

## Structural Response in the Frequency Domain using LabView

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### Abstract

This paper describes the implementation of LabView, in an experiment in an instrumentation laboratory in the mechanical engineering department, to allow the acquisition of real time data for display, analysis, control and storage. The system is set in motion with a calibrated impact hammer. This hammer produces a voltage, which is proportional to the impact force. This force is sent to the LabView VI for analysis. Similarly, the accelerometer produces a voltage that is proportional to the acceleration of the club (this represents the *response* of the club). This signal is also sent to the computer via the signal conditioners and the DAQ board. The goal is to carry out real-time measurements and displays acquired waveforms on a PC screen and also store data associated with these waveforms for later use. The objective of this lab is to examine the response of a structure in the frequency domain, as opposed to the typical time domain. In particular, the vibration characteristics of a golf club are examined by applying an impulsive load using a calibrated impact hammer. The time domain signals are then analyzed using LabView software to obtain a spectrum response.

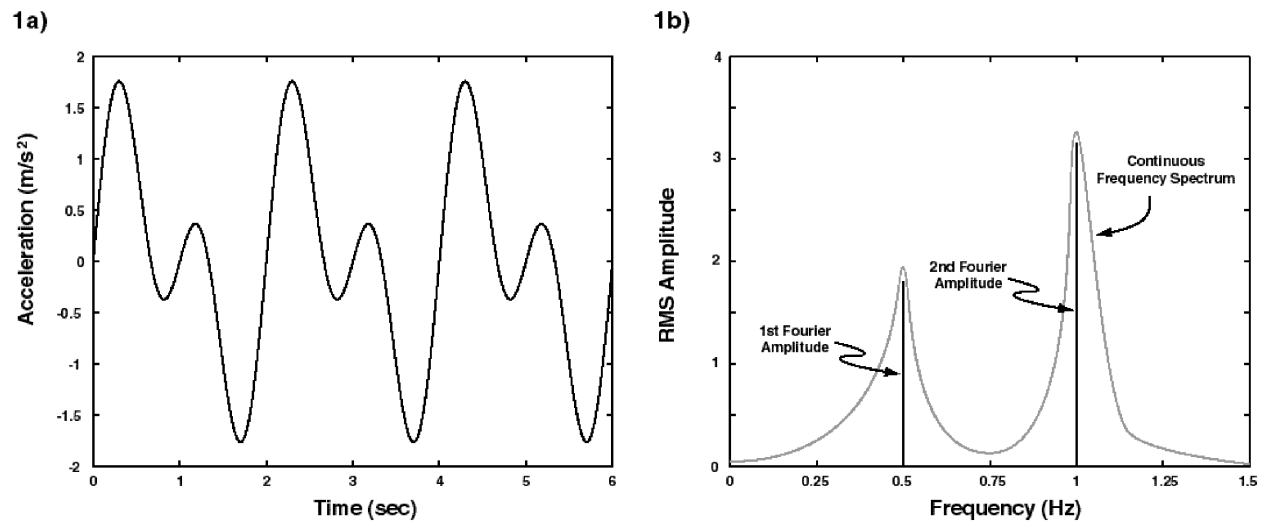
### Introduction

The use of a computer to imitate an instrument or device is known as virtual instrumentation. One software development package used to create virtual instruments is LabView (Laboratory Virtual Instrument Engineering Workbench). LabView is a graphical programming language that, when used in conjunction with a data acquisition device and personal computer, allows the user to control devices, collect, manipulate and display data. Written code is not used in LabView instead graphical representations of the circuits are constructed which are called virtual instruments (VI's). These VI's are manipulated so that they will perform the desired tasks at hand. The VIs (virtual instruments) in LabView are run from their front panels. This is the panel with all of the controls and displays. Each front panel has an associated block diagram. This block diagram is built using the graphical programming language G. The components of the block diagram represent different structures, loops and functions. The wiring of the block diagram represents flow of data between these components. A VI becomes a sub VI when it is placed inside the block diagram of another VI. These sub VIs are analogous to sub routines, and allow layering and modularity of the VIs.

In this experiment, virtual instruments created with LabView were used to provide practical experience measuring a system's frequency response and provides insight on how certain measurement decisions (sampling rate, signal duration, etc.) influence the results.

