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SAGE off Stage: Teaching Electromagnetics with Symbolic Computation Tool

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SAGE off Stage: Teaching Electromagnetics with a Symbolic Computation Tool

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Abstract

Electromagnetics is often considered the most challenging undergraduate course for many students, and there are several challenges: mathematics, imagination, as well as new concepts and approaches. Instructors and students alike have trouble in dealing with so many new issues in a short period of time. Among these challenges, mathematics is the toughest, as it takes considerable time and effort to learn. In our school, the background of students is very diverse, and some of them even have trouble in doing simple integrals. Fortunately, technology comes to the rescue. SAGE© is an open source symbolic computation tool, and it can be used for symbolic derivation, so every student can find the derivative, integral, and even gradient of functions easily. In addition, it also supports programming in Python© style. With the challenge of mathematics alleviated, more time is available to cope with the challenges of other issues, such as new concepts and approaches. At the end of the semester, students were tested with *Conceptual Survey of Electricity and Magnetism*, as well as surveyed on experience using SAGE©. The results were pretty encouraging.

I. Introduction

Electromagnetics is a special course in the curriculum of undergraduate engineering majors. Before learning this course, students are used to solving problems with algebra and time-related differential equations, where only a couple of variables are involved. However, field concepts play the major role in *Electromagnetics*, where 3D distributions of scalar and vector fields are involved and spatial differential equations are needed. Although vector calculus is a prerequisite course, in our school only a small portion of students can really understand the concepts and use the theorems to solve problems, and it is also a challenge for students in many other schools.[1]

Besides the challenge in mathematics, students also find it difficult to imagine the spatial distribution of a field in space. For over twenty years, there has been a consistent effort to use software tools to alleviate this problem.[2] We adopted this approach several years ago, but it was found that some students became technology dependent and lost interest in learning the theories of this course.[3] Therefore, we refrained from introducing very powerful electromagnetic field simulators to the students, which could become a distraction for their study.

There is a middle path in engaging technology in education, where students need to play the leading role and computer software tools play the supporting role. One such tool is Mathematica©, which has the capability of symbolic derivation and powerful functionality of graphic display.[4] However, the price tag of this software is pretty high, and thus it is not accessible for the students in our department. Fortunately, there is an open source software available called SAGE©, and its online version is available to everyone even without the trouble of downloading and installation. In addition, Python© is the required computer language in our department, so students here have no trouble in programming in SAGE©.

II. Symbolic Computation Tool

Since the early time in human history calculation with numbers has been an important skill, and its early applications include management and trading. However, human's capability in this area is not very strong, and thus various tools were invented. The first widely used calculation tool was the abacus, which can do the basic arithmetic operations quite efficiently. In the seventeenth century a mechanical calculator was built by Blaise Pascal, but its functionality was confined to adding and subtracting. Many science and engineering applications need to deal with multiplication and division operations, so the slide rule was invented, which can convert these operations to adding and subtracting. Slide rule was widely used until the 1960s, when pocket sized electronic calculators became available.

Now students in elementary school no longer spend so much time practicing their basic arithmetic skills, the emphasis of the mathematics curriculum is shifted to more advanced skills, such as symbolic derivations. Now comes another revolution, even such operations can also be done by computer. Mathematica© is the first widely used tool for symbolic computation, now SAGE© provides such a tool for everybody with no cost. Unfortunately, the math curriculum in most universities is not reformed accordingly, and the result is the very high attrition rate in the college of engineering due to the demanding mathematical skill.

We are a regional university with moderately selective admission standards. Consequently, the range of academic ability of our engineering students is quite diverse. When they take the course of *Electromagnetics*, a lot of students are unable to calculate the area of a circle by integral, let alone the advanced operations in vector calculus. Therefore, the introduction of a symbolic computation tool is essential to overcome the insurmountable barrier in mathematics. In addition, with less time and effort on mathematics, students can pay more attention on concepts, theorems and approaches in solving problems.

The textbook used in this course was *Electromagnetics* written by Branislav M. Notaros, and we covered the first nine chapters in this three credit hour course. The first chapter is on electrostatic field in free space, and three different approaches in finding electric fields are covered: Coulomb's law, gradient of potential and Gauss' law. Students have learned these laws in General Physics, but the depth in this course is deeper. In this case, the major challenge was on mathematics, so SAGE© was very helpful to the students. In the past, students with poor mathematics background usually lost confidence at this point, and then they would rely on solution manual to finish the homework. Now these students can overcome the challenge in mathematics with SAGE©, and everybody is at a level playing field. In addition, students' attention was shifted to the physics, instead of the mathematics.

