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Student performance on drawing Free Body Diagrams and the effect on Problem Solving

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Abstract

Using data involving free-body diagrams on final exams from a first year Mechanics I course, a broadly defined rubric was created which assesses free body diagrams in six separate categories: overall quality, forces/moments, body, axes, dimensions, and resulting equations. Data from 238 free-body diagrams and equilibrium equations were then assessed. Results of the study found that 45% of the equilibrium equations had errors in them of which 67% of those errors were a direct result of errors from the free-body diagrams. Quantitative data on frequency and types of errors found in students' free-body diagrams are provided and discussed.

Background

Mechanics problems in first year engineering are typically in the form of initial and boundary value problems. Students are provided with a physical situation from which they apply the equations $\Sigma \vec{F} = m\vec{a}$ and $\Sigma \vec{M} = I\vec{\alpha}$ with the aid of other auxiliary equations and physical properties to solve for the unknown variables. Initially, some forces and moments are not obvious to the students and they must use their conceptual understanding of the situation to construct a visual representation of the problem. For simple problems with few forces/moments a student may be able to write down the equations with ease. However, as the number of forces increases the cognitive load [1] on the student increases making it difficult to setup the equations directly from the problem description. To help remedy this, an intermediate step (or additional representation) is often taken where a free body diagram (FBD) is drawn showing the forces and/or moments which act on a body. Literature has suggested that the use of multiple representations helps develop problem solving skills for students [2,3]. Formally a FBD is a schematic representation of a particle or rigid body that is isolated from its surroundings and uses vectors to represent external forces and couple moments. It includes both magnitude and angle of the force and any relevant dimensions on the location of the applied forces and the coordinate system used [4,5].

Errors reported in free-body diagrams (FBD) include: the inability to recognize differences between particles and rigid bodies [6], forces drawn at centroid [6], incorrect or missing friction forces [3,6,7], incorrect direction of weight [3,7], missing arrows [1], missing axes [3], and misaligned or unlabeled vectors [8]. Currently, evidence based literature correlating the benefits of the use of FBDs for student success remains unresolved. Some studies found that students who drew a correct FBD were more likely to solve problems correctly [7,9,10] and found that drawing a FBD incorrectly led to more incorrect solutions [10] or led to failing the course [7]. On the other hand, other researchers noted the opposite result; that the quality [2] or correctness

[1] of the diagrams did not impact student performance significantly. Furthermore, similar studies noted that students who created high quality FBDs were likely to produce low quality equations [3]. The result of the literature survey leaves uncertainty on whether FBDs are helpful to students in the construction of the equations required for solving equilibrium problems. In addition, there is little information on how different types of errors in FBDs contribute to the errors resulting in the student's equilibrium equations.

The goal of the current paper is to gain insight on the effectiveness of FBDs by studying the errors that students make on FBDs and then correlating the errors made to the effects that they have on the solution process. In addition, preventative measures on how to help students past these difficulties will be discussed.

Methodology

Data from questions on a final exam involving free-body diagrams was used from a first year calculus based Engineering Mechanics I (Statics) course. Before moving on to equilibrium concepts, students are taught vector concepts and the associated mathematical operations useful for vectors before moving onto equilibrium concepts. Instructors who taught several sections of the course adopted a "strong" teaching approach [2] when teaching FBDs. This includes defining them, explaining the importance of each step with respect to problem solving and showing the full procedure in solving equilibrium problems. Students were able to practice and receive feedback on their FBDs through weekly assignments and seminars in which their methodology for solving problems was assessed by the marker and teaching assistants as opposed to just their final answers.

The final exam from which the data was collected consisted of 5 questions each equally weighted (out of 20) focusing on: Trusses, Frames & Machines, Shear & Bending Moments, Friction, and Moment of Inertia. The exam was problem solving based and students were strongly encouraged to show both their free-body diagrams and all of their work. The problem assessed in this study was from the friction component which consisted of a wedge and block as shown in Figure 1.

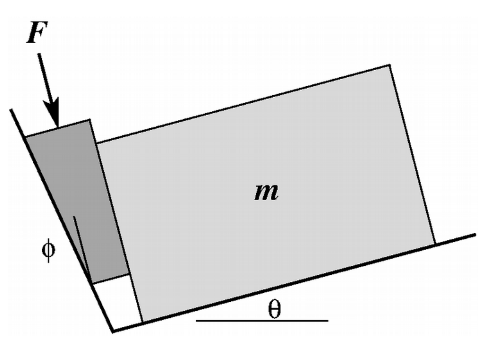
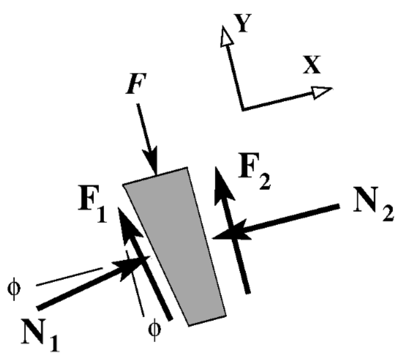


