

AC 2007-1919: STUDENT UNDERSTANDING IN SIGNALS AND SYSTEMS: THE ROLE OF INTERVAL MATCHING IN STUDENT REASONING

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Student Understanding in Signals and Systems: The Role of Interval Matching in Student Reasoning

Abstract

This study was designed to investigate student understanding in signals and systems, particularly the study of continuous-time linear, time-invariant systems. In this paper, we report on a principal finding of this investigation, namely, the importance of the *interval matching* reasoning resource in accounting for the faulty reasonings that students invoke in reasoning about central topics in signals and systems. The qualitative method of clinical interviewing was employed for probing into student understanding. Fifty-one undergraduate students majoring in aerospace engineering at the Massachusetts Institute of Technology volunteered to participate in this study. Data was analyzed with the aim of identifying the faulty reasonings that participants invoked in their response to different signals and systems problems, and the cognitive structures of reasoning resources that describe and explain the origin of these faulty reasonings. Results indicate that there is a consistency across student faulty reasonings related to three different signals and systems topics — superposition, convolution, and the Laplace transform. This consistency is ascribed to the systematicity in student invocation of the reasoning resource of the interval matching readout strategy.

Introduction

This study was designed to investigate student understanding in the undergraduate engineering course, signals and systems. Signals and systems is a core discipline in Electrical Engineering departments, and in other engineering departments, such as aerospace. Signals and systems is the study of signals and how they interact with systems, particularly linear, time-invariant (LTI) systems. Generally, the breadth and context of presentation of topics in signals and systems varies among institutions. The main variations are whether the course covers both continuous- and discrete-time systems, or only continuous-time, and whether the context of application is electric circuitry. Despite these variations, a central theme that cuts across introductory signals and systems courses is the study of continuous-time LTI systems, which constitutes the focus of this research.

It has been maintained in the science education literature that learners frequently express conceptions that are in discord with expert understanding.¹ Such conceptions could hinder student learning if not appropriately addressed and refined through instructional approaches. Research has shown that traditional modes of instruction which do not take into account students' initial knowledge state result in small gains in student understanding^{2,3} In fact, an effective means for improving student understanding is through the implementation of active learning methods.⁴ Such methods are most effective when designed based on an understanding of the nature of student understanding and the difficulties they encounter in their study of a particular discipline.

There have been few studies on student conceptual understandings in signals and systems with the notable exception of the research conducted by Wage, Buck, Welch and Wright⁵⁻⁷ who developed the Signals and Systems Concept Inventory (SSCI) to measure student understanding of core concepts in the study of linear, time-invariant systems. Despite these efforts, there is a dearth of

in-depth qualitative research on student understanding in signals and systems. The lack of empirical evidence on student learning of signal and systems has made it difficult to develop effective instructional materials that support student learning in signals and systems. This difficulty, together with the realization of student conceptual difficulties in signals and systems, surfaced through the implementation of active learning techniques,⁸ has motivated faculty from the Department of Aeronautics and Astronautics at the Massachusetts Institute of Technology to conduct a thorough investigation to better understand the conceptual problems that students encounter in their study of the discipline.

Particularly, this study was conducted with the aim of identifying students' faulty reasonings in signals and systems, and the cognitive resources that explain the origin of these faulty reasonings. In an earlier paper,⁹ we reported our findings on student reasoning about LTI electrical circuits as this constitutes the context in which signals and systems is taught in the aerospace engineering program at MIT. In a subsequent paper,¹⁰ we reported on our preliminary results and analysis of student reasoning about continuous-time LTI systems. In this paper, we report on a central finding of this study — the role of the interval matching reasoning resource in student reasoning in signals and systems.

Theoretical Framework

In this study, we adopt a complex systems perspective of knowledge representation that explains student thinking and reasoning in terms of cognitive structures of reasoning resources.^{11–15} These resources constitute the elements of a dynamic cognitive system in which they continuously interact and are reconfigured in associational patterns in response to a particular problem situation. When students are presented with a problem, a set of reasoning resources is activated and arranged bringing forth a response to the problem situation.

Reasoning resources can be defined as knowledge elements that are used to describe thinking and reasoning. They can range from fine-grained knowledge elements to larger knowledge structures. A fine-grained knowledge element is referred to as a reasoning primitive; it is what constitutes the basic units or building blocks of more complex knowledge structures. A reasoning resource can also refer to a more complex knowledge element that can be represented as a robust network of other resources.

