AC 2010-900: SOURCES OF STUDENTS' DIFFICULTIES WITH COUPLES AND MOMENTS IN STATICS

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Sources of Students' Difficulties with Couples and Moments in Statics

Introduction

This study was conducted in response to observations of students' difficulties in understanding and applying moments and couples during our previous work in statics[1]. Over the past several years, we have been engaged in an interdisciplinary effort to help students understand the proper problem solving skills required to draw free body diagrams, an important element for developing a model of a real problem in statics. Through this effort, we have found that two key problems for students are the conceptual understanding of moments and couples and the ability to apply a moment equation during equilibrium analysis.

We believe that part of the confusion for students results from definitions that are inconsistent across textbooks and courses. In sampling a few textbooks, we found that the terms, moment, couple, and torque, have different definitions, some of which are in conflict with each other. For example, Meriam and Kraige define a couple as _the moment produced_ by two equal, opposite, and non-collinear forces[2]. Other texts define a couple as _a pair of equal, opposite, and non-collinear forces_ whose moment is defined as the moment of a couple[3,4]. This was the definition used as far back as 1874, and arrived at using analytical geometry[5]. Another issue that arises in terminology is the use of the same symbol (M) to represent both a moment and a couple across all texts that we sampled. This seems to create confusion about whether the reaction couple should be included in the sum of moments equation, and also what word should be used to describe it, and what symbol should be used to represent it.

Compounding the issues that students face in engineering mechanics courses, terms such as force, couple, and moment are used in everyday language with meanings that are often very different than the meaning in the engineering domain[6]. Adding to the confusion generated by these imprecise terminologies in statics are issues that arise in more advanced mechanics courses. In sampling a few mechanics of materials texts, for example, we found that the descriptions of torque, moment, and couple also vary. The internal reactions resisting 'twisting' in shafts are referred to as torques[7,8,9,10] and moments[7] and couples[9]. In the beam bending portion of the texts, in all cases the internal force is referred to as a 'bending moment'[7,8,9,10], however during bending stress calculations, the internal load is sometimes referred to as a couple[8,9]. This potential for confusion is further compounded in introductory physics courses where moments of forces are referred to as torques, a term usually reserved in mechanics for axial moments[11].

These issues have led us to try and understand why students are confused about the use of moments and couples, and how we might address this in future studies. Here we report on

our findings with regard to the ability of students to define and use moments and couples in a statics setting through the use of think aloud sessions and analysis.

Our goals for this study were:
- To understand the difficulties that students have with the concepts of couples and moments and in doing calculations involving couples and moments.

- To develop a set of exercises that would improve students understanding of couples and moments and their ability to use them correctly in calculations.

Methodology

Five students, three females and two males, majoring in engineering at a large university participated in this study. The students were recruited from a statics class late in the semester so they had completed nearly 13 weeks of the course by they time they participated in the think-aloud sessions. To encourage participation, students were offered a credit of 1% towards their final grade. We do not believe that this skewed participation in the project because the instructor placed a cap on the total extra credit each student could earn and this was a small extra credit opportunity amongst many others that were available throughout the semester for the students.

Individually each student was asked to solve problems involving moments and couples. Students were also asked questions about moments and couples and about their problem solving process. Because one of the goals of this study was to improve the materials that were being used, the students were also asked about what worked well for them and what could be improved. During the sessions the students' were recorded with a digital camcorder.

Before beginning the think-aloud session, each participant read and signed an informed consent that outlined the purpose of the study and the basic procedure. The interviewer then introduced the task by asking the students to think out loud while they were working on each problem. He offered to answer questions of clarification. He also indicated that there would be a chance to discuss the problems after the students had completed all of them.

After explaining the verbal protocol procedure, the experimenter gave participants the problems. After the students completed the problems the interviewer asked them questions to probe their understanding and to follow-up on statements made during the think-aloud problem solving. No time limit was imposed; the students could spend as much time as they needed in order to complete the problems. The average amount of time for the entire procedure was approximately 40 minutes.