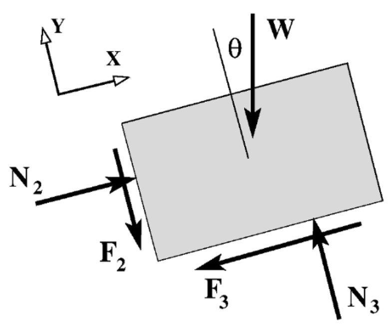
Figure 1: Diagram of the friction-wedge problem

The exam question asked students to solve for the magnitude of the point force, *F*, needed to keep the system in equilibrium at the verge of slip, given the mass of the block, the angles, and the coefficients of static friction. The problem solving approach adopted in teaching the course (the expected outcome from the students) involves the use of two separate FBDs to develop the equations in which the forces (friction and normal forces) are coupled between the diagrams. From the FBDs (Figure 2 left) the anticipated equations are presented on the right hand side of Figure 2 below.



Wedge:

$$\overset{+}{\rightarrow} \sum F_x = 0; N_1\cos(\phi) - \mu_1 N_1 \sin(\phi) - N_2 = 0$$
$$+\uparrow \sum F_y = 0; \mu_2 N_2 - F + \mu_1 N_1 \cos(\phi) + N_1 \sin(\phi) = 0$$



Block:

$$\overset{+}{\rightarrow} \sum F_x = 0; -W\sin(\theta) - \mu_3 N_3 + N_2 = 0$$
$$+\uparrow \sum F_y = 0; -W\cos(\theta) + N_3 - \mu_2 N_2 = 0$$

Figure 2: Solution to Problem with FBD and equations

We consider this an intermediate difficulty level problem as the number of steps required to reach the solution and the number of forces that are involved are greater as compared with standard problems involving only one FBD [7,11].

To assess students' use of FBDs a rubric was created (see Appendix 1) which evaluates the students' FBDs in six separate categories: overall quality, forces/moments, body, axes, dimensions, and correctness of the resulting equations. A review of the literature on rubrics for the assessment of FBDs shows two different techniques. The first, more qualitative than quantitative, used a scale [7,9,10,12,13,14] which focused on the overall correctness of the FBDs. This rubric typically broke the assessment into groupings of 0,1,2, and 3 where 0 represented no diagram present, 1 and 2 express a varying degree of mistakes in the FBD and 3 indicated that the FBD was error free. The second rubric type found to assess FBDs was more quantitative [1,2,12] and kept track of specific errors such as: forces missing, incorrect direction of forces, labels missing , missing axes, and errors in the resulting equations.

The rubric used in this study to assess students' FBDs is a mixture of the two methods described above. Our rubric is quantitative in the sense that we track all of the errors that the students make. In addition, qualitative information was also gathered. The rubric was developed starting with the formal definition of a FBD using categories (body, forces/moments, dimensions, and axes) linked to this formal definition. In order to correlate the results of the FBDs with the resulting equations a category rating the correctness of equations was added. Qualitative measures were added through a category on general impressions, keeping track of qualities such as neatness and size. In each category information on whether objects were missing or incorrect was also collected. It should be noted that there was no validation of the rubric used for the study. The rubric was developed based on information typically reported in the literature and personal experience marking FBDs; making sure that all information from the students' FBDs was captured. It was expected that some data on the usefulness of information gathered would result from the study.

Results

Free Body Diagram Quality

Results of the study showing the mean, μ, and standard deviation, σ, of the quality of the students free body diagrams are provided in Table 1.

Topic	Description	All Data , S_1 (N=238)		Errors in Eqns, S_2 (N=105)		Errors in Eqns due to FBD, S_3 (N=70)	
		Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
General	Quality	3.74	1.09	3.39	1.01	3.33	0.93
	Size	4.04	1.11	3.70	1.23	3.77	1.19
Forces	Quality	3.70	0.87	3.38	0.79	3.31	0.75
Body	Quality	4.01	1.08	3.79	1.13	3.73	1.15
Axis	Quality	3.70	1.06	3.47	1.10	3.58	1.05

Table 1: Results of Mean and Variance of different subsets of analysis using Likert Scale where poor(1)-good(5)

Here a Likert scale (1-5) was used to subjectively assess parts of the students' FBDs where a value of 1 represented that it was done poorly and 5 represented that it was of good quality. Results are shown for the complete set and two subsets of the data collected: all of the data (S_1), data with errors in the equations (S_2), and data with errors in the equations and where those errors were associated with mistakes from the FBD (S_3). The table also provides the sample sizes (N) of each sample set. For the all of the data, results indicate that students drew fairly good quality FBDs (μ=3.74, σ=1.09). This result was found to be similar for the quality of forces, bodies, and the FBD's axes. A comparison between each set of data show that mean values do not change significantly between data sets S_1 and S_2 (5.8%) and even less between data sets S_2 and S_3 (1.5%). With the lack of variability in the mean and standard deviations between sample sets, the results suggests that the quality of the FBDs may not be a good assessment indicator for solving equilibrium problems.

