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Solving Schrodinger Equation with Electronics Engineering Students and Analyzing Their Feedback

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Solving Schrodinger equation using to Electronics Engineering students and analyzing their feedback.

Abstract:

The Importance of teaching Quantum Mechanics to Engineering Students was discussed in last year's Conference in Seattle, WA, so in this present research the author has made a survey of how this material was perceived by senior year Electronics Engineering Technology students. It was taught to a class of 22 students who took their 2nd Physics class, in their senior year. The outcomes analysis show that for most advanced or even average students majoring in Electrical Engineering programs, the subject is very consumable and exciting and understandable so long as the calculus by which the equation is solved is kept at minimal level.

It was also thought that the possibility of providing a 2 credit hours course entitled "Basic Quantum Mechanics ", or could be given any other course label, even though as an elective course, to Engineering student would be very beneficial. This latter will be discussed in the presentation with audience at the conference.

Introduction :

In quantum mechanics, the Schrodinger Equation is a partial differential equation that describes how the motion of subatomic particles as a wave function. Teaching quantum mechanics on an introductory level was suggested by Rainer Muller and Hartmut Wiesner[2] but their approach was so detailed and long and involved mathematically that was not suitable for Engineering Students and rather was designed for physics majors. Last year at ASEE conference a paper was presented by Bryan A Lynn[3] et.al which was investigating learning quantum mechanics via computer simulations. In this research also there was no emphasis on student's perception and comprehension of the concept of quantum mechanics. There are two types of solutions to Schrodinger Equation. One solution is for Time Dependent Equation and the other solution for Time – Independent Equation. Here the solution of Time – Independent Equation is considered with a simple mathematical procedure that Student with the Basic Calculus knowledge would understand. In fact the solution would yield the quantum states that changes with energy.

In classical mechanics, Newton's 2nd Law, (F = m*a) is used to mathematically predict what is the system is doing given some initial conditions. In quantum mechanics, analogous to Newton's law, Schrodinger's Equation would give the state of the particle at the given potential and kinetic energy. It is not a simple algebraic equation. It is a linear partial differential equation that students know how to solve, given that they have taken required Calculus courses.

Time – Independent Equation[4]:

Recall from Last year paper[1] that Time – Independent Schrodinger equation is obtained directly from the applying De Broglie principle to a wave equation:

From the very basic classical mechanics, General Physics I Class students already know the Work – Kinetic Energy Theory:

$$W = \Delta k = -\Delta U \qquad (1)$$

and at the same time for all conservative Forces we have:

$$\mathrm{F} = -\frac{\partial u}{dx} \qquad (2)$$

Where ΔU is change in Potential Energy

Now Newton's second Law states that: . $\frac{\partial^2 \mathrm{x}}{\mathrm{d}t^2} = \frac{\mathrm{F}}{\mathrm{m}}$ (3)

So Combining the 2 formula: we have

$$\frac{\partial^2 \mathrm{x}}{\mathrm{d}t^2} = \frac{\mathrm{F}}{\mathrm{m}} = -\frac{1}{m}\frac{\partial u}{dx} \qquad (4)$$

This is the basis of Schrodinger Equation where Erwin Schrodinger made use of De Broglie Principal and obtained All he did , took faith in De Broglie Theorem[6] and considered the motion of the particles to be shown by a sine wave , or what we call a harmonic wave with wavelength defined by De Broglie :

$$\psi(x) = A\, Sin\ \frac{2\pi}{\lambda} x \qquad (5)$$

And when taking the 2nd derivative of the above equation:

$$\frac{\partial^2 \psi}{\mathrm{d}x^2} = -\ \frac{4\pi^2}{\lambda^2}\ \psi(\mathrm{x}) \qquad (6)$$

But by introducing $\hbar = \frac{h}{2\pi}$ and reminding De Broglie Wavelength[6] $\lambda = \frac{h}{mv}$

Then, $\hbar^2 = h^2/4\pi^2$ and at the same time $\lambda^2 = \frac{h^2}{(mv)^2}$ and $\frac{4\pi^2}{\lambda^2} = \frac{2km}{\hbar^2}$

Easily we found that Schrodinger Equation is obtained which is:

$$\frac{\partial^2 \psi}{\mathrm{d}x^2} = - \frac{2m}{\hbar^2} (\mathrm{K} - \mathrm{V})\psi(\mathrm{x}) \qquad (7)$$

The right hand side is the total energy of the particle and the left hand side is the 2nd derivative of the wave function with respect to space. The interesting point is that Newton's 2nd law, (4) also involves the 2nd derivative of the position of the particle but with respect to time. The right hand side in Newton's 2nd law is the gradient of potential energy and the right hand side of Schrodinger equation is the total mechanical energy, potential and kinetic.

