and $v_B = -5$ m/s. Let v' be their common velocity after the impact. Conservation of linear momentum requires that

$$m_A v_A + m_B v_B = (m_A + m_B)v':$$

$$(4\text{ kg})(10\text{ m/s}) + (4\text{ kg})(-5\text{ m/s}) = (4\text{ kg} + 4\text{ kg})v'.$$

Source: Bedford & Fowler[1]

We note that we have discovered one exercise in the Solution Manual of Beer, Johnston, & Clausen[2] in which the assumption of a massless rod is stated and directly employed in the solution. This problem is discussed further in the Appendix, as it not only involves the analysis of a massless object, but also treats a problem with angular momentum in an appropriate manner, and provides an opportunity for multiple-method problem-solving.

Toward Measuring Effectiveness Despite Inconclusive Data

We provided a rationale that careful attention to the targeted issues – solving problems by multiple means, organizing systems of equations in a standard form, and paying close attention to assumptions – will promote deeper understanding, long-term retention of concepts, and students' ability to apply sound problem-solving techniques in future engineering courses, even in areas other than mechanics. We have some anecdotal evidence to add weight to our convictions, including informal comments from students and colleagues, and we sought to determine whether any student performance data exists to substantiate this.

Providing quantitative evidence was a questionable task from the outset, for the development of our instructional techniques and materials to address the targeted issues did not include any controlled studies to measure their influence. We have only aggregate, post hoc student performance data from which to infer the effectiveness of our strategies. Although our results appear to be inconclusive, we report them here to illustrate an assessment method that might work if it were applied to data collected from a controlled experiment.

Because the targeted strategies are transferable and applicable to most other engineering courses, we sought to measure whether students who are exposed to these strategies form habits which are retained in later courses. We thus chose to measure student performance (grades) in advanced courses as a function of prior instruction in mechanics. In particular, we collected data to measure (1) performance in Fluid Mechanics versus prior instruction in Dynamics and (2) performance in Structural Analysis versus prior instruction in Statics. These relationships were chosen by reasoning that in each case, the more advanced course depends critically on concepts developed in the prior course. For simplicity, we restricted study to students who, as of the date of the data query (November 2006), had completed degrees in or were currently enrolled in the fields of Civil Engineering or Mechanical Engineering [1].

We developed an "Influence Factor" to measure the difference in student performance between students taught by the first author ("Author") and students taught by another instructor ("Other") [2, 3]. The rationale for distinguishing student performance on this basis is that, because the targeted issues are central and highly emphasized in courses taught by the Author, performance of the Author's students is possibly correlated with the effectiveness of the targeted methods.

The influence factor is computed as

$$\text{Influence Factor} = \text{RSPI}_{\text{author}} - \text{RSPI}_{\text{other}},$$

where RSPI = "Relative Student Performance Index". The RSPI is a measure of student performance in a given class (e.g. Fluid Mechanics or Structural Analysis) compared to baseline student performance. We chose the baseline datum to be a weighted average of the students' overall GPA (67%) and prior performance in calculus (33%) [4]. The RSPI is then calculated as

$$\text{RSPI} = \text{Grade in Fluids or Structural Analysis} - \text{Baseline Grade}$$

Because grading patterns are highly dependent on instructor (e.g. the average assessed grade differs from instructor to instructor), we partitioned our data into subsets according to instructor for Fluid Mechanics (A, B, C) and Structural Analysis (D) [5]. No attempt was made to separate data by instructors for courses that contributed to the baseline grade; it is assumed that on average, grading patterns over all courses are uniform for each category of data in our study. The results of our analysis are reported in Table 4.

Table 4. Influence of Teaching in Courses Taught by the Author and Other Instructors.