Errors Found in Free Body Diagrams

Quantitative data from the rubric shows that 45% of the equations developed by the students had errors in them. Furthermore, 67% of those errors in the equilibrium equations were a direct result of errors in the FBD. Other errors in the equations not directly linked to the FBD include: incorrect choice of sine/cosine, sign errors, or mistakes in translation from FBD to equations. The data was further analyzed to determine the different types of errors found in the students' FBDs and these results are shown in Table 2 where the frequency of each error was noted for each data set. First with respect to the body, students did not have problems drawing them and a majority of the students drew bodies as opposed to particles as expected.

Topic	Description	Count for each Data Set		
		S_1 (N=238)	S_2 (N=105)	S_3 (N=70)
Forces/ Moments	Missing	24	15	10
	Label Missing	11	7	3
	Direction Incorrect	43	37	35
	Angle Missing	88	48	28
	Angle Incorrect	38	29	27
	Label Incorrect	4	3	3
Body	Missing	2	0	0
	Point instead of Body	7	5	4

Axis	Missing	71	31	21
Equation	Missing	27	0	0
	Errors in Equations	108	105	70
	Errors due to FBD	80	70	70

Table 2: Results of analysis showing the frequency of errors for different subsets of data

A significant number of FBDs were missing axes (30%) of which 30% also had errors in their equations which was directly caused by the incorrect FBD. It was also noted that when drawing forces on FBDs students did not have a problem labeling them. The largest issue found was due to angles missing on the FBD (37%) of which 32% resulted in incorrect equations. The next most prevalent error was due to having incorrect direction of forces (18%) of which 71% resulted in incorrect equations. This is rather interesting as it suggests that for students who drew the direction wrong but wrote the resulting equations correctly that the error was a slip in judgment as opposed to a mistake. Next, 16% of the FBD's were found to have incorrect angles of which 71% again resulted in incorrect equations. Finally, only 10% of the FBD's had forces that were missing of which 42% resulted in errors in the resulting equilibrium equations.

Issues noted in Student's Free-Body Diagrams

Recurring issues in students' FBDs were also studied and categorized. These results are shown in Table 3.

Angle	Quality	Missing, Incorrect
Force	Friction	Wrong direction, missing, double direction
	Normal	Missing, wrong direction
	Weight	Missing, wrong direction
Axes	Quality	Too far away, missing, messy
	Direction	Poorly chosen
Body	Quality	Messy, too small
	Shape	Shape incorrect

Table 3: Specific Errors found in FBDs

Here, it was noted that students have a difficult time with the frictional force: either have it in the wrong direction, forgot it entirely, or labeled it in both directions. Arising from the choice of the friction-wedge problem used in this study, other issues were noted namely with the normal force: some students either missed it entirely or gave the vector the wrong direction. Since the block was on an incline, some students had difficulty with the direction of the weight being either normal to the surface or at the incorrect angle. There were also issues with coordinate system (axes) specifications, and in certain cases no coordinate system was specified. Other issues with quality involved the coordinate system being too far from the body for it to be useful or in some cases poorly drawn set of coordinates. In addition, some students had difficulty choosing the direction of their axes. Finally, drawing the body itself was an issue for some students. In this case, the most common issue was that the wedge was drawn in the shape of a block which in some cases resulted in the students forgetting about the angle of the wedge. Other issues noted were that the body was messy or too small.

Discussion and Conclusions

Free body diagrams are used as a visual representation of forces to help students write governing equations for mechanics problems. Our results have shown that 67% of the errors resulting from equations written incorrectly are caused by errors drawn in students' FBDs. These results help support the literature [7,9,10] that claim FBDs help students solve statics problems. This result is unique as the problem studied is different from those reported in the literature. Here the current problem has equations that are coupled, a wider variety of forces, and includes more forces which are not aligned with the axes.

The most prevalent errors observed in data collected in this study were due to angles missing from force vectors, incorrect force direction, and angles incorrectly drawn. Although there were fewer errors in FBDs due to incorrect angles and incorrect directions, these errors typically resulted in incorrect equations when they occurred. Another component missing in the students' FBDs was the coordinate system (similar to results of [3]). This was found not have a large effect on the resulting correctness of the equations. All other less significant errors noted in this study were similar to those reported in the literature. Due to the differences in the problems considered and the lack of detailed information of specific errors in earlier work, a full comparison of errors was not possible.