### Theory: The Frequency Domain

In order to monitor the vibrations of a structure, a transducer (strain gage, accelerometer, etc.) is mounted on the structure and the system is set in motion. The output of the transducer, plotted against time, is the easiest and most physically intuitive way to view the response. As an example, the acceleration response of a fictitious structure is shown in Figure 1a. Quite clearly, this waveform is not a simple sine wave oscillating at a single frequency. However, because it is



**Figure 1:** a) The response viewed in the time domain. b) The discrete Fourier amplitudes and the continuous frequency spectrum.

periodic it can be expressed as a Fourier series, i.e. it can be expressed as an infinite sum of sine and cosines. This representation takes the form:

$$f(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos\left(\frac{2n\pi t}{T}\right) + b_n \sin\left(\frac{2n\pi t}{T}\right) \right] \quad (1)$$

where the coefficients  $a_n$  and  $b_n$  can be determined through integration:

$$a_o = \frac{2}{T} \int_0^T f(t) dt \quad (2)$$

$$a_n = \frac{2}{T} \int_0^T \left[ f(t) \cos\left(\frac{2n\pi t}{T}\right) \right] dt \quad (3)$$

$$b_n = \frac{2}{T} \int_0^T \left[ f(t) \sin\left(\frac{2n\pi t}{T}\right) \right] dt \quad (4)$$

At this point, it is worth making an observation: the  $n^{\text{th}}$  coefficient (either  $a_n$  or  $b_n$ ) is associated with the  $n^{\text{th}}$  frequency,  $\omega_n = 2n\pi/T$ . In fact, you can think of the frequency  $\omega_n$  as having an RMS amplitude of  $C_n = \sqrt{[a_n^2 + b_n^2]}$ .

Now consider the following question: “How much does the  $n^{\text{th}}$  frequency contribute to the signal?” The answer lies in the RMS Fourier coefficient  $C_n$ . For example, if  $C_n = 0$ , then the frequency associated with  $C_n$  (i.e.,  $\omega_n$ ) plays no part in the signal, see Equation (1). The other extreme is if  $C_n$  is finite but all of the other coefficients are zero. Then the signal will be periodic with frequency  $\omega_n$ . To gage the relative importance of the different frequencies, the RMS coefficients are often plotted against their discrete frequencies - creating what is commonly known as a *discrete frequency spectrum*. The discrete frequency spectrum associated with Figure 1a is shown in Figure 1b. Here the  $C_1 \approx 1.8$  (associated with  $\omega_1 = 0.5\text{Hz}$ ) and  $C_2 \approx 3.2$  (associated with  $\omega_2 = 1.0\text{Hz}$ ). All of the other  $C_n$  are zero. The relative contribution, or importance, of the two different frequencies is readily evident.

The shortcoming of this approach is that only discrete frequencies can be considered. What happens at intermediate frequencies, say  $\omega = 0.75\text{Hz}$ ? To remedy this problem, the continuous Fourier spectrum has been devised. It acts very much like the discrete spectrum but is defined as

$$C(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad (5)$$

Note the similarity of this formula to the definitions of the Fourier coefficients in Equations (3) and (4). The similarity can be seen more clearly by remembering that sine and cosine can be expressed in terms of a complex exponential function (using Euler’s identity). The continuous Fourier spectrum  $C(\omega)$ , as calculated by Equation (5), is shown in Figure 1b with the gray line. Obviously, the most important frequencies are still at  $\omega = 0.5\text{Hz}$  and  $1.0\text{Hz}$  but now other frequencies can be considered.