By solving this differential equation (7) we will find the solutions of the wave function whose squared value , $|\psi|^2$ would show the probability of the position of the particle at position x. We already know that in classical mechanics would give the location and well defined position of the particles. Some times the potential energy is moved to the left hand side and this equations is shown as:

$$E\Psi(x) = [\frac{-\hbar}{2\mu} \nabla^2 + V(x)]\psi(x) \qquad (8)$$

As you see in this equation the Kinetic Energy is shown by "*E*". The right hand side operator is called Hamiltonian and is shown by "*H*", so at times the Schrodinger Equations is shown simply by:

$$E\boldsymbol{\psi}(\mathrm{x}) = H\boldsymbol{\psi}(\mathrm{x}) \qquad (9)$$

The Solution:

Engineering students at their junior year need to take calculus II class which deals with solving Differential Equations. They are already familiar with solving linear homogeneous and non – homogeneous differential equation. The above time-Independent equation is a second order homogeneous differential equation, when written as equation (7) and the solution is as follows which is really the steady state solution:

In equation (7) in absence of Potential energy, V = 0, which means the particle is not under constraint and said to be free particle solution:

$$\frac{\partial^2 \psi}{\mathrm{d}x^2} + \frac{2mE}{\hbar^2} \psi(\mathrm{x}) = 0 \qquad (10)$$

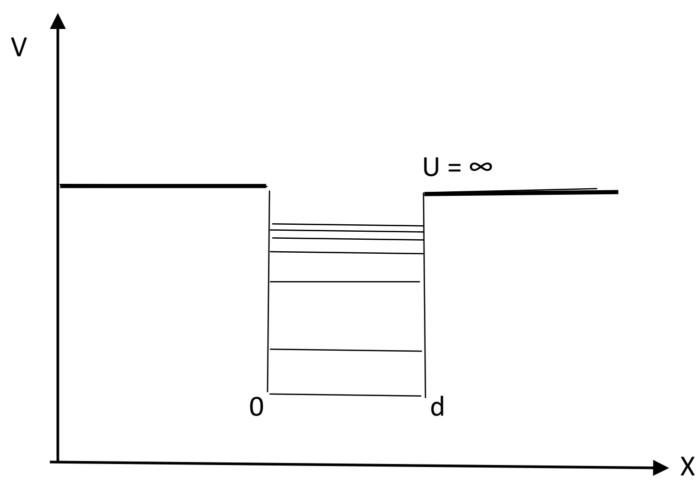
The solution is wave function which is going to the right and left:

$$\psi(x) = A\, e^{-ikx} + Be^{ikx} \qquad (11)$$

Where in time dependent domain is written as

$$\psi(x) = A\, e^{-i(\omega t+kx)} + Be^{i(\omega t-kx)} \qquad (12)$$

This is a representative of a wave that propagates to the left and right indefinitely , that why in quantum mechanics the position of a free particle is not known unless there is a constraint with potential well. This is the most important part that engineering students should pay attention because they will find out what happens to the energy levels in an energy band gap. Now we look at the solution with potential U(x) over a distance d applied to the particle. This situation is called Particle in a box. The solution is standing waves that are given by:



Particle in a quantum well

$$\psi(x) = Ae^{-ikd} + Be^{ikd} = -2iA\, Sin\, kx$$

$$Considering\ that\ the\ initial\ condition\ exerts\ that:\quad at\ \ x = 0\ and\ \ x = d\ , No\ displacement\ , so$$

$\text{Sin}(K_n d) = 0$, $K_n d = n\pi$, $K_n = \frac{n\pi}{d} = (\frac{2mE_n}{\hbar^2})^{1/2}$