Influence of Experience in Dynamics on Performance in Fluid Mechanics									
N	GPA	Prior Math	Effective Baseline	Dynamics	Instruct	Fluids	Instruct	RSPI	Influence Factor
51	3.109	2.529	2.721	2.553	Author	3.431	A	0.711	+0.330
53	2.942	2.554	2.682	2.874	Other	3.063	A	0.381	
21	3.094	2.683	2.819	2.651	Author	3.032	B	0.213	+0.138
16	3.302	2.771	2.946	3.042	Other	3.021	B	0.075	
35	3.013	2.452	2.637	2.495	Author	2.524	C	-0.113	-0.039
62	2.982	2.598	2.725	2.927	Other	2.651	C	-0.074	
Influence of Experience in Statics on Performance in Structural Analysis									
N	GPA	Prior Math	Effective Baseline	Statics	Instruct	Struct. Anal.	Instruct	RSPI	Influence
18	3.179	2.951	3.103	3.020	Author	3.274	D	0.171	-0.185
29	2.985	2.722	2.897	2.609	Other	3.253	D	0.356	

N = number of students in category. GPA = cumulative GPA in all courses at University. Prior Math = average grades in last math course. Effective Baseline = 0.67*GPA + 0.33*Prior Math; Dynamics (Statics) = grade in last dynamics (statics) course completed prior to taking Fluids (Structural Analysis); Fluids (Struct. Anal.) = grade in Fluid Mechanics (Structural Analysis) course attempted. RSPI = Fluids (Struct. Anal.) - Baseline. Influence = difference in RSPI for Author's students and Other's students. Data includes records of students who took Fluid Mechanics from Fall 2002 to Fall 2005, and Structural Analysis during Fall 2006; records of students with grades W or F in these courses removed.

To illustrate how the data is compiled and how the influence factor is computed, consider data corresponding to category "A", which appears in the first two data rows. Category A consists of student performance data for students who took Fluid Mechanics with instructor A. Category A is further broken into two subsets: one subset (Author) for students whose last prior Dynamics course was taught by Author, and the other subset (Other) for students whose last prior Dynamics course was taught by another instructor. The Author's students had an average baseline grade of 2.721, and an average grade in Fluid Mechanics of 3.431. The RSPI for this case is thus 3.431 – 2.721 = 0.711. [Note that the grade in Dynamics itself is excluded from the calculation because it is highly dependent on instructor and cannot be interpreted to have absolute meaning. It is provided for reference]. Similarly, the RSPI for students with other instructors is 3.063 – 2.682 = 0.381. The influence factor is then computed to be 0.711 – 0.381 = 0.330.

As indicated by the positive values of influence factor for Categories A and B, students of the Author in these segments were highly successful. Performance of students in Category C was roughly equal for students of the Author and Other instructors. Students of the Author in Category D were not as successful as Others' students. Overall, little inference can be made due to the small data sizes (data is highly sensitive to outlying performance of small numbers of students), although it is perhaps plausible that the Author's teaching in Dynamics is effective on the basis of this measure.

More broadly, even had the data itself been conclusive questions remain as to the overall validity of the influence factor developed here as a measure of teaching effectiveness. And even if the method itself is valid, only in highly controlled circumstances could the positive measures of teaching influence imply effectiveness of the targeted strategies, for teaching outcomes are influenced by a variety of factors. Despite these significant shortcomings in both the results and the methodology, we decided to present the data because it illustrates a possible direction of assessment that has the potential to measure effectiveness of introductory instruction based on outcomes in future courses. We think this is useful.

Discussion and Conclusions

We presented three issues in mechanics education (referred to as the "targeted issues) – multiple-method problem-solving, systematically organizing equations, and paying special care to discussing assumptions – that we believe to be important. We have taken special care in our teaching to emphasize these matters and to design teaching strategies that effectively engage students in these questions. Our feeling is that on the whole, these strategies effectively promote student understanding of not only mechanics, but of analytical problem-solving in general. And we believe that the approaches to problem-solving learned in mechanics are applicable to problem-solving in many other fields of engineering.

We provide some insight into the degree to which our identified issues are addressed in textbooks. By identifying sample problems that are both universally used and illustrative of the targeted issues, we documented that on the whole, textbooks do not consistently address the targeted issues, although several examples exist in which there is good treatment. Our review of the texts is not exhaustive, but based on our overall perusal of several textbooks, our sense is that the examples that we selected are useful and representative of the overall approaches adopted by the texts.