III. Example

One of the homework assignments is the calculation of the electric field distribution of a line charge with limited length. Instead of using Coulomb's law directly, one can calculate the electric potential first and then find the gradient of this function. Although the closed form solution can be derived by hand calculation, working out the integral is pretty challenging, let alone the gradient. However, this can be done easily by using SAGE, and the code is shown below (the linear charge density and the factor of 1/(4πε) are skipped). The charges are uniformly distributed along z-axis and its length is 2 centimeters, with the coordinates of the two ends at -0.01 and 0.01, respectively.

```
sage: d = 0.01
sage: x,y,z,zp = var('x, y, z, zp')              # zp is source variable for integral.
sage: func = 1/sqrt(x^2 + y^2 +(z-zp)^2)
sage: assume(x^2 + y^2 > 0)
sage: assume(z != 0)
sage: V(x, y, z) = integral(func, (zp, -d, d))   # Electric potential function
sage: show (V)                                   # Show the expression.
sage: E_F = -V.gradient()                        # Electric field function in vector form
sage: plot_vector_field3d(E_F, (x, 0.01, 10), (y, 0.01, 10), z(0.01, 10))
```

The derived electric potential function is shown below, and the expression of electric field is too complicated to present here. However, the field distribution in space is displayed underneath.

$$(x, y, z) \mapsto \operatorname{arcsinh}\left(\frac{\sqrt{4\,x^2 + 4\,y^2}\left(0.02\,x^2 + 0.02\,y^2 + 2\left(x^2 + y^2\right)z\right)}{4\left(x^4 + 2\,x^2y^2 + y^4\right)}\right) - \operatorname{arcsinh}\left(\frac{\sqrt{4\,x^2 + 4\,y^2}\left(-0.02\,x^2 - 0.02\,y^2 + 2\left(x^2 + y^2\right)z\right)}{4\left(x^4 + 2\,x^2y^2 + y^4\right)}\right)$$

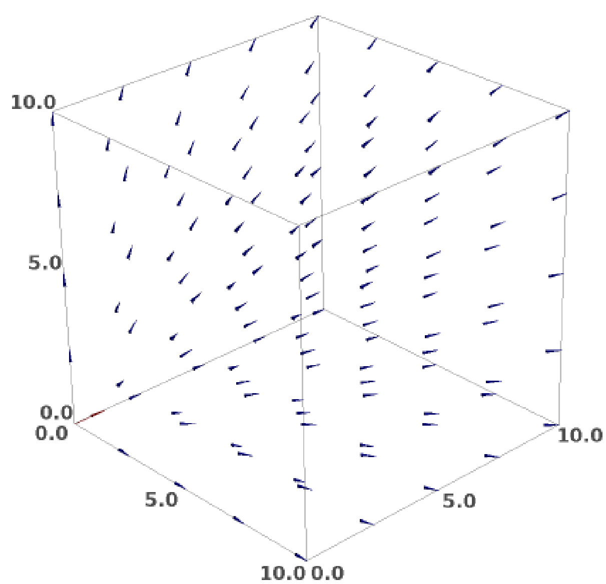

Fig. 1. Electric field distribution in space.

IV. Assessment Result

Students are assessed in two different ways. First, the *Conceptual Survey of Electricity and Magnetism* [5] was administered at the end of the semester, which can be used as an objective evaluation of learning effectiveness. Second, students were surveyed on the adoption of SAGE©.

1. Testing Result

The *Conceptual Survey of Electricity and Magnetism* has 32 questions, 20 students took the test and the average is 19.1; the individual scores of each student are shown in Fig. 2. The enrollment of this class is 21 and they are from different programs: 7 in Physics, 4 in engineering physics-electrical application, 8 in engineering physics-mechanical application, 1 in biology, and 1 in computer science (exchange student from Mexico). Unfortunately, a pre-test at the beginning of the semester was not taken, so it is a little hard to make a comparison. However, this survey was done in the 2009 and 2011, and the class average post-test scores were 15.1 and 21.1, respectively. The average in 2013 is much higher than the result in 2009, but a little lower than the result in 2011. The pre-test average scores in 2009 and 2011 are 10.6 and 12.7, respectively. Even if the higher score is used as the projected pre-test score, there is a significant increase in the average, which indicates that students did make substantial progress in conceptual understanding. As this course is offered every other year, the survey results in even years are not available.