Several theoretical constructs have been posited to describe the fine-grained knowledge elements of reasoning; these include *phenomenological primitives*,¹² *reasoning resources*,¹⁵ and *intuitive rules*.¹⁶ Despite their variations, these theories hold a common view of naive knowledge as consisting of loosely-connected knowledge elements, the activation of which is heavily dependent on context.^{16–18} A theoretically rigorous account of the nature of naive knowledge is one proposed by diSessa^{13,14} that is explicated in the remainder of this section.

diSessa^{13,14} has proposed a theory of intuitive knowledge that explains knowledge construction and development in terms of numerous and diverse interacting knowledge elements that are continuously rearranged into complex systems. Particularly, he hypothesizes the existence of two different knowledge types — *phenomenological primitives* and *coordination classes*. Phenomenological primitives or *p-prims* are abstract knowledge elements that constitute the base

level of our intuitive sense of physical mechanism. They are phenomenological because they are considered to originate from our interpretations of our experiences with the world, and are the basis upon which we make predictions and judgments of our experiences. They are primitive because they refer to irreducible mental structures that are viewed as self-evident needing no further justification.¹²⁻¹⁴ P-prims do not represent full-fledged conceptions, however, it is their organization and the appropriateness of their activation that generate conceptions or misconceptions.

An example of a p-prim is the *force as a mover* p-prim. This primitive refers to the notion that objects move in the direction they are pushed.¹² For example, in the case of a ball tossed in the air, *force as a mover* accounts for the toss causing the upward motion of the object. This primitive was coined by diSessa¹⁹ after conducting a study to investigate student understanding of Newton's laws. He examined the performance of eight sixth grade students on a computer game called *dynaturtle*, which simulates the behavior of objects obeying Newton's laws on a frictionless surface. The students were instructed to command the turtle to change direction onto a path that would cause the turtle to strike a predetermined target. For example, the *dynaturtle* was set to move upwards, and the students were asked to "kick" the *dynaturtle* in a direction that would cause it to hit the target located to its right. A typical mistake committed by the students was turning the turtle until it faced the target and then kicked the turtle in that direction. diSessa explains that this incorrect response is due to students inappropriately invoking the *force as a mover* p-prim by ignoring the effect of the previous motion of the turtle.

Based on a systems perspective of knowledge representation, diSessa^{11,14} explains that attaining expert-like understanding entails the reorganization of p-prims and the rearrangement of their priority in one's knowledge structure. He describes the process of learning science as the development of knowledge structures that consist of isolated or loosely linked p-prims into more coherent knowledge systems that he calls *coordination classes*. Unlike p-prims, which are sub-conceptual or sub-theoretical, a coordination class is a large complex system that can constitute a model of a certain type of scientific concept. It is an integrated way of coordinating multiple observations and processing information received from the world.

Generally, a coordination class consists of two knowledge substructures: a *readout strategy* and a *causal net*.^{13,14} Readout strategies define the perceived elements that form the focus around which the system is interpreted. They are the strategies that are employed when making observations and extracting information from a particular situation. A causal net is a connected set of p-prims that develop around readout strategies. It is a set of inferences associated with a coordination class that is sufficient to cover or span all contexts in which the information is needed.

This theory of coordination classes can be illustrated with an example from Wittmann's²⁰ analysis of student understanding of mechanical waves. Generally, the findings of Wittmann's study indicate that students tend to incorrectly reason about waves as object-like rather than as event-like. Students incorrectly invoked the readout strategies of *wave as solid* and *wave as object* bringing to bear a set of inappropriate reasoning resources all of which coordinate and are organized around the object-class. For instance, in a problem that Wittmann presented to students, a taut string attached to a wall is flicked creating a pulse that travels to the wall, and students were asked to provide ways through which one can create a slower pulse. A prediction that students

made is that the pulse will travel slower if the string is flicked slower, and it will travel faster if the string is flicked faster. This prediction is incorrect; the speed with which the wavepulse travels does not depend on the speed with which the hand is flicked but only on the properties of the medium through which it travels. Wittmann explains that this reasoning is consistent with the *working harder* p-prim: the greater the effort that is exerted, the greater the effect. This reasoning is correct if applied to an object, such as a ball. The faster one moves one's hand when throwing a ball, the faster the ball will travel.

In this study, we attempt to identify the reasoning resources that explain student faulty reasonings in signals and systems. We use the term reasoning resources to refer to any cognitive structure used to describe or explain student reasoning, and the term reasoning primitives²¹ to refer to the basic units of knowledge, such as p-prims, that are applied when reasoning about a particular problem situation. Reasoning resources can refer to either atomic elements or complex systems of elements of reasoning. They can refer to reasoning primitives, readout strategies, or coordination classes.