Three engineering experts received copies of the recordings for analysis along with copies of the students' written work. In each case, the experts were asked to note statements that they felt were important indications of students' understanding or lack thereof. The reviews of the recordings and written work were conducted independently, and the results were collated to arrive at the results reported here.
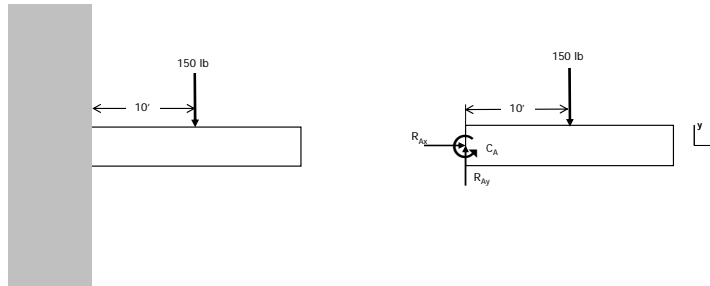
Materials

The problems used for the first student are presented in Figure 1. These problems were given to assess the student's ability to identify couples present in real supports, and to understand the connection between an idealized fixed support and a more accurate physical model. The third problem, however, did not work as intended because the student solved each problem independently rather than determining their equivalence. Therefore, the third problem was modified as shown in Figure 2. The text was also changed based on suggestions from the student. This modified version of the third problem was still not clear to the second student, who made suggestions to improve the text.

After completion of the think-aloud sessions with the first two students, the interviewer became concerned that the problems used were not naturally eliciting much discussion of couples and moments by the students.   He discussed this issue with the rest of the team and they jointly arrived at a decision to modify the problems further. The loading on the beam was changed to a pair of equal, opposite, non-collinear forces, .i.e., a couple. Problem 3 was modified to match the new loading and the wording was changed based on the suggestions of the second student; the new form of the problem is shown in Figure 3. In addition, a new problem was added at the beginning of the session that asked the students to calculate the moment about three different points on the beam. This problem is shown in Figure 4.
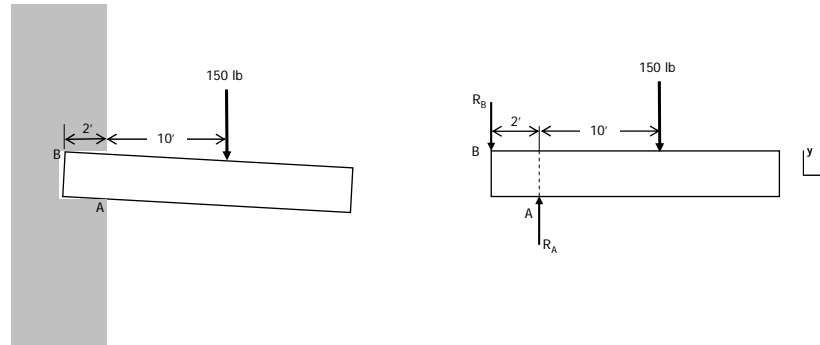
As a check on whether students were grasping the concept of a couple from a physical point of view, a final problem was added that asked them to show the free-body diagram for a tight bolt that was being acted on by a wrench, but not yet turning. This problem is shown in Figure 5.

A beam has been embedded in the wall as shown below, creating a fixed support. A load of 150 lbs acts on the beam. The weight of the beam is negligible compared to the load. Determine the reactions at the wall.
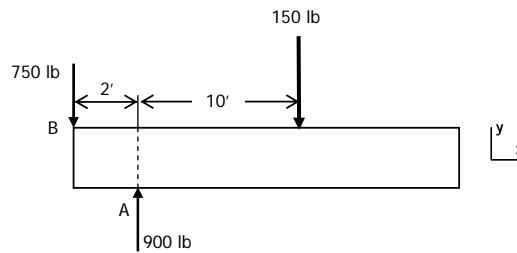
A beam is inserted into the loose slot as shown below. The angle has been <u>greatly exaggerated</u> to show that the beam only makes contact at two points (A and B).