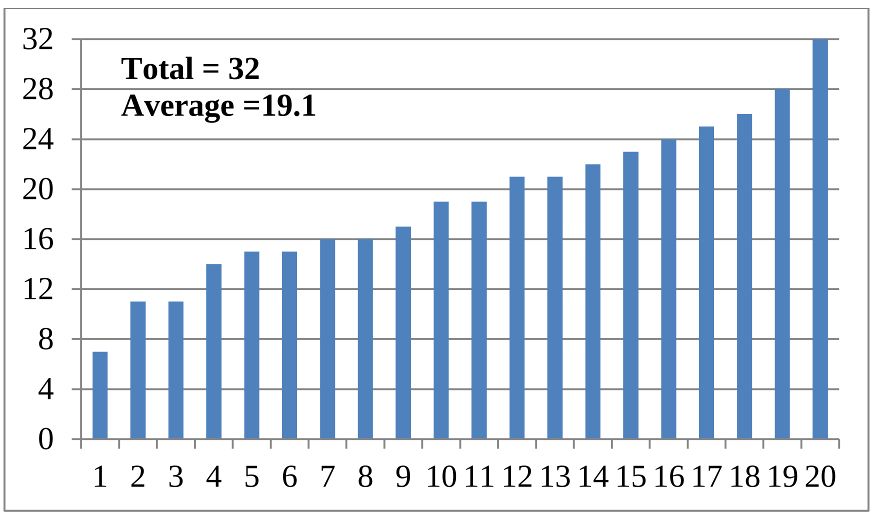

Fig. 2. Results of the *Comceptual Survey of Electricity and Magnetism*.

2. Subjective Perception

Students were surveyed at the end of the semester on their opinion about the helpfulness of SAGE© in learning this course. There are five options: 1) very helpful, 2) of some help, 3) neutral, 4) not helpful, 5) useless. The survey result is shown in Fig. 3; the overall response is on the positive side, though only two students consider it very helpful. Just like learning any new software tool, initially one feels rather awkward and stressful, and it takes time of practice until one feels at ease in using it. Due to the limited usage, most students are still in the learning phase, and the full benefit was probably not realized.

Besides the multiple choice questions, students were also asked to write some comments on using SAGE© in this course. A student with the option 2) wrote: "I was able to focus more on the concepts and not have to deal with the difficult math." A student with the option 3) wrote: "While it could be helpful if we were more familiar with the command prompts, I feel that it was not as helpful because it took me more time trying to put the problem in the correct format into SAGE than it did to do the problem by hand." Another student with the option 4) wrote: "A few times when we had very long, complicated integrations, it was helpful, and I think we did a few graphs where it came in handy. But overall I didn't use it much for the class, and when I did I had trouble getting it to do what I wanted."

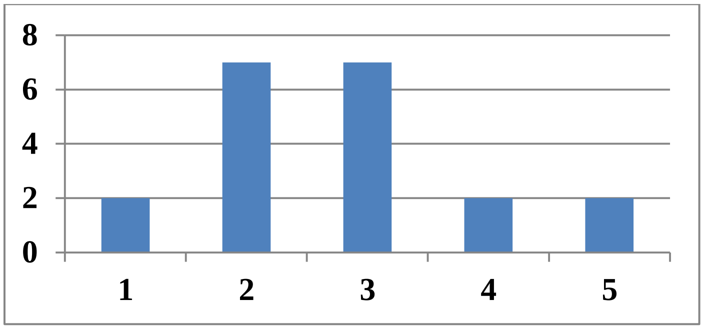

Fig. 3. Survey result on using SAGE.

V. Conclusion

Just like we rely on a calculator to do numerical calculations, it is suggested that we will rely on symbolic computing tools in derivations in the future. Because mathematics is a serious challenge for students learning Electromagnetics in our school, SAGE© was introduced in the fall 2013 offering of the course. The survey results show that its overall effect was positive, though not as significant as it might have if students could have more experience with SAGE©. If SAGE© or other symbolic computation tools were introduced earlier in the mathematics curriculum, we expect the benefit would be much more substantial.

Reference

[1] Johanna Leppavirta, "The impact of mathematics anxiety on the performance of students of electromagnetics," *Journal of Engineering Education*, vol. **100**, no. 3, pp. 424-443, 2011.

[2] Magdy F. Iskander, "Computer-based electromagnetic education," *IEEE Trans. Microwave Theory Tech*., vol. 41, pp. 920-931, 1993.

[3] Yumin Zhang and David K. Probst, "Blessing or curse, teaching with CAD software," *117th Annual ASEE Conference*, Louisville, Kentucky, June 20 - 23, 2010.

[4] Tianxia Zhao, "Introducing math software and EM software to enhance the vector field understanding in EM class," *114th Annual ASEE Conference*, Honolulu, Hawaii, June 24-27, 2007.

[5] David P. Maloney, Thomas L. O'Kuma, Curtis J. Hieggelke, Alan Van Heuvelen, "Surveying students' conceptual knowledge of electricity and magnetism," *Phys. Educ. Res. Am. J. Physics. Suppl.* **69**(7), pp. S12-S23, July 2001.

[6] Yumin Zhang and David K. Probst, "Stepping stones in learning electromagnetics," *119th Annual ASEE Conference*, San Antonio, Texas, June 10 - 13, 2012.