Methodology

The qualitative method of clinical interviewing was employed for probing into student understanding. Due to its methodological flexibility,²² clinical interviewing is valuable in allowing to delve into one's thought, and for capturing the crucial characteristics of one's thinking.^{22,23} Particularly, with respect to the goals of this study, it is the most adequate method as it provides the flexibility to uncover the reasonings underlying student responses to signals and systems problems and the elements of the student's mental context that generate these faulty reasonings. It allows the interviewer to spontaneously generate questions during the interview targeted to uncover students' natural mental inclinations and thought processes.²⁴

Signals and Systems, as taught in the Department of Aeronautics and Astronautics at MIT, is a part of a larger engineering course, *Unified Engineering*, that is offered as a requirement for sophomore students. Signals and Systems, as taught in *Unified*, consists of two parts: The first part, covered during the first five weeks of the Fall semester, involves the analysis of linear electrical circuits. The second part, offered during the last eight weeks of the Spring semester, involves the analysis of generic continuous-time, linear systems.

In 2002-2003, one-hour oral assessments were introduced as part of the requirements in Signals and Systems. These assessments were the interviews for this research. Each student enrolled in *Unified* was scheduled for a one-hour oral assessment session. In the Spring semester, students were divided into seven cohorts, and each cohort had their oral assessment sessions in the same week and they worked on the same oral problem. The oral assessment sessions were divided into two half-hour sessions. During the first half hour, students were given the problem statement and were required to prepare a preliminary answer in private. During the second half hour, students were interviewed on their solutions to the problem.

For the Spring semester, seven different oral problems (Oral Problems 5–11) were developed to assess student understanding of the following signals and systems topics: (1) superposition; (2) convolution; (3) Laplace transforms; (4) inverse Laplace transforms; (5) BIBO stability; and (6)

amplitude modulation. Students who volunteered to participate in this study agreed to have their oral sessions audio-taped and video-taped. Fifty-one out of the 69 students enrolled in *Unified* volunteered to participate in this study.

The interview tapes were transcribed, and the transcripts were analyzed qualitatively. The analysis was conducted with the aim of identifying the faulty reasonings that participants invoked in their response to each of the different signals and systems problems, and the cognitive structures of reasoning resources that describe and explain the origin of these faulty reasonings.

Results

The results of this study relate to student understanding of three central signals and systems topics: superposition, convolution, and the Laplace transform. Results indicate that student faulty reasonings can be ascribed to an intuitive knowledge base of reasoning resources that students inappropriately invoke in their response to a signals and systems problem. Moreover, a degree of consistency with which certain resources were invoked across the different oral problems was identified. This consistency is partly ascribed to the systematicity in student invocation of the reasoning resource of the interval matching readout strategy. This paper particularly focuses on the role of interval matching in student reasoning in signals and systems. Note that this discussion does not emphasize other resources that were elicited around the interval matching readout strategy.

Interval matching was commonly invoked by participants as a way to interpret and readout information from signals and systems problems. In signals and systems problems, students are generally given at least one time function u and are asked to determine another function that relates to u through some linear operator. Such an operator usually represents the system, the convolution integral, or the Laplace and Fourier transforms.

For instance, consider the case in which the operator represents the system. Typically, the given function u in a signals and systems problem is piecewise continuous. Consider u to be

$$u(t) = \begin{cases} u_1(t), & t \in I_1 \\ u_2(t), & t \in I_2 \\ \vdots & \vdots \\ u_n(t), & t \in I_n \end{cases}$$

where $I_1, I_2 \dots I_n$ are the domain intervals over which u is defined.

In a typical systems analysis problem, students are given the input $u(t)$ to an LTI system and are required to determine the response $y(t)$ of the system to this input. A system establishes a relationship between its input $u(t)$ and its output $y(t)$ that is given by

$$y = \mathcal{H}[u]$$

where \mathcal{H} is an operator that represents the system and that defines the operations that the system performs on $u(t)$ to produce $y(t)$. By invoking the interval matching readout strategy, the function $y(t)$ would be determined by first dissecting the problem into intervals that match the domain

intervals over which the function $u(t)$ is defined. Then only the expression of $u(t)$ over a particular interval would be utilized to determine the expression of $y(t)$ over that interval. That is, the function $y(t)$ would be expressed as

$$y(t) = \begin{cases} y_1(t), & t \in I_1 \\ y_2(t), & t \in I_2 \\ \vdots & \vdots \\ y_n(t), & t \in I_n \end{cases}$$

where

$$\begin{aligned} y_1(t) &= \mathcal{H}[u_1(t)] \\ y_2(t) &= \mathcal{H}[u_2(t)] \\ &\vdots \\ y_n(t) &= \mathcal{H}[u_n(t)] \end{aligned}$$

Students adopted this readout strategy in various problem situations related to three signals and systems topics: superposition, convolution, and the Laplace transform: in Oral Problem 5, they invoked interval matching when attempting to determine the step response of a linear, time-invariant (LTI) system, given the input and the corresponding output of the system; in Oral Problems 6 and 7, they also appealed to this readout strategy when attempting to determine the output of an LTI using convolution; and in Oral Problems 8 and 9, students invoked the interval matching readout strategy to determine the Laplace transform of a function and the region of convergence. These cases are discussed below.

