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Resistors, Capacitors and Inductors Are Not as They Appear

Dr. Paul Benjamin Crilly, U.S. Coast Guard Academy

Paul Crilly is a Professor of Electrical Engineering at the United States Coast Guard Academy. He received his Ph.D. from New Mexico State University, his M. S. and B.S. degrees at Rensselaer Polytechnic Institute, all in Electrical Engineering. He was previously an Associate Professor of Electrical and Computer Engineering at the University of Tennessee and was a Development Engineer at the Hewlett Packard Company. His areas of interest include laboratory development, antennas, wireless communications, signal processing, and instrumentation.

Dr. Tooran Emami, U.S. Coast Guard Academy

Tooran Emami is an associate professor of Electrical Engineering at the U. S. Coast Guard Academy. She received M.S. and Ph.D. degrees in Electrical Engineering from Wichita State University in 2006 and 2009, respectively. Dr. Emami was an adjunct faculty member of the Department of Electrical Engineering and Computer Science at Wichita State University for three semesters. Her research interests are Proportional Integral Derivative (PID) controllers, robust control, time delay, compensator design, and filter design applications, for continuous-time and discrete-time systems.

Abstract

This paper presents an analysis of the basic elements of an electrical circuit in order that undergraduate engineering students will experience, and thereby understand the non-ideal nature of electrical components. It is motivated by the fact that many electrical engineering students, after they have completed their first circuits course believe that the assigned or measured values of a given resistor (R), inductor (L) or capacitor (C) are within the manufacturer's stated tolerances and are in fact pure Rs, Ls and Cs. They also assume these components when connected to form a circuit will behave as a lumped parameter, time invariant system whose response can be predicted using a mathematical model based on measured or stated values. This paper demonstrates a practical experience that shows this is not always the case at frequencies above a few MHz. In a junior level laboratory, students discover that a coil will have a resonant frequency that is caused by parasitic or stray capacitance, that a resistor or capacitor lead whose length, l, is greater than 0.01 times the wavelength (i.e. $l > 0.01\lambda$) will have a significant inductive component that cannot be ignored, and that an iron core choke's inductance is affected by its input signal's frequency. The objective is to provide some practical, hands on experiences so that students can experience for themselves that resistors, inductors and capacitors are not at all what they seem and thereby develop deeper insight into the behavior of electrical components. The ultimate goal of this understanding is to make them more competent at design and analysis of electrical systems.

Introduction

Occam's Razor states that the simplest explanation is best when explaining a system's behavior. In the case of electrical engineering education, students start out with elementary circuit theory with ideal components in order to characterize and analyze a time invariant, lumped parameter system. As they design, and test these systems, they quickly find out that basic circuit theory is inadequate and thus they need more sophisticated methods to account for higher order effects. For example, at low frequencies, the simplest explanation using basic theory may be adequate. Whereas, this is definitely not the case at higher frequencies where an ordinary wire starts to behave as a transmission line; that a resistor and capacitor will also have an inductive component, and an inductor, due to stray capacitance may exhibit resonance. In other words, there is no such thing as a pure resistor, inductor or capacitor. As the student progresses, they then take upper division courses such as electromagnetic theory and quantum mechanics to learn about these higher order effects.

This lesson was especially brought to home by the first author when using a toroidal inductor as a band-pass-filter for a 5 MHz radio circuit. The inductance value of a particular toroid as measured using a standard LCR (inductor, capacitor, and resistor) meter operating at 1 kHz was vastly different when used in a 5 MHz circuit. The difference was way beyond simple experimental error. Investigation showed that the relative permeability of the iron core was frequency dependent and thus its inductance was not simply a function of geometry, number of turns, and turn density. For this reason, choke manufacturers specify frequencies where the rated inductances are valid.

The lessons learned in this paper are not just for those who will do radio frequency (RF) engineering, but apply to other Electrical Engineering (EE) areas as well. In the case of modern high speed digital systems where clock frequencies are in excess of 200 MHz, a seemingly short lead length may have to be treated as a transmission line to in order to minimize glitches caused by standing waves. Similarly, in order for bypass capacitors to effectively function, their lead lengths must be a minute fraction of a wavelength.

In this paper we describe some simple experiments that demonstrate the limitations of basic circuit theory and thereby enable students to experience a more accurate and higher order model of a resistor, capacitor and inductor. Our objective is to develop in students more critical thinking and an instinctive sense of when a component no longer functions as expected. The techniques described have been used in our EE program's electromagnetics course at the U.S. Coast Guard Academy. However we may incorporate these into our circuits and digital design courses. Our focus will be primarily on the parasitic reactive components of resistors, capacitors and inductors as well as describe the frequency dependence on the relative permeability of an iron core choke inductor.

