Stability of Simultaneously Triangularizable Switched Linear Systems on Time Scales

John Miller
B.S./M.S. Electrical and Computer Engineering Candidate
Baylor University

Ian Gravagne
Faculty Advisor
Department of Electrical and Computer Engineering
Baylor University

Abstract

Switched linear systems are a class of dynamical systems in which linear, time-invariant (LTI) dynamics closely model system behavior for a period of time, after which the system parameters suddenly (discontinuously) change, or "switch," and the system continues to evolve under new LTI parameters until another switching instant. Switched linear systems are often found wherever a dynamical system is coupled with supervisory control logic that can abruptly change the operating mode of the system, e.g. the transmission of a vehicle or the regulatory dynamics of a biological cell[3]. Until now, it has always been assumed that the underlying dynamics evolve naturally on $\mathbb{R}$ or $h\mathbb{Z}$ ($h > 0$). However, many systems constructed with complex layers of computers and communication networks do not fit the standard choices for time domain, so recently, researchers[1,2] have been investigating the use of *dynamic equations on time scales* to enable system modeling on unusual time domains $\mathbb{T}$, where $\mathbb{T}$ is any closed subset of $\mathbb{R}$. In this paper, we examine the stability of switched systems on discrete time scales, with arbitrary switching instances. We show that there exists a *temporal region of stability* for switched systems with simultaneously triangularizable system matrices, and that this region is identical to the region for simultaneously diagonalizable system matrices. It is henceforth assumed that the reader has a working knowledge of the field of time scale mathematics.

Introduction

Consider the set of matrices $A_1, A_2, \ldots, A_m \in \mathbb{R}^{n\times n}$ with switching signal $s : \mathbb{T} \to 1, 2, \ldots, m$, where

$$x^{\Delta}(t) = A_{s(t)}x(t) \qquad \text{where } t \in \mathbb{T}, t \geq 0 \text{ and } x(0) = x_0 \tag{1}$$

We make the following assumptions about this system and the underlying time scale

A1 The switching signal s is arbitrary over $\mathbb{T}$.

A2 The eigenvalues of all of the A_i are strictly within the Hilger circle for all $t \in \mathbb{T}$. (This means each A_i is "stable," or $x^{\Delta}(t) = A_i x(t)$ has $\|x(t)\| < \infty$ for all $t \geq 0$.)

A3 All of the A_i's are regressive. (Meaning that $(I + \mu(t)A_i)^{-1}$ exists $\forall\, t \in \mathbb{T}$.)

A4 All of the A_i's commute, i.e. $A_i A_j = A_j A_i \;\forall\, i \neq j$.

A5 $\mathbb{T}$ has the following properties: (i) $0 \in \mathbb{T}$, (ii) $\mathbb{T}$ is unbounded above, (iii) $\mathbb{T}$ has graininess $0 < \mu_{\min} \leq \mu(t) \leq \mu_{\max}$ for all $t \in \mathbb{T}$.

Note that all quantities can be assumed to be time-varying except for the A_i, unless otherwise indicated. Without loss of generality, we restrict the following analysis to $m = 2$.

To investigate stability of (1), we define a Lyapunov candidate

$$V = x^* P x \tag{2}$$

where $P = P^* > 0$ and $*$ denotes conjugate transpose. To ensure stability, we need

$$\begin{aligned} & V^{\Delta} < 0 \\ \Rightarrow\quad & A_i^* P + P A_i + \mu A_i^* P A_i + (I + \mu A_i^*) P^{\Delta} (I + \mu A_i) < 0. \end{aligned} \tag{3}$$

We now set

$$A_i^* P + P A_i + \mu A_i^* P A_i = -M_i \tag{4}$$

where $M_i = M_i^* > 0$ and solve for P. It can be shown[4] that P solves (4) for all i if

$$P(t) = \int_{\mathbb{S}} \Phi_{A_i}(s,0)^* M_i(t) \Phi_{A_i}(s,0) \Delta s \tag{5}$$

and

$$M_1(t) = \int_{\mathbb{S}} \Phi_{A_2}(s,0)^* Q(t) \Phi_{A_2}(s,0) \Delta s \tag{6a}$$

$$M_2(t) = \int_{\mathbb{S}} \Phi_{A_1}(s,0)^* Q(t) \Phi_{A_1}(s,0) \Delta s \tag{6b}$$