Oral Problem 5

Problem Statement. A system G is tested in the lab with an input given by

$$u_t(t) = \begin{cases} 0, & t < 0 \\ 2, & 0 \leq t \leq 1 \\ 0, & t > 1 \end{cases}$$

(The subscript “ t ” stands for “test.”) For this input, it is found that the output is given by

$$y_t(t) = \begin{cases} 0, & t < 0 \\ 4 - 4e^{-2t}, & 0 \leq t \leq 1 \\ (4e^2 - 4)e^{-2t}, & t > 1 \end{cases}$$

Sketch the signals above, and come prepared to answer the following questions:

1. Is it possible to determine the response $y(t)$ of the system to an arbitrary input $u(t)$? Explain.

2. Can you find the step response of the system?
3. If your answer to (2) is “yes”, what is the step response?
4. If your answer to (2) is “no”, explain why.

Description of the problem. Oral Problem 5 was used to test student understanding of the system properties of linearity and time-invariance, and their ability to apply the superposition principle in the analysis of a linear, time-invariant (LTI) system. In this problem, students were given the response, $y_t(t)$, of a system to a finite duration pulse, $u_t(t)$, and were required to determine the step response of the system. Because the unit step function, $\sigma(t)$, can be built up as a superposition of scaled and delayed versions of the test input, and the system is linear and time-invariant, then the system obeys the superposition principle, and hence the step response $g_s(t)$ can be found by summing scaled and shifted versions of $y_t(t)$.

Participants. Out of the ten students who sat for Oral Problem 5, eight participated in this study. Three were female (S42, S61, and S67) and five were male (S06, S48, S49, S66, and S87).

Results from Oral Problem 5. Four participants — S48, S61, S66, and S67 — provided an incorrect reasoning about this problem that could be partially ascribed to the invocation of the interval matching readout strategy. Three of these participants — S48, S61, and S67 — concluded in their written response or during the oral session that the step response of the system is

$$g_s(t) = (2 - 2e^{-2t})\sigma(t).$$

In their written responses, these participants derived an expression for the step response of the system that is equivalent to $2 - 2e^{-2t}$. Also, during the oral session, participants S61 and S67 graphed $g_s(t)$ as $(2 - 2e^{-2t})\sigma(t)$. Even though this answer is correct, participants’ reasoning underlying this response was faulty. They incorrectly reasoned that because the step function can be obtained by extrapolating the function $u_t(t)/2$, defined over $0 \leq t \leq 1$, to infinity, then the system’s step response can be obtained by extrapolating $y_t(t)/2$ for $0 \leq t \leq 1$ to infinity.

The origin of this response can be attributed to the invocation of the interval matching readout strategy. These participants applied the interval matching readout by matching the input $u_t(t)$ over the time interval $0 \leq t \leq 1$ with the output $y_t(t)$ for that time interval. They incorrectly assumed that the input defined over $0 \leq t \leq 1$ is only responsible for the system’s response for $0 \leq t \leq 1$, and hence they disregarded the system’s response for $t > 1$. Then, they utilized the expression for $y_t(t)$ only over the time interval $0 \leq t \leq 1$ to derive the step response of the system $g_s(t)$. They argued that because the step function is a scaled and extrapolated version of the input function $u_t(t)$, as defined over $0 \leq t \leq 1$, then the system’s step response should be a scaled and extrapolated version of $y_t(t)$ as defined over $0 \leq t \leq 1$, hence obtaining the answer $(2 - 2e^{-2t})\sigma(t)$. The following excerpts clearly reflect how participants S61 and S67 invoked the interval matching readout when justifying that the step response of the system is $g_s(t) = (2 - 2e^{-2t})\sigma(t)$:

S61: Well, I'm thinking you only use only the $y_t(t)$ on the range from 0 to 1? Yeah, I ...

I: Yeah, but you have to tell me what happens for $t > 1$ then.

S61: Oh, for t greater ... [...] this $y(t)$ is for $t > 0$ like all time. For $t > 1$ this is true. For t is 1000 this is true. Umm ...

I: Okay, so in the problem statement ... [...] Roughly you can say that you got $2 - 2e^{-2t}$. [...] Okay. So you're telling me that this part [$y_t(t)$ for $t > 1$] [...] has no importance whatsoever?

S61: Right.

.....

S67: We're given the response, so we want to find the unit step response, and for this problem it's actually easy, because for, between $t = 0$ and $t = 1$ [...] the input is just two times the step input, so then the response is two times the step response.