Previous work and theory

For many EE students, their first introduction to the higher order effects on circuit behavior is in their first electromagnetics course where they learn about transmission lines, distributed parameter systems, and Maxwell's equations. Ulaby et. al. [1] does an excellent job of introducing these topics at the theoretical level and reinforces the theory with practical engineering problems. But as often the case, students really only understand or know about these higher order effects when they experience them while testing their circuits.

In circuits whose wire lengths are a significant portion of a wavelength, it may be necessary to account for transmission line effects [2], [3]. Patel [4] provides an in-depth method of calculating the inductance of a straight conductor with finite length and Wyatt [5] describes a model for a resistor that includes parasitic inductances and capacitances.

Kollman [6] and Johnson [7] discuss the parasitic inductances associated with bypass capacitors and under what conditions these have to be taken into account during the design phase. A rule of thumb often used by engineers is that a resistor or capacitor takes on inductance when its lead length, $l > 0.01\lambda$. Note the purpose of a bypass or decoupling capacitor is to provide a low impedance path for high frequency transients and thus suppress them. Thus having an unexpected inductor in series, may prevent full suppression of these transients and thereby cause the circuit to malfunction. Similarly, in high speed amplifier circuits, the bypass or decoupling capacitor serves to isolate one stage from another and thus prevent spurious signals from one stage affecting another stage (e.g. positive feedback).

While parasitic inductance may prevent the proper operation of high speed digital and analog circuits, parasitic capacitance associated with an inductor will cause unexpected resonances, or in the case of a tuned LC RF circuit, may have to be accounted for when implementing a circuit to achieve a specified resonant frequency. Toledo [8], and Anicin et. al. [9] discusses parasitic capacitances in RF coils, and Massarini [9] describes parasitic capacitance in power circuits.

Cohen [11] and Clark [12] discussed the effects of frequency on the inductance and resistance of solenoid (i.e. iron core) coils.

Experimental procedure and results

The following equipment is used for the experimental procedure: (a) Tenma model # 72-10465 LCR meter, (b) Agilent DSO-X-4164A O-Scope, (c) Agilent model 33220A Waveform generator, and (d) Agilent 9912A portable RF Analyzer. The equipment and measurement setups are shown in the Appendix.

In order to confirm their stated values and provide a benchmark, the Tenma LCR meter functions to measure the values of the various resistors, inductors and capacitors used in our experiments. It was found that the measured values using the Tenma were within the manufacturer's tolerances. Note that the Tenma operates at either 100 kHz or 300 kHz.

Parasitic capacitance of a coil. During construction of radio circuits, it was observed that the tuner coil acts as a parallel resonant LC circuit with a resonant frequency of f_0 even when there is no external capacitor connected in parallel. Thus the RF coil in Figure 1 can be modeled as shown in Figure 2.

Figure 1. RF coil

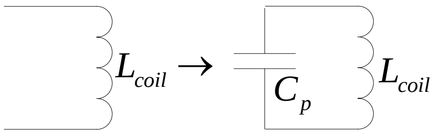

Figure 2. Parallel LC circuit caused by wire wound inductor having parasitic capacitance.

A more sophisticated model of the circuit in Figure 2 would also include a resistor in series with the inductor. However, this model is adequate to explain and observe parallel LC resonance.

The Tenma LCR meter was used to measure the inductance of the coil with the result being $L_{coil} = 59\mu H$. Identical results were obtained at both 100 kHz and 300 kHz settings of the Tenma. To determine the resonant frequency of the LC circuit, we connected the waveform generator to the coil via a series 5 pf blocking capacitor and then varied the generator's frequency to get a peak response on the scope. The 5 pf capacitor [1] was necessary to isolate the generator from the LC circuit and thereby minimize the generator's effect on the LC circuit's resonant frequency. After noting the resonant frequency, f_0, the parasitic capacitance, C_p is determined using the below equations.

$$f_0 = \frac{1}{2\pi\sqrt{(C_p)L_{coil}}} \Rightarrow C_p = \frac{1}{59\times10^{-6}\left(f_0\times2\pi\right)^2} \tag{1a}$$

To further validate our measurement, an additional 95 pf of additional parallel capacitance (C_i) is added to the coil, re-measured the value of f_0 and then re-calculated C_p using Eq. (1b).

$$f_0 = \frac{1}{2\pi\sqrt{(C_p+C_i)L_{coil}}} \Rightarrow C_p = \frac{1}{59\times10^{-6}\left(f_0\times2\pi\right)^2} - C_i \tag{1b}$$

The results are tabulated below.