So it will yield the Energy levels as:

$$E_n = \frac{\pi^2 n^2 h^2}{2md^2} \qquad (13)$$

Normalization states that the probability of the existence of the particle over x = 0 and x = d must be 100% or: $\int_0^d \Psi\Psi^* dx = 1$

That would yield $A = i\sqrt{\frac{1}{2d}}$ and that means $-2iA = \sqrt{\frac{2}{d}}$

So our final solution is $\Psi_n(x) = \sqrt{\frac{2}{d}}\sin(\frac{n\pi x}{d})$

The square of this wave function give the probability of the position of the particle and there is still uncertainly depending on the momentum. This is known as the Heisenberg's uncertainty principle which is given by:

$$\Delta x * \Delta p \geq \hbar/2 \qquad (14)$$

The above simple solution and the energy levels obtained in a potential quantum well is the basis of the design of semiconductor's laser diodes. It seems that because quantum mechanics is not offered to engineering students they lack such a precious knowledge. As it was presented in last year conference all semiconductor lasers, LED's, Infrared detectors, optical fiber couplers, and so many other devices use this quantum well technology to produce desired emission.

Students Feed Back:

This solution was presented to senior Electronics Engineering Technology students last Semester.(Summer of 2015). The class consisted of 22 students in their final year attending, their College Physics II course, Phys320 Class. I have used the opportunity of the last week's course schedule in their syllabus which is allocated to Modern Physics. The introduction to quantum mechanics and the Solution of Schrodinger equation was very much welcomed by our students. So I decided to carry a survey. The survey included 9 questions asking graduating students if they are comfortable with the mathematics and understood the procedure and above all the outcome of the solution. Some of the questions in the survey are given below but are not limited to the followings:

- Would the conceptual understanding of QM would remain with you long after graduation?
- Would QM skill, if well developed, be useful in your Engineering Career?
- Would you think that QM should be taught in all Engineering disciplines programs?
- If a student can understand Basic QM mathematical formulations well, then would you think dealing with other physical concepts such as electromagnetism, thermodynamics, classical mechanics, etc…. be easier ?

- Would knowing QM be enabling you to communicate more effectively in any physical arguments?
- Don't you think that knowing QM as an intellectual tool would impress your interviewer and generally in your resume for job application would show an outstanding advantage?
- At some stages during physics class some students feel so overwhelmed by the comprehension of the behavior of small subatomic particles, i.e. QM that at times they consider changing their major, t how likely would you think this is with respect to yourself?

Only 0.85%, that is < 1% of the students were totally lost with comprehending the QM lesson.

For 6.5 % the concept and the mathematical method was somewhat comfortable but with some confusion.

22% could comfortably understand the material with no confusion.

71% of the students understood the concept and the mathematical procedure very comfortably and express willingness to take a basic QM course. In total 93% of the students were comfortable or very comfortable with the understanding the Schrodinger's equation and its solution. This last statistics was indicative that there shouldn't really be a gap in the understanding QM with in Engineering students and physics students. This gap is often felt by instructors who teach physics at an Engineering college. The above statistics indicates that students are eager to learn QM and they think that this will empower their understandings to discuss any physical subjects with less confusion, (question 4 above, which overwhelmingly received positive responses). Another question that was predominantly responding positively was question 6 above, where they were asked if knowing QM would empower them for a better job interview. The response was impressively positive.

In conclusion the above survey showed that it would be very appropriate to offer a short course to engineering students at final year on introductory quantum mechanics and that would be very welcomed even if it is an elective course. With the nano-technology progressing so rapidly in our industry and in our lives, certainly the research, design and fabricating technologists must bear the basic knowledge of QM. It is expected that an engineer working in the above said industry to master this subject. After all the basic information in laser technology is the energy levels in quantum potential well of a thin semiconductor layer,[1] (nano-layer). I would certainly welcome any suggestion from our colleagues at the colleges and universities nationwide and would be willing to participate in writing the curriculum and the syllabus for it. That would be a breakthrough for all our engineering graduates particularly for Electronics Engineering Technology graduates. I am looking forward to that time when quantum mechanics is not exclusively a property of Physics major students only.

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