(Note that a bracketed ellipsis is used to indicate an instance when irrelevant utterances are omitted for the purpose of including in the quote the most concise and relevant verbalizations. An ellipsis is used to indicate that the speaker did not finish a sentence.) Participant S67 further explained later in the interview:

S67: Because, like, regardless of what happens after $t > 1$, like, between $t = 0$ and $t = 1$, the input is twice the step input. Like, that's, like, a defined thing [...] So for the output then it's got to be, like, between 0 and 1, and it's got to be twice the other output.

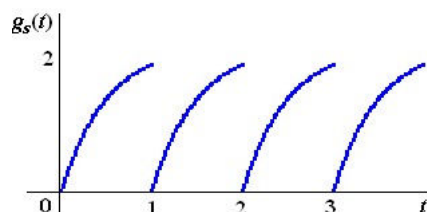
I: And let me repeat back your argument to you.

S67: Okay.

I: And you tell me if this is your argument. [...] Your argument, I think, is that if this input [$\frac{1}{2}u_t(t)$] produces that [$\frac{1}{2}y_t(t)$], this input [$\sigma(t)$] produces that [$g_s(t)$] because if I cover up before $t = 1$, these [$\sigma(t)$ and $\frac{1}{2}u_t(t)$] are identical.

S67: Right.

Participant S66 similarly reasoned about the problem when asked to deduce the step response of the system, however, he arrived at a different answer. He concluded the step response, $g_s(t)$, to be



This response can be also ascribed to the invocation of the interval matching readout strategy. However, participant S66 appealed to the notion of extrapolation not in the sense of an extension but as a projection or replication of a given pattern. He argued that because the step function can be obtained by extrapolating the scaled version of the input function $\frac{1}{2}u_t(t)$ as defined over $0 \leq t \leq 1$ to infinity, then the system's step response can be obtained by "carrying on" $\frac{1}{2}y_t(t)$ for $0 \leq t \leq 1$ to infinity:

S66: Well, if you have a step, and you just kind of basically just carry this $[\frac{1}{2}u_t(t)]$ on, so you should just carry this on $[\frac{1}{2}y_t(t)]$.

Oral Problem 6

Problem Statement. An LTI system G has impulse response

$$g(t) = \begin{cases} 0, & t < 2 \\ e^{-2t}, & t \geq 2 \end{cases}$$

The input to the system is given by

$$u(t) = \begin{cases} 1, & -1 < t < 3 \\ 0, & \text{otherwise} \end{cases}$$

Find the output of the system, using the convolution integral. Sketch the signals $g(t)$, $u(t)$, and $y(t)$, and come prepared to discuss your answer.

Description of the Problem. This problem was designed to test student understanding of the convolution integral, and their ability to apply the integral in determining the response of a linear, time-invariant (LTI) system to an input, given the impulse response of the system. Because graphical convolution was not yet introduced in the course when this problem was administered, students were not expected to be able to solve this problem using graphical methods.

Participants. Out of the eight students who sat for Oral Problem 6, seven participated in this study. Six were male (S02, S50, S54, S59, S63, and S73) and one was female (S24).

Results from Oral Problem 6. Five participants — S02, S24, S50, S59, and S63 — provided incorrect responses to this problem that could be ascribed to the inappropriate invocation of the interval matching readout strategy. Four of these participants — S02, S24, S59, and S63 — incorrectly deduced that the output of the system occurs over the time domain in which both functions, the input $u(t)$ and the impulse response $g(t)$, are non-zero. They represented this reasoning in their graphical representations of $y(t)$ and in their expressions of the convolution integral.

These participants graphed the output of the system G as having non-zero values exclusively in the time interval $[2, 3]$. They determined the time intervals over which the non-zero regions of the

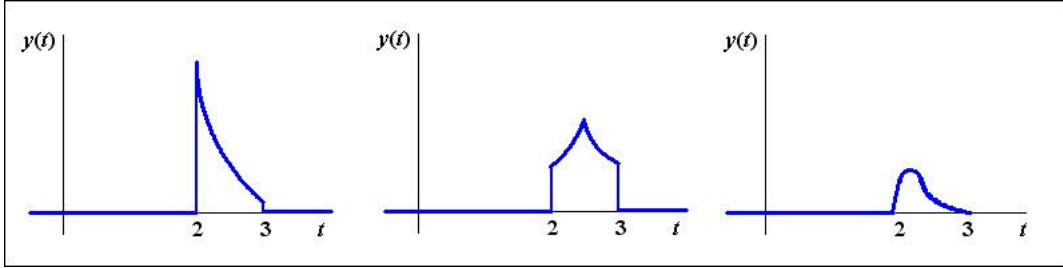


Figure 1: Participant graphical representations of the output $y(t)$.