where $\mathbb{S} = \mu(t)\mathbb{N}_0$, $Q = Q^* > 0$ is an arbitrary "seed" matrix, and $\Phi_{A_i}(s,0)$ is the transition matrix that solves $y^{\Delta}(s) = A_i y(s)$ with $s \in \mathbb{S}$ and an initial condition $y(0) = y_0$. Because $\mathbb{S}$ is a constant-graininess time scale,

$$\Phi_{A_i}(s,0) = e_{A_i}(s,0) = (I + \mu(t)A_i)^{\frac{s}{\mu(t)}}. \tag{7}$$

Substituting (4) into (3) yields

$$(I + \mu A_i)^* P^{\Delta} (I + \mu A_i) - M_i < 0 \qquad \text{for } t \in \mathbb{T} \tag{8}$$

where $P^{\Delta} = \frac{P^{\sigma} - P}{\mu}$. Note that the only terms in (8) which depend on t are μ and μ^{σ}.

We can now pose the following question: given a time $t \in \mathbb{T}$, what is the region $\mathcal{R} \in \mathbb{R}^2$ such that $\{\mu(t), \mu^{\sigma}(t)\} \in \mathcal{R}$ satisfies (8) for all i?

Results have been derived[2] for the diagonal and simultaneously diagonalizable cases, and we make note of them here. For simplicity, let

$$K_i := [2 \text{ Re } J_i + \mu J_i^* J_i] \tag{9a}$$
$$K_i^{\sigma} := [2 \text{ Re } J_i + \mu^{\sigma} J_i^* J_i] \tag{9b}$$

Then $\mathcal{R}$ is defined as the set of all $\{\mu(t), \mu^{\sigma}(t)\}$ such that

$$K_1^{\sigma} K_2^{\sigma} K_1^{-1} K_2^{-1} > (I + \mu J_i)^* (I + \mu J_i) \quad \text{for } i = 1, 2 \tag{10}$$

with $0 < \mu_{\min} \leq \mu(t) \leq \mu_{\max}$ for all $t \in \mathbb{T}$ and $A_i = S^{-1} J_i S$, where J_i is diagonal. Note that K_i^{-1} always exists because K_i is diagonal and has non-zero eigenvalues as a result of A2.

Our objective is to investigate the simultaneously "triangularizable" case.

Jordan Epsilon Form

We next explore the case where the A_i's are no longer simply diagonalizable. Thus, we need a more general form of the similarity transformation, $A = S^{-1} J S$. Therefore, let $A = S_{\epsilon}^{-1} J_{\epsilon} S_{\epsilon}$ with

$$J_{\epsilon} = \begin{bmatrix} \lambda & \epsilon & & \\ & \lambda & \epsilon & \\ & & \lambda & \epsilon \\ & & & \ddots \end{bmatrix}$$

where ϵ is arbitrary and $0 < \epsilon < 1$. J_{ϵ} comes from the generalized eigenvector equations

$$\begin{aligned} & A\bar{x}_1 = \lambda \bar{x}_1,\ A\bar{x}_2 = \lambda \bar{x}_2 + \epsilon \bar{x}_1,\ A\bar{x}_3 = \lambda \bar{x}_3 + \epsilon \bar{x}_2,\ \ldots \\ \Rightarrow & (A - \lambda I)\bar{x}_1 = 0,\ \frac{(A - \lambda I)}{\epsilon}\bar{x}_2 = \bar{x}_1,\ \frac{(A - \lambda I)}{\epsilon}\bar{x}_3 = \bar{x}_2,\ \ldots \\ \Rightarrow & (A - \lambda I)\bar{x}_1 = 0,\ \frac{(A - \lambda I)^2}{\epsilon}\bar{x}_2 = 0,\ \frac{(A - \lambda I)^3}{\epsilon^2}\bar{x}_3 = 0,\ \ldots \end{aligned}$$

Thus, let $S_{\epsilon} = [\bar{x}_1\ \epsilon \bar{x}_2\ \epsilon^2 \bar{x}_3\ \ldots]$ where the $\bar{x}_i$ are the eigenvectors of A. This is a valid similarity transform (as multiplication by a scalar doesn't change the fact that the $\bar{x}_i$ are eigenvectors). We term J_{ϵ} the "Jordan-epsilon form." From here on, let $S = S_{\epsilon}$ and $J = J_{\epsilon}$.