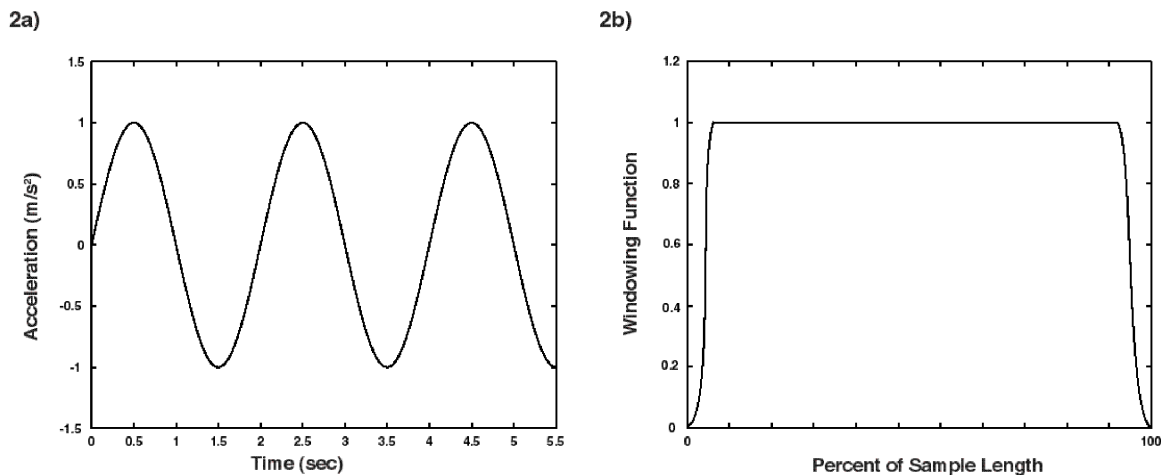
So how does this all tie into this lab? To begin, frequency domain techniques are extremely powerful in identifying natural frequencies and damping. However, it is not always easy to compute  $C(\omega)$  from experimental data. As a result, the objectives of this lab are (1) to identify the difficulties associated with computing the frequency spectrum, (2) to understand how these difficulties may distort the results (if not handled properly), and (3) to measure successfully the natural frequencies and modal damping by carrying out some simple tests on a structure.

## Factors Influencing the Spectra

### a. Windowing

When calculating the Fourier coefficients for a signal, the integrals were over *exactly* one period of the motion,  $T$ . When calculating the continuous frequency spectrum with Equation (5), there is a similar requirement. Hence, in an experimental context, it is important to record an integer multiple of the period of the motion - from which the spectrum may be computed. However, it is almost impossible to know a-priori what the period of your experimental signal is such that you can choose the duration of the record appropriately. This is particularly true when you have extremely complex signals with many frequencies participating.

So the notion of recording exactly one period of the response (or an integer multiple thereof) should be abandoned since it's thoroughly impractical. To make this issue more clear, consider Figure 2a. This signal is periodic but its record length is not exactly an integer



**Figure 2:** a) The record length of this periodic function is not an integer multiple of the period of the response. b) This window function ensures periodicity and suppresses spurious frequencies.

multiple of the response (there are about 2.75 cycles recorded ... not exactly 2 or 3). If the frequency spectrum is computed directly from this signal, it would assume that this function was periodic with period 5.5sec since it is always assumed that the sample duration is a multiple of the period  $T$ . As a result, the response would have to jump from -1 to 0 at the instant 5.5sec. This is clearly *not* what the system actually does. This causes erroneous frequencies to appear in the spectrum. Thus the actual frequencies will “leak” into a number of fictitious frequencies - giving rise to the nickname for this problem: *leakage*.

Leakage can be corrected to a large degree by using a *window function*, such as the one shown in Figure 2b. Windowing a signal involves multiplying the original signal (Figure 5a) by a weighting function, which forces the signal to be zero outside the sampling period. This enforces periodicity in the signal and significantly reduces the problem of leakage.

## b. Sampling rate

Another factor that plays a critical role in calculating the frequency spectrum is the rate of sampling (i.e., how fast you take the data in samples/sec). If the sample time is improperly chosen, a problem called *aliasing* may occur. Aliasing results from A/D conversion and refers to the misrepresentation of the analog signal by the digital recorder. Basically, if the sampling rate is too slow to catch the details of the analog signal, the digital representation will cause high frequencies to appear as low frequencies.

To avoid aliasing, the sample interval ( $\Delta t$ ) must be chosen small enough to provide at least two samples per cycle of the highest frequency to be calculated. In other words, to recover a signal from its digital samples, the signal must be sampled at a rate of at least twice the highest frequency in the signal. This is known as *Shannon's sampling theorem*.