We agree with other mechanics educators that course outcomes are unlikely to be functions of the textbook used, that no textbook is perfect or all-encompassing, and that any shortcomings of textbooks can be bridged by instructors. However, repeated patterns in textbooks, such as those that we indicate here, serve to indicate and influence general approaches to teaching and what information students accept as important. At a minimum, we hope that our survey will provide other educators with a sense of what some of these general patterns are. More broadly, as new editions of textbooks appear very frequently, yet with few substantive changes, the issues that we present in this paper provide suggestions of how textbooks can innovate to call students' attention to broader issues and critical thinking.

With regard to assessment, the attempt that we made to quantitatively demonstrate the effectiveness of our recommended approaches proved inconclusive. However, we believe that the idea behind our approach – to measure effectiveness of certain strategies based on future outcomes in subsequent courses – is sound and will complement other existing methods of assessment. For example, the use of force concept inventories is useful to provide immediate feedback of student knowledge, and if used for both pre- and post-tests, they can be used to measure effectiveness of teaching strategies. However, this approach does not indicate the duration over which students retain knowledge, or the ability of students to apply fundamental concepts in future situations. Approaches that do study long-term student success seem to be mostly concerned with measuring retention and graduation rates, and while valuable for assessing overall program outcomes, the also do not measure effectiveness of specific teaching strategies. Our attempt to measure the influence factor is a sketch of what such an approach might look like.

Although our study is limited, we hope that it provides some insight into some issues that are important in mechanics pedagogy, but perhaps neglected. We believe that the strategies that we present here are tractable for educators to try, and will serve to foster better problem-solving skills during students' early formative years.

Endnotes

[1] Fluid Mechanics is required only for students in Civil Engineering and Mechanical Engineering.

[2] The first author was a principal instructor for the basic mechanics courses during since Spring 2001. The second author worked as a grader and research assistant for the first author from 2003-2005. The third author has a similar philosophy of teaching from prior experience.

[3] A total of five different instructors, including the first author, have taught Dynamics to students included in this study. No attempts were made to determine distinct influences of "other" instructors.

[4] The prior math course for students in Fluid Mechanics is the grade in the last calculus course taken prior to enrolling in Dynamics. The prior math course for students in Structural Analysis is the average of the last two calculus courses taken.

[5] Because the Author had not taught Statics prior to Fall 2004, meaningful student performance data from Structural Analysis was available only from Fall 2005 and Fall 2006, both of which sections were taught by a single instructor "D".

Appendix

We discuss Problem 12.134 from Beer, Johnston & Clausen[2] because the prepared solution in the Solution Manual (1) invokes the idea of a massless object is nontrivially and with adequate attention, and (2) provides an appropriate solution when a blanket assumption about conservation of angular momentum could have been asserted instead. The problem statement is provided in Figure 5, and an excerpt of the solution is provided in Figure 6. This problem is presented as a Review Problem in the chapter on Particle Dynamics.

Figure 5. Problem 12.134 from Beer, Johnston & Clausen[2]

A 1-lb ball A and a 2-lb ball B are mounted on a horizontal rod which rotates freely about a vertical shaft. The balls are held in the positions shown by pins. The pin holding B is suddenly removed and the ball moves to position C as the rod rotates. Neglecting friction and the mass of the rod and knowing that the initial speed of A is $v_A = 8$ ft/s, determine (a) the radial and transverse components of the acceleration of ball B immediately after the pin is removed, (b) the acceleration of ball B relative to the rod at that instant, (c) the speed of ball A after ball B has reached the stop at C.

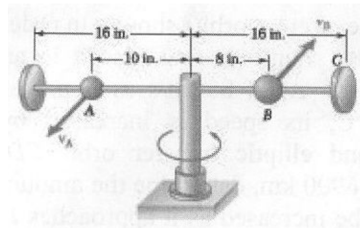


Figure 6. Excerpt of Solution of Problem 12.134 from Beer, Johnston & Clausen[2].