Parallel capacitance $(C_p + C_i)$	Measured f_0 (MHz)	$C_{p(calculated)}$ (pf)
C_p	1.60	168
$C_p +$ 95 pf	1.27	171

As observed from the table, with or without the extra capacitor, the value parasitic capacitance varied from $C_p \cong 168 \rightarrow 171$ pf. The variation in C_p is probably due to the LC network not having a having a sharp resonance point (i.e. high Q).

Parasitic inductance due to excessive lead length. Theory states that any length of wire can be modeled as an inductance, and the rule of thumb for design engineers is lengths become excessive at $l > 0.01\lambda$. Hence; Figure 3 illustrates an equivalent high frequency model for a resistor.

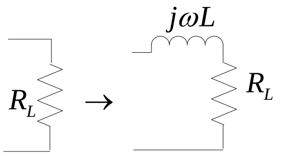

Figure 3. High frequency model of a resistor consisting of an inductor in series with a pure resistor.

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[1] The 5 pf capacitor consists of 2 pieces of wire, one inch in length twisted together to form a "gimmick" capacitor.

Again, a more sophisticated model of the circuit in Figure 3 would also include parallel capacitance across the inductor. But, in this case, the additional capacitance is small enough to be neglected.

As Figure 3 illustrates, there is a frequency where the resistor goes from being purely resistive to being a resistor in series with an inductor. That is there is some frequency where we can no longer ignore the inductive reactance. Stated mathematically, $Z_L = R_L \rightarrow Z_L = j\omega L + R_L$.

To determine when the load is no longer a pure resistance, we connect the load resistor is connected to the 9912A RF analyzer operating in CAT mode, and then measure its standing wave ratio (*SWR*). SWR is calculated as:

$$SWR = \frac{1 + \left| \dfrac{Z_L / Z_0 - 1}{Z_L / Z_0 + 1} \right|}{1 - \left| \dfrac{Z_L / Z_0 - 1}{Z_L / Z_0 + 1} \right|} \tag{2}$$

Where Z_0 is the characteristic impedance of the transmission line or the generator's output impedance.

At relatively low frequencies where there is minimal reactance due to stray reactance, the results show $Z_L = Z_0 = R_L \Rightarrow SWR \cong 1,$ and thus the load is a perfect match, hence no observable reactive component. On the other hand, as the input frequency is increased we get an $SWR > 1$ indicating that the load has a noticeable amount of inductive reactance. In this case, it was arbitrarily decided that the inductive reactance is significant enough when the $SWR \geq 2.$ We chose a $SWR = 2$ threshold since many radio engineers consider an SWR less than 2 to be an acceptable match.

The testing consisted of the following: (a) Obtain an almost perfect resistive load with nearly zero lead lengths. This will be the standard. (b) Test other resistors with lead lengths of 15 mm and 60 mm to determine at what frequency the SWR starts to exceed 2. The resistors used in the experiment are shown in Figure 3 with the results tabulated below.

Figure 3. Load resistors used to determine parasitic inductance. Lead lengths are 0 mm, 15 mm and 60 mm.

Lead length, l	Frequency (MHz) Where $SWR \geq 2$	Wavelength, λ	lead length/wavelength, l / λ
0 mm	1772	-	- -
60 mm	100	3.00	0.060/3=0.02
15 mm	432	0.69	0.015/0.69=0.022

You will note that the lead length in which the SWR exceeds 2 corresponds to a lead length of $l \cong 0.02\lambda$. Thus the rule of thumb of lead length being $l > 0.01\lambda$ is validated.

Similarly, capacitors with excessive lead lengths can also exhibit an inductive component. This is especially noteworthy in the case of bypass capacitors because the objective is to present a low impedance path to high frequency components and thus eliminate (i.e. short circuit) them.

Inductance as a function of frequency: Although air core inductors may have parasitic capacitances, their inductance is independent of the operating frequency. This is generally not the case for iron core choke inductors. This presents a design challenge for tuned RF circuits in the 3-30 MHz range. This is also why the manufacturer will specify a frequency range where the stated inductance or relative permeability of the core is valid.

Using the Tenma LCR meter operating at 100 kHz, we measured the values of our iron core inductor to be 308 μH and measured our capacitors to have values of 953, 300 and 208 pf. These components are shown in Figure 4. The measured values were within the manufacturers tolerances.

Figure 4. Iron core choke inductor with the “gimmick capacitor” and the a 300 pf capacitor.