### c. Sampling duration

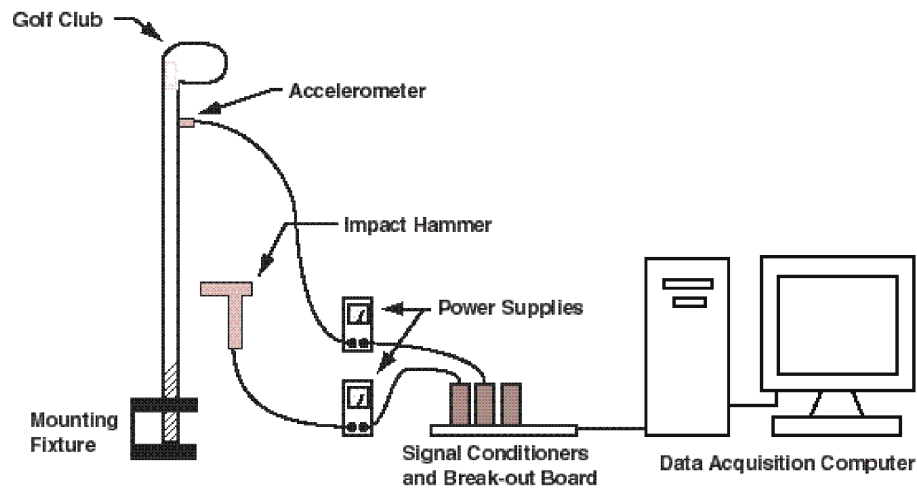
The duration of your sample may also impact the results of your frequency spectrum calculations. Specifically, if the sampled signal is too short, low frequency information may be lost. To make this more clear, consider a beam that is vibrating at 2Hz. If one second of data is acquired from an accelerometer, then the motion of the beam will not have had a chance to repeat itself. This 2Hz frequency will be lost in the spectrum since the sample duration was too short.

### d. Filtering Data

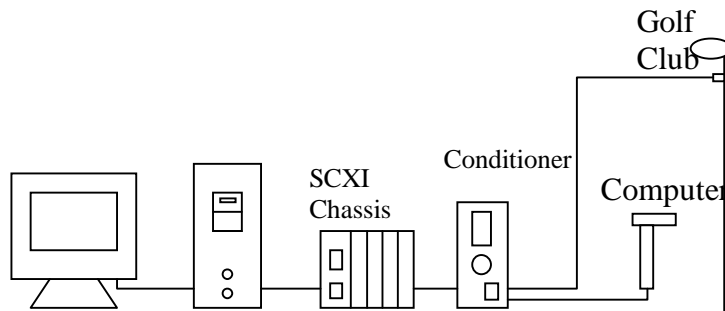
Data filters act very much like oil filters in a car - they let certain information pass through unobstructed (like the oil in your engine) but hold other information back (like the dirt in the oil). Filters are designed to work in the frequency domain. A *low-pass filter* will allow all of the low frequency components of the signal to continue unobstructed while all high frequency components are removed. A *high-pass filter* does the opposite. A *band-pass filter* removes low and high frequency components but leaves an intermediate band of frequencies to survive. The purpose of such filters is to remove unwanted frequencies from a signal. For example, suppose a signal contained significant 60Hz noise from a power supply. This spike in the spectrum could easily be removed by passing the signal through a low pass filter set at 59Hz before computing the spectrum.

## The Test Set-Up

The structure under consideration is a golf club and the objective is to measure accurately the first two natural frequencies and the modal damping for the club. To accomplish this the set-up, shown in Figure 3, has been developed. It consists of the golf club and a base fixture, in which the club is mounted. The system is set in motion with a calibrated impact hammer. This hammer produces a voltage, which is proportional to the impact force. This force is sent to the computer for later use. Similarly, the accelerometer produces a voltage that is proportional to the acceleration of the club (this represents the *response* of the club). This signal is also sent to the computer via the signal conditioners and the DAQ board.



**Figure 3:** A schematic of the test set-up, including the golf club, impact hammer, accelerometer, power supplies, signal conditioners and a data acquisition computer.



**Figure 3-1:** A schematic of the test set-up

A LabView VI has been developed, which calculates the frequency spectrum from the acceleration and force signals. In this VI, the experimentalist can easily change the sampling rate and the sampling duration to determine their influence on the resulting spectrum. The experimentalist is also responsible for going into the LabView code and changing the filter type (low-pass, band-pass, high-pass), to determine how these change the results and which one works best for this system (note: the experimentalist will have to explain *why* one filter is better than the rest. Keep this in mind during the experiments.).

### Issues to Address

1. To begin, look over the LabView VI and make sure you know roughly what's going on inside the VI. Note that on the front panel, you have complete control over the sampling rate and the sampling duration.