A load of 150 lbs acts on the beam. The weight of the beam is negligible compared to the load. Determine the reactions at each of the contact points.

Your calculations should have resulted in a reaction at B equal to 750 lb and a reaction at A equal to 900 lb as shown below.

This pair of reaction forces can be modeled as the force and couple reactions shown below. Please calculate R_{Ax}, R_{Ay}, and C_A.
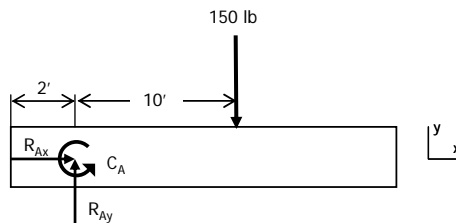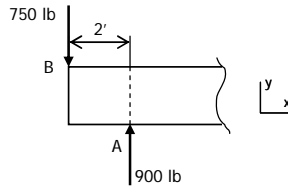
Figure 1. Problems used with first student

Your calculations for Problem 2 should have resulted in a reaction at B equal to 750 lb and a reaction at A equal to 900 lb as shown below.

750 lb

2'

B

y
x

A

900 lb

This pair of reaction forces can be modeled with the force and couple reactions shown below.
Please calculate R_{Ax}, R_{Ay}, and C_A.

2'

R_{Ax}

C_A

y
x

R_{Ay}

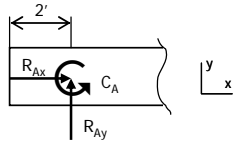Figure 2. Modified third problem for student 2

Your calculations for Problem 3 should have resulted in a reactions at A and B equal to 750 lbs.

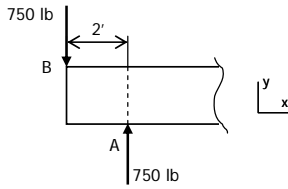750 lb

2'

B

y
x

A

750 lb

Figure 1

These reaction forces can be modeled with the force and couple reactions shown in Figure 2.
Please calculate R_{Ax}, R_{Ay}, and C_A so that they have the same effect as the forces in the Figure 1.

2'

R_{Ax}

C_A

y
x

R_{Ay}

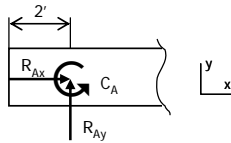Figure 2

Figure 3. Modified third problem for students 3, 4, and 5

Two loads of 150 lbs act on the beam as shown below.

  i)   What is the sum of the moments about point A caused by the two loads?
  ii)   What is the sum of the moments about point D caused by the two loads?
  iii)   What is the sum of the moments about point E caused by the two loads?

Figure 4. Problem added at beginning of session for students 3, 4, and 5

An open-end wrench, illustrated below, is being used to loosen a tight bolt. Please sketch the forces imposed on the bolt by the wrench.

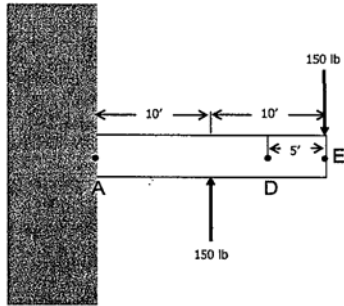Figure 5. Problem added at end of session for students 3, 4, and 5

Results

Our analysis of the think aloud sessions showed that four of the five students encountered difficulties in problem solving that we believe resulted from a poor understanding of the underlying materials. In reviewing the video of these four students, the governing approach seemed to be algorithmic. Each student when presented with the need to solve for reaction supports, or magnitudes of moments about a point, would simply write out the appropriate sum of moments equation, and go about the business of algebraic manipulation to solve the problem. This was particularly evident in the problem shown in Fig. 4. In this case all four of the students treating the problem algorithmically summed moments at each point without considering the nature of the applied couple. An example of this work is shown in Fig. 6.