It will be useful to look at J as the sum of two matrices, $J = L + N$, where

$$L = \begin{bmatrix} \lambda & 0 & & \\ & \lambda & 0 & \\ & & \lambda & 0 \\ & & & \ddots \end{bmatrix}, \quad N = \begin{bmatrix} 0 & \epsilon & & \\ & 0 & \epsilon & \\ & & 0 & \epsilon \\ & & & \ddots \end{bmatrix}.$$

N is a nilpotent matrix with ϵ on the superdiagonal, and L is a diagonal matrix of the eigenvalues. Note that N being nilpotent of order n means that $N^n = [0]$, where N (and A, L, and J) is $n \times n$.

Triangular Transition Matrix

Remembering that $s \in \mathbb{S} := \mu(t)\mathbb{N}_0$, we may now rewrite (7) with $h = \mu(t)$ and $z = \frac{s}{h} \in \mathbb{N}_0$ to obtain

$$\Phi_A(s,0) = e_A(s,0) = (I + hA)^{\frac{s}{h}} = (I + hA)^z. \tag{11}$$

Applying the Jordan-epsilon similarity transform to A then gives

$$\begin{aligned} \Phi_A(s,0) &= (I + hA)^z \\ &= (I + hS^{-1}JS)^z \\ &= S^{-1}(SS^{-1} + hJ)^z S \\ &= S^{-1}(I + hJ)^z S \\ &= S^{-1}\Phi_J(s,0)S. \end{aligned} \tag{12}$$

Expanding Φ_J as a binomial series we get

$$\begin{aligned} \Phi_J(s,0) &= (I + hJ)^z \\ &= (I + h(L + N))^z \\ &= ((1 + h\lambda)I + hN)^z \\ &= \sum_{k=0}^{z} \binom{z}{k} (1 + h\lambda)^{z-k}(hN)^k. \end{aligned} \tag{13}$$

If $z < n - 1$, then this is the final form of the binomial series, and the series is finite. If $z \geq n - 1$, then the series is truncated at $n - 1$ because N is nilpotent, and we get

$$\begin{aligned} &= (1 + h\lambda)^z I + \binom{z}{1}(1 + h\lambda)^{z-1}(hN) + \dots \\ &\quad + \binom{z}{n-1}(1 + h\lambda)^{z-n+1}(hN)^{n-1} + [0] + [0] + \dots \\ &= (I + hL)^z + \sum_{k=1}^{n-1} \binom{z}{k}(1 + h\lambda)^{z-k}(hN)^k \end{aligned} \tag{14}$$

which is still finite for any z. Now, let

$$\Phi_J(s,0) = (I+hL)^z + E(s) = e_L(s,0) + E(s) \tag{15}$$

where

$$E(s) := \sum_{k=1}^{z} \binom{z}{k} (1+h\lambda)^{z-k}(hN)^k \qquad \left(\text{recall } z = \frac{s}{h}\right). \tag{16}$$

In other words, E is an error matrix that depends on ϵ, is upper-triangular with zeros on the diagonal, and is at most of order ϵ^{n-1}. Note that $\|E\| \to 0$ as $\epsilon \to 0$.

Error Terms

In the following discussion, many such error terms will appear, so we list them here for the sake of clarity:

$$E_i(s) := \sum_{k=1}^{z} \binom{z}{k} [1+\mu\lambda_i]^{z-k}[\mu N]^k \tag{17a}$$

$$E1_i := \int_{\mathbb{S}} \left[e_{\bar{L}_i}(s,0)E_i(s) + E_i^*(s)e_{L_i}(s,0) + E_i^*(s)E_i(s) \right] \Delta s \tag{17b}$$

$$E2 := -E1_1[2 \text{ Re } L_2 + \mu L_2^* L_2]^{-1} - E1_2[2 \text{ Re } L_1 + \mu L_1^* L_1]^{-1} + E1_1 E1_2 \tag{17c}$$

$$E3_i := \mu(I+\mu J_i)^{-*} E1_j (I+\mu J_i)^{-1} - E2^\sigma + E2 \tag{17d}$$

$$E4_i := \mu(1+\mu\lambda_i)N^* + \mu(1+\mu\bar{\lambda}_i)N + \mu^2 N N^* \tag{17e}$$

$$E5_i := E4_i[(I+\mu J_i)(I+\mu J_i^*)]^{-1} + K_1 E3_i K_2 \tag{17f}$$

where $j = 2, 1$. Note: $E2$ is the same for $i = 1, 2$.