To the left: Graph of $y(t)$ sketched by participants S59 and S63. In the middle: Graph of $y(t)$ sketched by participant S02. To the right: Graph of $y(t)$ sketched by participant S24.

graphs of $u(t)$ and $g(t)$ overlap, and concluded that the output $y(t)$ is non-zero only in the interval $[2, 3]$. Accordingly, they sketched $y(t)$ as having non-zero values only in this time domain as shown in Figure 1.

With regards to their analytical responses, all four participants — S02, S24, S59, and S63 — incorrectly expressed the convolution integral. Three of these participants — S02, S24, and S59 — defined the limits of integration of the convolution integral to match the limits of the time domain in which both $g(t)$ and $u(t)$ are non-zero. That is, they defined the limits of integration to be from 2 to 3, where 2 corresponds to the time the impulse response begins, and hence the time they predicted the output to begin, and 3 corresponds to the time the input ends, and hence the time they predicted the output to end:

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} g(t - \tau)u(\tau) d\tau \\ &= \int_2^3 e^{-2(t-\tau)} \cdot 1 d\tau \end{aligned}$$

In fact, the domain of integration should be the domain for which both $g(t - \tau)$ and $u(\tau)$ are non-zero.

Participant S59's response could better illustrate this participant reasoning. Participant S59 obtained the same expression that participants S02 and S24 obtained for $y(t)$, but his written solution was more elaborate and indicative of the reasoning underlying his response. He wrote:

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} g(t - \tau)u(\tau) d\tau \\ &= \int_{-\infty}^{-1} g(t - \tau)u(\tau) d\tau + \int_{-1}^2 g(t - \tau)u(\tau) d\tau \\ &\quad + \int_2^3 g(t - \tau)u(\tau) d\tau + \int_3^{\infty} g(t - \tau)u(\tau) d\tau \end{aligned}$$

The participant matched the output of the system $y(t)$ for a given time interval to the input $u(t)$ and the impulse response $g(t)$ for that same time domain. Then, he incorrectly reasoned that because $g(t)$ and $u(t)$ are zero over the time interval $(-\infty, -1]$, and $g(t)$ is zero over the time

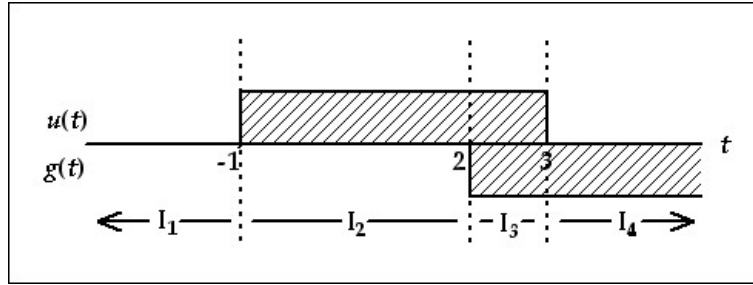


Figure 2: Interval matching readout strategy applied to Oral Problem 6.

interval $(-1, 2)$, and $u(t)$ is zero over $[3, \infty)$, then the first, second and fourth integrals evaluate to zero respectively, thereby reducing the expression of $y(t)$ to

$$y(t) = \int_2^3 e^{-2(t-\tau)} \cdot 1 \, d\tau.$$

Participant S59 explained:

S59: I broke it [the convolution integral] up into [inaudible speech] sections where you can definitely say if it's either going to be 0 or not. And you can definitely calculate it for each individual section. Because $u(t)$ goes from -1 to 3 , and $g(t)$ is only greater than 2 , the sections from $-\infty$ to -1 where both $g(t)$ and $u(t)$ were both 0. From -1 to 2 where $g(t)$ was 0, where $u(t)$ was 1. And from 2 to 3 , where both of them were non-zero. From 3 onto ∞ where $u(t)$ was 0 again. And after doing that, I noticed that there was only one integral from 2 to 3 .

The reasoning these participants invoked to solve this problem could be explained as partly due to the invocation of the readout strategy of interval matching. They matched the output of the system for a given time interval to the input and the impulse response as defined over that time interval. According to this readout, the problem could be dissected into the following four different time intervals: $I_1 = (-\infty, -1]$; $I_2 = (-1, 2)$; $I_3 = [2, 3]$; and $I_4 = [3, \infty)$. (See Figure 2.) Participants then utilized the expressions for both $g(t)$ and $u(t)$ over a particular interval to determine the expression for the output $y(t)$ over that interval. Analytically, they expressed the output over a certain time interval $[t_1, t_2]$ as the integration over that interval of the product of $g(t - \tau)$ and $u(\tau)$, where the expressions for these correspond to those of $g(t)$ and $u(t)$ respectively over the time interval $[t_1, t_2]$.