To determine to what degree the inductance of an iron core choke is affected by the operating frequency, a parallel LC circuit is configured using the 308 μH choke with various capacitors of known value, and then measured the corresponding resonant frequencies as we did with the air core inductor experiment previously described. We then determined the effective inductance of our choke by measuring the circuit's resonant frequency and then using Eq. (3), the effective inductance of the choke was determined when the operating at the measured resonant frequency. The results are tabulated below.

$$f_0 = \frac{1}{2\pi\sqrt{C_{known}L_{choke}}} \Rightarrow L_{choke} = \frac{1}{C_{known}\left(f_0 \times 2\pi\right)^2} \qquad (3)$$

Parallel capacitance $C_{measured\_LCR\_meter}$ (pf)	$L_{measured\_LCR\_meter}$ (μH)	Calculated f_0 (kHz)	Measured f_0 (kHz)	$L_{choke\_from\ f_0}$ (μH)
953	308[1]	293	293	308
300	308[1]	524	444	428
208	308[1]	629	496	495

[1] The nominal value as stated by the manufacturer is 300 uH.

As readily observed, at frequencies below 300 kHz, the manufacturers stated value of inductance is within their stated tolerance. However, this is not the case at frequencies above 444 kHz where the measured inductance is well in excess of the 300 μH expected value.

Conclusion

This paper described some simple experiments so students can experience the non-ideal nature of real electrical components. The instrumentation is commonly available in many if not most undergraduate EE programs. From these experiments, students in our electromagnetics course have observed and experienced resonance and parasitic capacitance of an air-core RF coil, the parasitic reactance of a resistor and how the relative permeability of an iron core reactor is no longer constant with frequency. Students in this course have gained a greater understanding and greater insight about the higher order effects that affect a system's response. These experiments could be incorporated in the latter portions of the circuits and digital design courses.

Future work will show to what degree R's, L's and C's are affected by ambient temperature, and how the value capacitors that use a thin film dielectric are also affected by the applied voltage and frequency.

References

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Appendix

The below figures illustrate the various experimental setups for measuring the non-ideal behavior of resistors, capacitors and inductors.

Figure A-1: Tenma Model 72-10465 LCR meter used to measure component values.

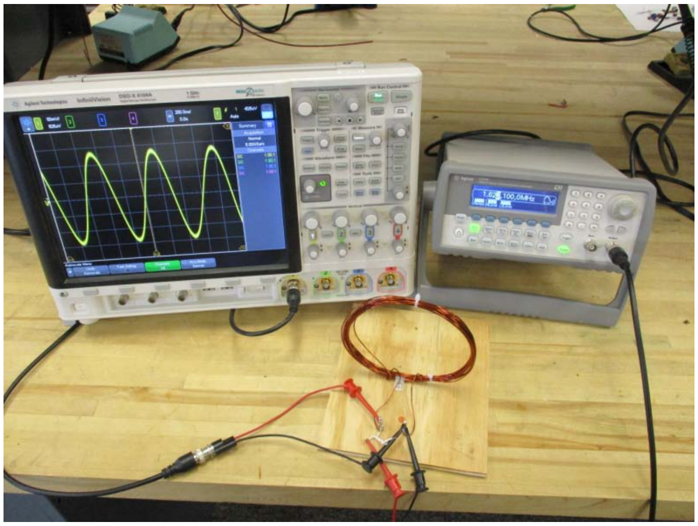

Figure A-2: Experimental setup to measure the parasitic capacitance of an RF coil by measuring its resonant frequency.

Figure A-3: Experimental setup used to determine parasitic reactance of a resistor using an RF analyzer that measures SWR.

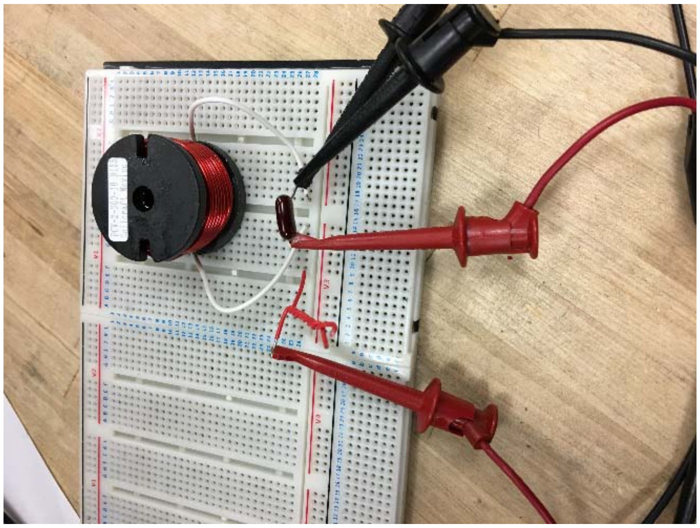

Figure Figure A-2: Experimental setup to determine how inductance of an iron core choke is affected by its operating frequency by measuring its resonant frequencies when combined with various capacitors.