Generalizing the Region of Stability

Substituting (6a) into (5) for $i = 1$, we get

$$\begin{aligned} P &= \int_{\mathbb{S}} \Phi_{A_1}(s,0)^* M_1 \Phi_{A_1}(s,0)\Delta s \\ &= \int_{\mathbb{S}} \Phi_{A_1}(s,0)^* \left(\int_{\mathbb{S}} \Phi_{A_2}(r,0)^* Q \Phi_{A_2}(r,0)\Delta r \right) \Phi_{A_1}(s,0)\Delta s \\ &= \int_{\mathbb{S}} \int_{\mathbb{S}} \Phi_{A_1}(s,0)^* \Phi_{A_2}(r,0)^* Q \Phi_{A_2}(r,0) \Phi_{A_1}(s,0) \Delta r \Delta s. \end{aligned} \tag{18}$$

Since Q in (6) may be any arbitrary, positive definite matrix, we choose $Q = S^*S$. Substituting this and applying the Jordan-epsilon similarity transform $A_i = S^{-1}J_iS$ to (18) gives

$$\begin{aligned} P &= \int_{\mathbb{S}} \int_{\mathbb{S}} \left(S^* \Phi_{J_1}(s,0)^* \Phi_{J_2}(r,0)^* S^{-T} \right) S^*S \left(S^{-1} \Phi_{J_2}(r,0) \Phi_{J_1}(s,0) S \right) \Delta r \Delta s \\ &= S^* \left[\int_{\mathbb{S}} \Phi_{J_1}(s,0)^* \Phi_{J_1}(s,0)\Delta s \int_{\mathbb{S}} \Phi_{J_2}(r,0)^* \Phi_{J_2}(r,0) \Delta r \right] S. \end{aligned} \tag{19}$$

We now need to investigate the integrals in this equation.

Transition Matrix Quadratic Integral

Lemma 1 (Quadratic Integral)**.** *Let* $A_i = S^{-1}J_iS$*, where* $J_i \in \mathbb{C}^{n\times n}$ *is a Jordan Epsilon Form matrix, and be stable on time scale* $\mathbb{S} = h\mathbb{N}_0$*. Then*

$$\int_{\mathbb{S}} \Phi_{J_i}(s,0)^*\Phi_{J_i}(s,0)\Delta s = -[2\,\mathit{Re}\; L_i + hL_i^*L_i]^{-1} + E1_i \tag{20}$$

where $J_i = L_i + N$*, with* L_i *diagonal and* N *nilpotent, and* $E1_i$ *is defined as in* (17b).

Proof. Using the definition of Φ_J from (15) gives

$$\int_{\mathbb{S}} \Phi_{J_i}(s,0)^*\Phi_{J_i}(s,0)\Delta s = \int_{\mathbb{S}} [e^*_{L_i}(s,0) + E_i^*(s)][e_{L_i}(s,0) + E_i(s)]\Delta s. \tag{21a}$$

By assumption A2, the left hand side of (21a) converges. We can expand it to see

$$= \int_{\mathbb{S}} \left[e_{\bar{L}_i}(s,0)e_{L_i}(s,0) + e_{\bar{L}_i}(s,0)E_i(s) + E_i^*(s)e_{L_i}(s,0) + E_i^*(s)E_i(s)\right]\Delta s$$

$$= \int_{\mathbb{S}} e_{\bar{L}_i}(s,0)e_{L_i}(s,0)\Delta s + E1_i \tag{21b}$$

where $E1_i$ is defined as in (17b). $E1_i$ must be finite because the integral on the left side of (21a) converges and the first term of (21b) is positive definite.