Two loads of 150 lbs act on the beam as shown below.

    i)   What is the sum of the moments about point A caused by the two loads?
    ii)   What is the sum of the moments about point D caused by the two loads?
    iii)   What is the sum of the moments about point E caused by the two loads?

$$\sum M_A = 0$$

$$-150\,(20) + 150\,(10) = \sum M_A$$

$$\sum M_A = -1500 \; lb \cdot ft$$

$$-150\,(5) - 150\,(5)$$

$$= -1500 \; lb \cdot ft$$

$$-150\,(10) = -1500 \; lb \cdot ft$$

Figure 6: Algorithmic calculations of moments about several points due to an applied couple.

In this case, and several others, the student begins thinking about the equilibrium condition where the sum of moments at a point is equal to zero, but goes on to successfully calculate the moment due to the couple at each point separately. Only one student was able to identify the pair of forces as a couple, and knew before going through the algebra at each point that the magnitude of the moment about each point would be equal due to the couple.
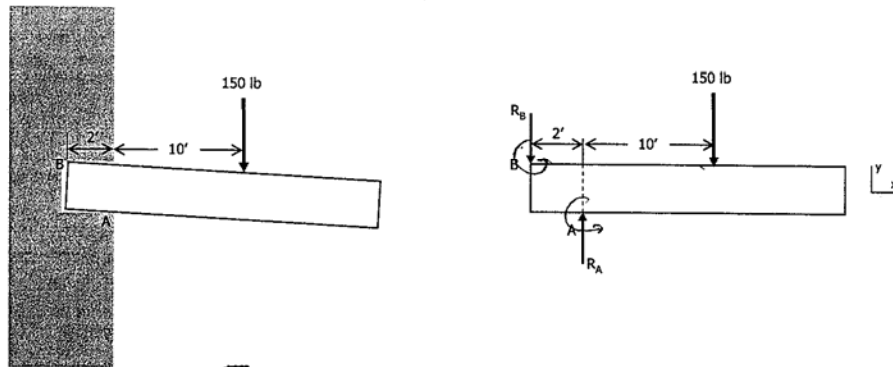
Another problem we observed with the students' solutions to the given problem set was frequent sign errors contained within their work. All of the students interviewed made sign errors at one point or another in the set of problems where they were calculating moments. Only one was able to diagnose his error based on reasoning out the solution to the problem. In calculating the support reaction at point A from Fig. 1 he knew that the reaction couple had to balance the moment due to the applied loading. We attribute the failing of most students to recognize sign errors to be a consequence of the algorithmic approach that they used[6]. By approaching the problem without thinking or applying higher reasoning skills the students are more prone to make errors in problem solving.

One student correctly recognized the concept that support couples restrict rotation of the body but went on to say that the contact points with the smooth slot in Fig. 1b have "two moments" one at each of the contact points with the slot, in addition to the normal forces present on the beam as shown in Fig. 7. Several problems are evident in this student's work. First, he has arrived at a statically indeterminate model of the problem. After

working on this problem for several minutes he concluded that he had too many unknowns, and that it didn't seem right. He also had errors in his sum of moments equation, which during the problem solving process he pointed to. He said that he could include M_A, but if he did that, he wouldn't have enough equations to solve the problem. Although the students were provided with a free body diagram, he added his own additional support reactions, because he believed that a couple was required to prevent the rotation of the beam.

A beam is inserted into the loose slot as shown below. The angle has been greatly exaggerated to show that the beam only makes contact at two points (A and B).

A load of 150 lbs acts on the beam. The weight of the beam is negligible compared to the load. Determine the reactions at each of the contact points.

$$\Sigma F_x = 0$$

$$\Sigma F_y = -150 + R_A + R_B = 0$$

$$900 = R_A$$

$$\Sigma M_A = -2(R_B) + (-150)(10) = 0$$

$$\frac{1500}{-2} = R_B$$

$$-750 = R_B$$

$$M_B = 1500$$

Figure 7: Incorrect student model, and work for the beam in slot problem.