Let r and θ be polar coordinates with the origin lying at the shaft.

Constraint of rod: $\theta_B = \theta_A + \pi$ radians; $\dot{\theta}_B = \dot{\theta}_A = \dot{\theta}$; $\ddot{\theta}_B = \ddot{\theta}_A = \ddot{\theta}$.

(a) Components of acceleration

Sketch the free body diagrams of the balls showing the radial and transverse components of the forces acting on them. Owing to frictionless sliding of B along the rod, $(F_B)_r = 0$.

Radial component of acceleration of B.

$$F_r = m_B(a_B)_r: \qquad (a_B)_r = 0 ◀$$

Transverse components of acceleration.

$$(a_A)_\theta = r_A\ddot{\theta} + 2\dot{r}_A\dot{\theta} = ra\ddot{\theta}$$

$$(a_B)_\theta = r_B\ddot{\theta} + 2\dot{r}_B\dot{\theta} \qquad (1)$$

Since the rod is massless, it must be in equilibrium. Draw its free body diagram, applying Newton's 3rd Law.

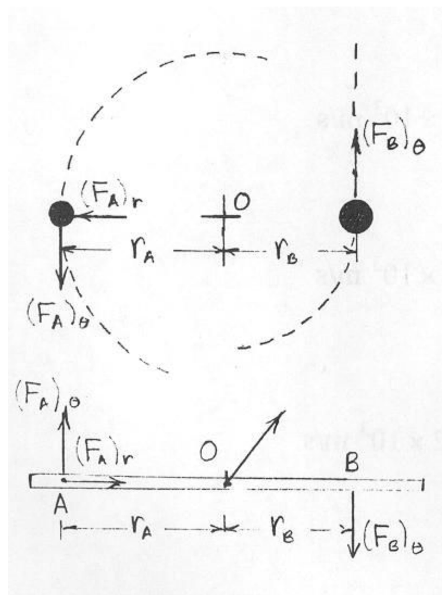
$$+\circlearrowleft\ \Sigma M_O = 0: \quad r_A(F_A)_\theta + r_B(F_B)_\theta = r_A m_A(a_A)_\theta + r_B m_B(a_B)_\theta = 0$$

$$r_A m_A r_A \ddot{\theta} + r_B m_B\left(r_B\ddot{\theta} + 2\dot{r}_B\dot{\theta}\right) = 0$$

$$\ddot{\theta} = \frac{-2r_B\dot{r}_B\dot{\theta}}{m_A r_A^2 + m_B r_B^2}$$

At $t = 0$, $\dot{r}_B = 0$ so that $\ddot{\theta} = 0$.

From Eq. (1), $(a_B)_\theta = 0$ ◀



The solution appropriately presents this problem in the context of particle dynamics, by providing Free Body Diagrams of each particle. In particular, the transverse force on particle B is properly not assumed to equal zero, even though this happens to be true (interestingly, this assumption was made in an earlier edition of the Solution Manual, and we applaud the authors for revising the solution). The assumption that the rod is massless (highlighted by a box added to Figure 6) is crucial in demonstrating that the transverse acceleration of particle B is zero. The solution could be slightly improved by further pointing out that this implies that the transverse force on B is zero; even though this is obvious, it calls attention to the fact that the force was not presumed to be zero.

This problem also provides an opportunity for multiple-method problem-solving, for it can also be solved by first considering the entire system. A Free Body Diagram of the entire system reveals that the rate of change of angular momentum about the vertical shaft is zero, from which the result in the given solution follows. This problem can thus be repeated in a later section on dynamics of systems.

One further technical comment: one could argue that the use of the massless rigid *body* in the given solution (Figure 6) is technically out of bounds for a chapter on *particle* dynamics. Even though students have already been exposed to statics of rigid bodies, the dynamic treatment of rigid bodies – even if massless – has not yet been presented. The treatment of massless cables is more palatable for a chapter on particle dynamics, since for a given segment of cable, the forces are collinear, and hence pass through a common point.

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