Based on this readout of the problem situation, participants invoked the reasoning primitive of *temporal overlap*, when determining the time interval over which the output $y(t)$ occurs. This resource is an abstraction of the phenomenon of the co-occurrence of two events. During the time intervals in which at least one of the two events does not occur, no overlap of events occurs, and hence the product of their interaction is annulled. Accordingly, participants inferred that the limits of integration of the convolution integral are the limits of the time interval over which the

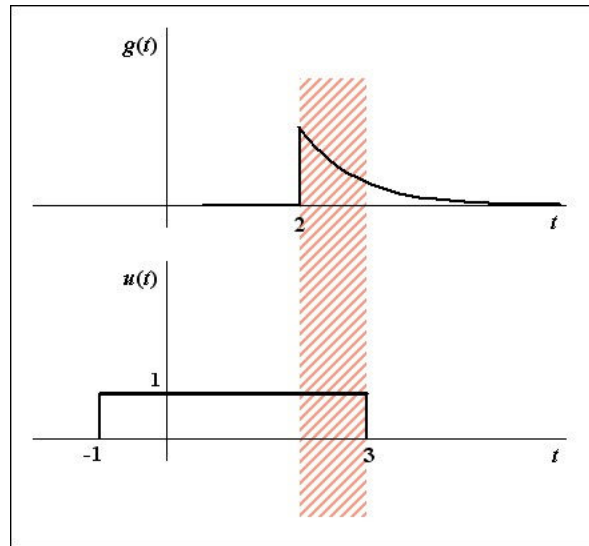


Figure 3: Novice use of the reasoning primitive of temporal overlap as the domain of overlap of the two curves $g(t)$ and $u(t)$.

occurrence of $g(t)$ and $u(t)$ overlap; that is, over the interval $[2, 3]$ in which the values of the functions $g(t)$ and $u(t)$ are non-zero. This reasoning is evident in the following excerpts:

S24: I said it only happens between 2 and 3 because the impulse response didn't start until 2 and then the input ended at 3.

.....

S02: So I just laid one on top of the other and then I said, well [...] from $-\infty$ to 2 it's going to be 0 because u is 0 up to 2, and then so from 3 to ∞ it will be 0 as well [...] and then u after 3 it's 0, so that cancels here and back here, so I just winded up with this.

.....

S63: Convolve the two [$u(t)$ and $g(t)$]. It's combining the impulse response for the interval that the input is valid and it's zero from -1 to 2. And then from 2 to 3 there's this section here of g is the impulse response, which would show up in the output.

Participant S50 analyzed the problem similarly as participants S02, S24, S59, and S63 when determining the time at which the output begins; however, he derived that after $t = 3$, the output drops exponentially and gradually dies away. He dissected the problem into three time intervals: (1) $t < 2$, (2) $2 \leq t < 3$, and (3) $t \geq 3$. Then he predicted that the output would be zero for $t < 2$ because the impulse response is zero for $t < 2$, and that at $t = 3$ the output would change because the input ends at that point. For $t < 2$, the participant stated that the output $y(t)$ would be zero, and for each of the time intervals $2 \leq t < 3$ and $t \geq 3$, he derived an expression for $y(t)$ using the convolution integral. However, unlike the other participants, participant S50 did not define the

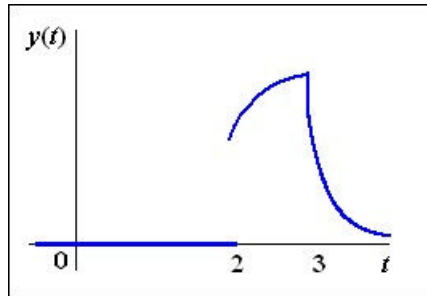


Figure 4: Graph of $y(t)$ sketched by participant S50.

limits of integration of the convolution integral to match the limits of the respective time interval with which the convolution integral is associated. Participant S50 wrote

$$\begin{aligned}
 t < 2 : \quad y(t) &= 0 \\
 2 \leq t < 3 : \quad y(t) &= \int_0^t e^{-2t+2\tau} \sigma(t-2) d\tau \\
 t \geq 3 : \quad y(t) &= \int_0^3 e^{-2t-2\tau} \sigma(t-2) d\tau + \int_3^t e^{-2t+2\tau} \cdot 0 d\tau
 \end{aligned}$$

Participant S50 then graphed the expressions he obtained for $y(t)$ over the different time intervals as shown in Figure 4. He explained that the graphical representation he obtained for $y(t)$ was consistent with his conceptual understanding of the problem, and that the discontinuities in the graph of $y(t)$ at $t = 2$ and $t = 3$ are due to the fact that $g(t)$ is zero for $t < 2$ and $u(t)$ ends at $t = 3$:

S50: I was just thinking through conceptually making sure this all made sense that you know it's obviously going to be 0 because there's no input or response at that point up until 2.