The first term of (21b) is an integral of a diagonal matrix. We can use the diagonal Quadratic Integral Lemma[2] to say that

$$\int_{\mathbb{S}} e_{\bar{L}_i}(s,0)e_{L_i}(s,0)\Delta s = -[2\,\mathrm{Re}\; L_i + hL_i^*L_i]^{-1}. \tag{22}$$

Substituting this into (21), we have

$$\int_{\mathbb{S}} \Phi_{J_i}(s,0)^*\Phi_{J_i}(s,0)\Delta s = -[2\,\mathrm{Re}\; L_i + hL_i^*L_i]^{-1} + E1_i \tag{23}$$

which is the lemma statement. □

The result of Lemma 1 closely parallels the result for the diagonalizable systems. In other words,

$$\lim_{\epsilon\to 0} \int_{\mathbb{S}} \Phi_{J_i}(s,0)^*\Phi_{J_i}(s,0)\Delta s = -[2\,\mathrm{Re}\; L_i + hL_i^*L_i]^{-1}. \tag{24}$$

The Region of Stability

Motivated by (9), we generalize K_i by redefining it as

$$K_i := [2\,\mathrm{Re}\; L_i + \mu L_i^*L_i] \tag{25a}$$

$$K_i^\sigma := [2\,\mathrm{Re}\; L_i + \mu^\sigma L_i^*L_i] \tag{25b}$$

(recall that, in (9), the J_i were diagonal, while now, the L_i are the diagonal of J_i; also, K_i^{-1} still exists because of the same reasoning as before).

Theorem 2 (Region of Stability)**.** *Given $A_i = S^{-1}J_iS$ where the J_i are Jordan-epsilon form matrices with $J_i = L_i + N$ and the A_i are stable on time scale $\mathbb{T}$, there exists a region $\mathcal{R} \in \mathbb{R}^2$ consisting of pairs $\{\mu^\sigma, \mu\}$ such that*

$$K_1^{-1}K_2^{-1}K_1^\sigma K_2^\sigma > (I + \mu L_i^*)(I + \mu L_i) \quad \textit{for } i = 1, 2 \tag{26}$$

with $0 < \mu_{\min} \le \mu(t) \le \mu_{\max}$ *for all* $t \in \mathbb{T}$.

Proof. Beginning with (19), we use Lemma 1 and (25) to say

$$\begin{aligned} P &= S^*\left[\int_{\mathbb{S}} \Phi_{J_1}(s,0)^*\Phi_{J_1}(s,0)\Delta s \int_{\mathbb{S}} \Phi_{J_2}(r,0)^*\Phi_{J_2}(r,0)\Delta r\right] S \\ &= S^*\left[\left(-[2 \text{ Re } L_1 + \mu L_1^* L_1]^{-1} + E1_1\right)\left(-[2 \text{ Re } L_2 + \mu L_2^* L_2]^{-1} + E1_2\right)\right] S \\ &= S^*\left[\left(-K_1^{-1} + E1_1\right)\left(-K_2^{-1} + E1_2\right)\right] S \\ &= S^*\left[K_1^{-1}K_2^{-1} + E2\right] S \end{aligned} \tag{27}$$

where $E2$ is defined as in (17c). Similarly

$$P^\sigma = S^*\left[K_1^{\sigma^{-1}} K_2^{\sigma^{-1}} + E2^\sigma\right] S \tag{28}$$

Inserting P and P^σ from (27) and (28) into (8) for $i = 1$ and eliminating S gives

$$\frac{1}{\mu}(I + \mu J_1)^*\left[K_1^{\sigma^{-1}} K_2^{\sigma^{-1}} + E2^\sigma - K_1^{-1}K_2^{-1} - E2\right](I + \mu J_1) + K_2^{-1} - E1_2 < 0 \tag{29}$$

$$\frac{1}{\mu}(I + \mu J_1)^*\left[K_1^{\sigma^{-1}} K_2^{\sigma^{-1}} - K_1^{-1}K_2^{-1} + E2^\sigma - E2\right](I + \mu J_1) < -K_2^{-1} + E1_2$$

$$\begin{aligned} K_1^{\sigma^{-1}} K_2^{\sigma^{-1}} - K_1^{-1}K_2^{-1} + E2^\sigma - E2 &< \mu(I + \mu J_1)^{-*}\left(-K_2^{-1} + E1_2\right)(I + \mu J_1)^{-1} \\ K_1^{\sigma^{-1}} K_2^{\sigma^{-1}} - K_1^{-1}K_2^{-1} &< -\mu(I + \mu J_1)^{-*}(I + \mu J_1)^{-1}K_2^{-1} + E3_1 \end{aligned} \tag{30}$$