Other generally confusing factors for students included the right hand rule, and its proper application, and whether or not to include applied couples in the calculation of the sum of moments about a point. When asked about the inclusion of C_A in the calculation of sum of moments about A on Fig. 4 the student says "But then when I think about the couple, you don't multiply that by anything, you just add it on there, so like I have to second guess myself by thinking should I put it on there or not, but you do." The fact that the reaction couple is applied at the point where she is summing moments adds confusion for this student because she lacks a fundamental understanding of what the couple is.

Amongst the students we interviewed one student worked through the problems in a dramatically different way from the other four. In our analysis of the think aloud sessions this student was the only one to correctly identify the pair of forces acting on the beam in Fig. 4 as a couple, before doing calculations. This student used the right hand rule to replace the pair of equal and opposite forces with an equivalent couple-moment, and realized that the pair of forces will produce the same moment at any point along the beam. He said that, "Because it's a couple. It's a moment, the two forces together produce a couple and the couple can be placed anywhere along a body." He told us that he can replace the two forces with a couple acting anywhere along the body. He says he can "move the two forces along the body, and it wouldn't make a difference." This was in regard to the calculation of the moment due to the couple. All of the other students faced with this problem, simply summed moments at the point in question, and calculated the moment about that point due to the contributions from each of the forces without considering the pair of forces as a couple.

We also explored the students' ability to grasp that the couple support reaction at a fixed connection is in fact a modeling simplification used to produce a generic fixed support without the need to fully analyze how the beam was fixed in place. The same student who recognized the couple seemed to fully grasp the idea of using the fixed connection to model the complexity associated with the actual built in connection in a way that simplified the solution of the problem. In fact, during the interview he told us that he started statics trying to determine what was going on with the "foundation of the building" and that he was "a little equation happy" but that he now realizes that using the simplification is a "much nicer way of looking at it." While this student seemed to have the best comprehension of how couples and moments work he used the words couple and, moment imprecisely throughout the interview, stating at one point that he and a classmate that he studied with decided that the difference between moment and couple was just "book silliness."

We probed this student further about the differences between moments and couples in his understanding of the course material. During the think aloud process with the wrench shown in Fig. 5 the student drew a couple acting on the wrench as both a pair of forces, and then described it separately using the couple symbol of "a circle with an arrow". When asked if it was a moment or a couple, the student responded "both." When prompted to expand on his thoughts on moments and couples the student responded, "It was confusing … they started throwing the couple, the C, at us. And it was confusing at first, and then we dismissed it as book silliness to the point where we would understand that they meant a moment because it was easier for us to understand that two couples acting in the opposite direction produce a couple because it is a 'couple of forces. Is that couple a moment, yes! However, if we draw a swirly arrow, we'll call it a moment. Just for us personally. If the teacher says there is a couple of Nm then I understand that she is talking about a moment." This statement gets at the heart of many students' confusion. Competing sources and notation lead to confusion about what is a couple, and what is a moment, and when to use each term. But for some students the words used do not

undermine the conceptual understanding of the differences between summing moments at a point, and the application of a couple as a load.

Conclusions

This preliminary study has given us insights into the difficulties that students are encountering in solving statics problems involving couples and moments. Most of the students interviewed seem content to apply an algorithmic approach to problem solving which does not demonstrate a deeper understanding of the underlying physical reasons for why the approach is taken. This leads to difficulties in circumstances that have not been memorized from a table in the text, or in analyzing problems designed to probe conceptual knowledge. This approach also leads to more sign errors in calculations due to the lack of physical reasoning in assessing their solutions.

With respect to the precise use of terminology, the most compelling comment from a student was that the distinction between couple and moment was "book silliness". This comment, and the variation in definitions of the terms across courses and textbooks, led us to conclude that rigorous attention to precise use of the terms should be secondary to developing a conceptual understanding of the underlying fundamentals of moments and couples, and the ability to apply them. Consequently, our next round of think-aloud sessions will not have any elements designed to probe precise use of terminology. We anticipate having results from an additional twelve students by the time of the presentation at ASEE.

Acknowledgments

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