2. Based on your physical understanding of the golf club system, you should choose an appropriate sampling rate and a sampling duration. Choose a sampling rate based on the Shannon sampling theorem. Next, you will be using the impact hammer to initiate vibrations in the golf club (you don't have to hit it very hard! Be gentle). Save the spectrum information to a file. Do your results look reasonable or do you think aliasing may have occurred? Now choose a much higher sampling rate (say, 10 times higher) and recompute the spectrum. Save the results to a file. How does this new result compare to the previous? What does this tell you about the Shannon sampling theorem?
3. Carry out a series of tests to arrive at a "rule-of-thumb" for choosing a good sampling rate (for this golf club system).
4. Now that you've figured out a good sampling rate for this experiment, go into the LabView VI and remove the Hanning window. Now measure the frequency spectrum for an arbitrary sampling duration. Describe what the window does to the spectrum.
5. There are a variety of different windows that you can use (though the Hanning is by far the most common). Try one of the other windows available in LabView. How does your window impact your spectrum? Give specific details of the window chosen and how your results differ.
6. Intentionally alias the system (choose a large  $\Delta t$ ). Try putting a low-pass filter, then a high-pass filter, and then a band-pass filter in the VI. Which type of filter best removes the effects of aliasing. i.e., which spectrum looks most like your "unaliased" spectrum that you found earlier (step 2).
7. What are the natural frequencies of the golf club? Estimate the damping in the first and second mode based on the half power method (also known as the quality factor method).

### Acquiring Test Data -Frequency Response Measurements

In this exercise, the Golfclub2.VI is used to determine the natural frequency and the damping coefficient. The **GolfClub2** VI allows you to measure the frequency of the input signal directly as shown in Figure 4. After setting the parameters on the front panel of the VI, the golf club is impacted lightly and the VI is run. After sampling the signal from the accelerometer for a predetermined length of time, the sampled signal is displayed and the damped frequency and the damping coefficient can be found.

- Clamp the golf club to the fixture
- Connect the interface to the input/output board in the computer
- Connect the accelerometer and the impact hammer to the signal conditioner: connect the output from the conditioner to input channel 0 and channel 1 of the interface.

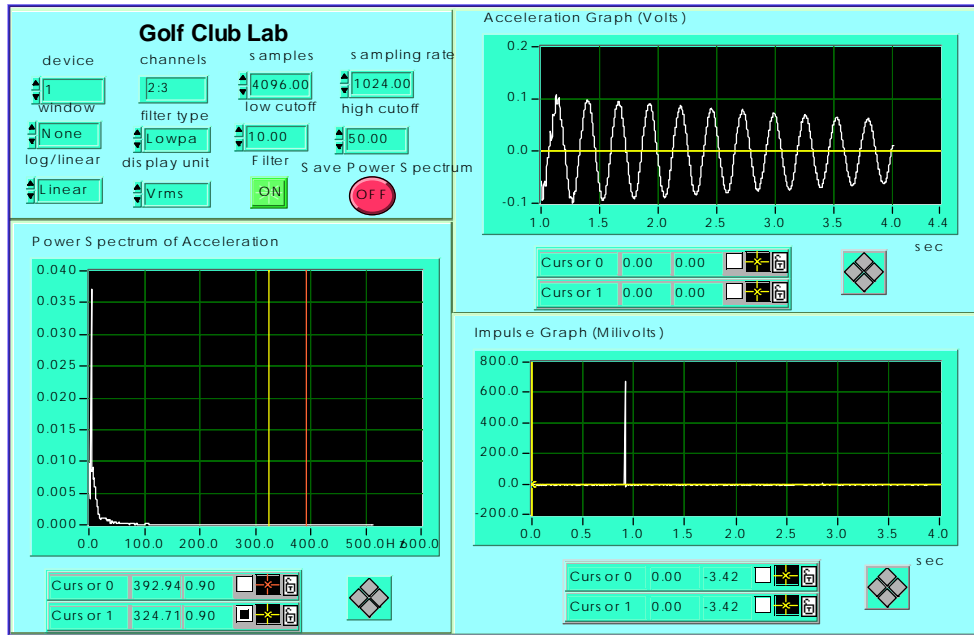


Figure 4: Front Pant VI

Below are the steps required for taking a frequency measurement.