where $E3$ is defined as in (17d). Continuing,

$$\begin{aligned} K_1^{\sigma^{-1}} K_2^{\sigma^{-1}} - K_1^{-1}K_2^{-1} &< -\mu[(I + \mu J_1)(I + \mu J_1^*)]^{-1}K_2^{-1} + E3_1 \\ K_1K_2K_1^{\sigma^{-1}} K_2^{\sigma^{-1}} - I &< -\mu[(I + \mu J_1)(I + \mu J_1^*)]^{-1}K_1 + K_1E3_1K_2 \\ K_1K_2K_1^{\sigma^{-1}} K_2^{\sigma^{-1}} &< I - \mu[(I + \mu J_1)(I + \mu J_1^*)]^{-1}K_1 + K_1E3_1K_2 \\ K_1K_2K_1^{\sigma^{-1}} K_2^{\sigma^{-1}} &< [(I + \mu J_1)(I + \mu J_1^*) - \mu K_1][(I + \mu J_1)(I + \mu J_1^*)]^{-1} + K_1E3_1K_2 \end{aligned} \tag{31}$$

Multiplying $(I + \mu J_1)(I + \mu J_1^*)$ out gives

$$\begin{aligned} (I + \mu J_1)(I + \mu J_1^*) &= I + 2\mu \text{ Re } L_1 + \mu^2 L_1L_1^* + \mu(1 + \mu\lambda_1)N^* + \mu(1 + \mu\bar{\lambda}_1)N + \mu^2 NN^* \\ &= I + \mu K_1 + E4_1 \end{aligned} \tag{32}$$

where we define $E4$ as in (17e). Thus we have

$$K_1K_2K_1^{\sigma^{-1}}K_2^{\sigma^{-1}} < [I+\mu K_1 + E4_1 - \mu K_1][(I+\mu J_1)(I+\mu J_1^*)]^{-1} + K_1E3_1K_2$$
$$K_1K_2K_1^{\sigma^{-1}}K_2^{\sigma^{-1}} < [I + E4_1][(I+\mu J_1)(I+\mu J_1^*)]^{-1} + K_1E3_1K_2$$
$$K_1K_2K_1^{\sigma^{-1}}K_2^{\sigma^{-1}} < [(I+\mu J_1)(I+\mu J_1^*)]^{-1} + E4_1[(I+\mu J_1)(I+\mu J_1^*)]^{-1} + K_1E3_1K_2 \quad (33)$$

Let $E5$ be defined as in (17f). Then

$$K_1K_2K_1^{\sigma^{-1}}K_2^{\sigma^{-1}} < [(I+\mu J_1)(I+\mu J_1^*)]^{-1} + E5_1$$
$$K_1^{-1}K_2^{-1}K_1^{\sigma}K_2^{\sigma} > \left([(I+\mu J_1)(I+\mu J_1^*)]^{-1} + E5_1\right)^{-1}$$
$$K_1^{-1}K_2^{-1}K_1^{\sigma}K_2^{\sigma} > \left([I+\mu K_1 + E4_1]^{-1} + E5_1\right)^{-1}$$
$$K_1^{-1}K_2^{-1}K_1^{\sigma}K_2^{\sigma} > \left([(I+\mu L_1^*)(I+\mu L_1) + E4_1]^{-1} + E5_1\right)^{-1} \quad (34)$$

$i = 2$ follows similarly.

We define a region $\mathcal{R}_\epsilon$ given by (34) for $i = 1, 2$. Then we note that

$$\lim_{\epsilon \to 0} \mathcal{R}_\epsilon = \mathcal{R} \quad (35)$$

because

$$\lim_{\epsilon \to 0} \left([(I+\mu L_i^*)(I+\mu L_i) + E4_i]^{-1} + E5_i\right)^{-1} = (I+\mu L_i^*)(I+\mu L_i). \quad (36)$$

Applying this to (34) yields the set of equations in Theorem 2:

$$K_1^{-1}K_2^{-1}K_1^{\sigma}K_2^{\sigma} > (I+\mu L_i^*)(I+\mu L_i) \qquad \text{for } i = 1, 2. \quad (37)$$

Thus, to guarantee stability of the switched linear system (1), it is sufficient to show that $\{\mu(t), \mu^\sigma(t)\} \in \mathcal{R}$, regardless of whether the system is simultaneously diagonalizable or only simultaneously triangularizable, which is the theorem statement. □

This is a favorable result, because $\mathcal{R}$ is significantly easier to compute than $\mathcal{R}_\epsilon$.