The connection resulting between the errors in FBDs and the errors in the derived equilibrium equations suggests that more instructional effort should be placed on the drawing of FBD representations. Effort should be placed on the concepts where the errors in the FBDs directly affect the resulting equations. The use of incorrect direction of forces might imply a conceptual misunderstanding based on the type of force (ie. friction and weight) but could also be due to of a lack of practice or understanding of Newton's third law. Issues with the incorrect direction of weight could be addressed by providing students more practice with problems where gravitational acceleration is not perpendicular to the direction of motion. With respect to the direction of the frictional force, errors may be caused by: the inability of students to visualize the potential direction of motion of the body, a lack of understanding of the guidelines used to determine the direction of the friction force, or a lack of understanding of the limitations of these guidelines. It may be beneficial to allocate more time in the course for the students to develop a better understanding of friction forces through a better description of how their direction is determined especially in problems where the direction of potential motion is not easily found. Issues with missing or incorrect angles could be caused by: a missing coordinate system, labeling an angle with respect to an incorrect axis, or by difficulty in the extraction of the angle from the diagram. These types of errors could be resolved through practice with problems where forces are not aligned with an axis or by prompting students to use protractors to check their results.

Finally, a significant number of coordinate axes in FBDs were missing. Although this did not have a significant direct effect on student performance it could have an indirect effect through misaligned angles or could result in coupled equations that were more difficult for students to solve if the axis was not aligned properly with the forces. Instruction focusing on why coordinate systems are important in developing equations and examples focusing on the differences in equations from aligned and unaligned axes would help students choose a coordinate system that best suits the problem being solved.

Adding more material to a course is oftentimes difficult and undesired. To improve a student's ability to draw FBDs "low stakes" exercises could be developed where students are asked to draw the FBD and then allow them to immediately check their solution to enhance "just-in-time learning". This exercise would allow students to get exposure to many different types of problems and difficulty levels in a single study session.

Future Work

The current study focuses on problem solving skills of a single cohort of first-year engineering students. The problem chosen was at an intermediate difficulty level which necessitated students drawing two FBDs and a system of 4 equilibrium equations. Future work on the effect of using FBDs on problem solving should focus on the effect of: problem difficulty, teaching styles, student background, student motivation, and student learning styles. A study on the effect of different difficulty levels of FBDs on performance may help determine the threshold at which cognitive load becomes a barrier for students and the use of FBDs become advantageous. In addition, more work is needed to account for factors of students' efficacy in problem solving. This should shed some light on why certain concepts cause some students to have errors in their equations but not for others. Finally, studies focusing on the results of different teaching approaches and exercises aimed at improving student's FBD drawing skills are needed.

Acknowledgments

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Appendix 1 - Rubric used for assessing the free-body diagrams

UUID:		Year	
Question Mark:		Question	
Exam Mark:			

Topic	Description	Body #1	Body #2
General	FBD Missing	no yes ☐ ☐	no yes ☐ ☐
	Quality	poor good ☐ ☐ ☐ ☐ ☐	poor good ☐ ☐ ☐ ☐ ☐
	Size	poor good ☐ ☐ ☐ ☐ ☐	poor good ☐ ☐ ☐ ☐ ☐
Forces / Moments	Forces Missing/Extra	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐
	Magnitude/Label missing	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐
	Magnitude/Label incorrect	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐
	Angle Incorrect	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐
	Angle missing	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐
	Incorrect direction	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐
	Quality	poor good ☐ ☐ ☐ ☐ ☐	poor good ☐ ☐ ☐ ☐ ☐
Body	Body Missing	no yes ☐ ☐	no yes ☐ ☐
	RB instead of Point	no yes ☐ ☐	no yes ☐ ☐
	Point instead of RB	no yes ☐ ☐	no yes ☐ ☐
	Quality	poor good ☐ ☐ ☐ ☐ ☐	poor good ☐ ☐ ☐ ☐ ☐
Axis	Axis Missing	no yes ☐ ☐	no yes ☐ ☐
	direction	Incor poor good cor ☐ ☐ ☐ ☐ ☐	Incor poor good cor ☐ ☐ ☐ ☐ ☐
	Quality	poor good ☐ ☐ ☐ ☐ ☐	poor good ☐ ☐ ☐ ☐ ☐
Dimensions	Missing Dimensions	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐
	Incorrect Dimensions	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐	no 1 2 3+ all ☐ ☐ ☐ ☐ ☐
	Quality	poor good ☐ ☐ ☐ ☐ ☐	poor good ☐ ☐ ☐ ☐ ☐
Equations	Missing	no yes ☐ ☐	no yes ☐ ☐
	Errors in Equations	no yes ☐ ☐	no yes ☐ ☐
	Errors due to Incorrect FBD	no yes ☐ ☐	no yes ☐ ☐