1. Open the **VI**. (c:\me260w\lab2 two degree of freedom lab\Golfclub2.vi)
2. Make sure the SCXI chassis is turned on.
3. turn on the PCB signal conditioner.
4. Check the connection from the PCB signal conditioner to your DAQ system. Make sure the signal can get through.
5. Press the **Save data** button on the VI front panel (Option). This stores the data in a file.
  - Set the following parameters on the VI:
    - Device:1
    - Channels :ob0!sc1md2!0:1
    - Sample rate: as desired ( e.g.1024)
    - Samples: as desired (e.g. 1024)
    - Filter: off ( or on)
    - Low cutoff ( 3 or as desired)
    - High cut off : ( 50 or as desired)
    - Filter type: ( select it only if the filter is ON)
    - Windows: None or as desired
    - Filter : select as desired (non, low pass filter, high pass filter)
  - Display Setting
    - Log/linear: linear
    - Display Units: Vpk

Further refinement of these three VIs could lead to increased timesavings and perhaps, greater precision. All VIs could be brought together as sub VIs in one VI that would perform the entire experiment. The Beam Data VI could be constructed to perform the experiment a number of



times, average the data, and then export the result to the Frequency Data VI. This VI could also be configured to sweep through its frequency range a number of times, recording the data as it proceeds.

## Conclusion

The use of virtual instruments created with LabView allows the user to quickly investigate and gather data on the response of a cantilever beam subject to harmonic excitations, and also serves to introduce many students to the use of virtual instruments. This work will demonstrate that the capability to rapidly acquire, display and analyze data provides a valuable tool to students. It is also believed that the time students' take to complete the experiments will be significantly reduced by using LabView.

## Bibliography

1. Rao, Singiresu S. Mechanical Vibrations. Addison-Wesley Publishing Company (1995).
2. Rieley, William F., Sturges, Leroy D., Morris, Don H. Statics and Mechanics of Materials. John Wiley and Sons (1995).
3. Craig, R.R., and McConnell, L., "Virtual Instruments Revitalize an Undergraduate Measurements and Instrumentation Course, 1999 annual ASEE Conference Proceedings, Charlotte, NC, June 20 - 23, 1999.
4. Neebel, D. J., Blandino, J. R., and Lawrence, D. J., " A Survey of Modern Computer-Based Experiments, Part No. 321884B-01, National Instruments, Austin, TX, March 1998.
5. Smith, C.C., Lund, E.K. and Dahl, J.F., "Instrumentation and Digital Data Acquisition in Mechanical Engineering," 1994 ASEE Annual Conference Proceedings, Vancouver, BC., 1994
6. Eibeck, P.A., " Multimedia Courseware Support for the Instruction of Engineering Experimentation," Proceedings of Frontiers in Education 23rd Annual conference, sponsored by ASEE and IEEE, Washington, D.C., Nov. 6-9, pp. 135-139, 1993.
7. Smith, C.C., Heaton, H.S., and Queiroz, M., "Integration of Computer-Based Data Acquisition Systems into Undergraduate Instrumentation Laboratories, 1992 ASEE Annual Conference Proceedings, Toledo, Ohio, 1992
8. Inman, D., Engineering Vibration, Prentice Hall, Englewood Cliffs, New Jersey, 1994.

## Biographical Information

Kevin Murphy, Assistant Professor of Mechanical Engineering at University of Connecticut. Dr. Murphy joined the faculty of the Mechanical Engineering Department in 1997. He received his B.S. and M.S. in Mechanical Engineering and Applied Mechanics from the University of Michigan (Ann Arbor) and his Ph.D. from Duke University in Mechanical Engineering in 1994. His research involves both theoretical and experimental studies in nonlinear vibrations and stability of structural systems. In particular, this work has focused on the acoustics and structural vibrations occurring in surface panels on aircraft, nonlinear resonance characteristics in rotating

beams (such as helicopter blades), and vibrations in manufacturing processes such as wire electro-discharge machining (EDM). Dr. Murphy has been awarded a NASA GSRP Distinguished Fellowship, an NSF Career Award and several other competitive grants in support of his research and teaching efforts. He has served on the review boards for several technical journals and for the National Science Foundation. He has received four awards for distinguished teaching.

Ismail I. Orabi, Professor of Mechanical Engineering at University of New Haven. He received his Ph.D. from Clarkson University, and his MS degree from the State University of New York and B.S. from Cairo Institute of Technology, all in Mechanical Engineering. In the past 10 years, he has established three Laboratories: the Materials Testing laboratory sponsored by the NSF, the Engineering Multimedia Laboratory supported by AT&T Foundation and the Space Dynamic Systems Lab funded by United Technologies and the Yankee Ingenuity of the State of Connecticut. He has published over 25 technical articles in refereed journals and conference proceedings. His research interests include dynamics of linear and nonlinear structural systems, numerical simulations and seismic analysis and design.