Examples

Example 1

Let

$$A_1 = \begin{bmatrix} -1.2 & 0.2 \\ 0 & -1.2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.6 & 0.2 \\ 0 & -0.6 \end{bmatrix},$$

which are already in irreducible, Jordan-epsilon form. Solving equation (37) for all $\{\mu(t), \mu^\sigma(t)\}$ pairs yields the region $\mathcal{R}$ in Figure 1, where it can be seen that the region is under the minimum of the four boundary curves (two curves exist outside the plot axes limits).

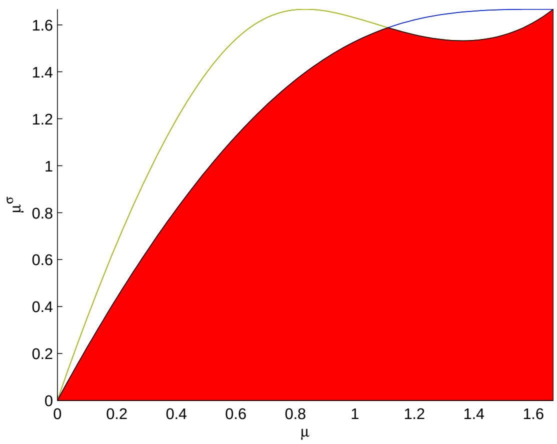

Figure 1: The temporal region of stability for Example 1. The two curves are the solution of (37) as if it were an equality, and the red area is under the minimum of the two curves. The upper limits on the axes are $\mu_{\max}$, which is the maximum graininess allowed by assumption A2.

Example 2

Let

$$A_1 = \begin{bmatrix} -0.78 & 0.8 \\ 0 & -0.78 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.1 & 0.8 \\ 0 & -0.1 \end{bmatrix}.$$

Similarly to Example 1, solving (37) yields Figure 2.

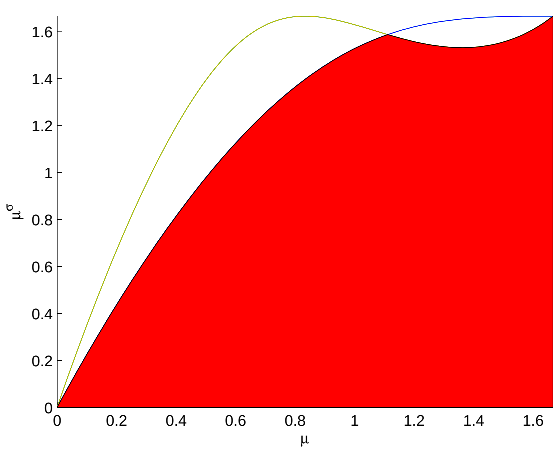

Figure 2: The temporal region of stability for Example 2. The two curves are the solution of (37) as if it were an equality, and the red area is under the minimum of the two curves. The upper limits on the axes are $\mu_{\max}$, which is the maximum graininess allowed by assumption A2.

References

1. DaCunha, J. "Stability for time varying linear dynamic systems on time scales," *J. Comput. Appl. Math.*, Vol. 176, Issue 2, April 2005. 381–410.
2. Davis, J., Miller, J., Gravagne, I., Marks, R., Ramos, A. "Stabilty of Switched Linear Systems on Non-Uniform Time Domains," submitted to *IEEE Trans. on Systems, Man, and Cybernetics - Part B*. 2008.
3. Liberzon, D. Switching in Systems and Control. 2003, Birkhäuser, Boston.
4. Ramos, Alice. Stability of Hybrid Dynamical Systems: Analysis and Design. Ph.D. Thesis. Baylor University, Aug 2009.

JOHN MILLER
John is a first year graduate student in Electrical and Computer Engineering at Baylor University. His research is focused on switched linear systems in time scales.

IAN GRAVAGNE
Dr. Gravagne is an Associate Professor in Baylor's Electrical and Computer Engineering Department. His research interests include solar energy, dynamic equations on time scales, and energy literacy education.