.....

S50: Well, I say g is the impulse function would govern what happens here because if you put an input in until 2, the response is effectively dead because I mean nothing comes out because $g(t)$ is just 0. So that's why it doesn't do anything until here [$t = 2$] in spite of the fact that there is an input starting at -1 , but then ...

I: So the fact that this [time at which the output starts] is a 2 is associated with the fact that this [time at which the input starts] is a 2.

S50: Yes.

I: And what about this discontinuity at 3?

S50: That's to do with the, the input gets cut off here [at $t = 3$] and it goes from about that point and drops down.

Participant S50's interpretation of the problem situation can be also explained as an instance of invocation of the interval matching readout strategy. He matched the output of the system for a given time interval to the input and the impulse response as defined over that time domain, and dissected the problem into three different time intervals: (1) $t < 2$, (2) $2 \leq t < 3$, and (3) $t \geq 3$. He also invoked the notion of temporal overlap, and incorrectly explained that the output $y(t)$ is zero for $t < 2$ because $g(t)$ is zero for $t < 2$, and is non-zero for $2 \leq t < 3$ since both $g(t)$ and $u(t)$ are non-zero over this time interval.

Oral Problem 7

Problem Statement. An LTI system G has impulse response

$$g(t) = \begin{cases} 0, & t < 0 \\ e^{-2t}, & t \geq 0 \end{cases}$$

The input to the system is given by

$$u(t) = \begin{cases} 0, & t < 1 \\ 1, & 1 \leq t < 2 \\ -1, & 2 \leq t < 4 \\ 0, & t \geq 4 \end{cases}$$

1. Sketch the signals $g(t)$ and $u(t)$.
2. Use graphical techniques to sketch $y(t) = g(t) * u(t)$.
3. Express $y(t)$ as one or more integrals, with appropriate limits of integration (which may depend on t).
4. Find $y(t)$.

Description of the problem. While Oral Problem 6 required students to determine the response of a linear, time-invariant (LTI) system analytically using the convolution integral, this problem requires finding the output of an LTI system using both the convolution integral and graphical methods. Oral Problem 7 was given to students after they had been taught graphical convolution.

Participants. All nine students who sat for Oral Problem 7 participated in this study. Two were female (S51 and S81) and seven were male (S01, S05, S07, S09, S55, S90, and S99).

Results from Oral Problem 7. All participants, except for participant S55, had difficulty determining the limits of integration of the convolution integral. Five of the nine participants — S01, S05, S07, S09 and S90 — wrote down similar expressions for $y(t)$. Like the participants who responded to Oral Problem 6, these participants also incorrectly reasoned that the limits of integration of the convolution integral should match the limits of the time domains over which the input and the impulse response are non-zero.

Four participants — S01, S07, S09 and S90 — expressed $y(t)$ as

$$y(t) = \int_1^2 e^{-2(t-\tau)}(1) d\tau + \int_2^4 e^{-2(t-\tau)}(-1) d\tau \quad (1)$$

and participant S05 expressed $y(t)$ as

$$y(t) = \begin{cases} 0, & t < 1 \\ \int_1^2 e^{-2(t-\tau)} d\tau, & 1 \leq t < 2 \\ \int_2^4 -e^{-2(t-\tau)} d\tau, & 2 \leq t < 4 \\ 0, & t \geq 4 \end{cases} \quad (2)$$

These participant responses can also be explained as due to an incorrect invocation of the interval matching readout strategy. Participants matched the time intervals over which the convolution integral is integrated with the different time domains over which the input and the impulse response are defined. According to the interval matching readout, $u(t)$ and $g(t)$ can be matched over the following five different time intervals: $I_1 = (-\infty, 0]$; $I_2 = (0, 1)$; $I_3 = [1, 2)$; $I_4 = [2, 4)$; and $I_5 = [4, \infty)$. (See Figure 5.) Based on this readout, participants invoked the reasoning primitive of temporal overlap and inferred that the limits of integration of the convolution integral are the limits of the time intervals over which the values of both functions $g(t)$ and $u(t)$ are non-zero. Hence, they inferred that the limits of integration are from 1 to 2 and from 2 to 4. The following quotes illustrate this participant reasoning:

S09: My convolution integral ended up being [. . .] [Participant writes $y(t) = \int_0^\infty e^{-2(t-\tau)}u(\tau) d\tau$.] The $u(\tau)$ changes over three intervals. So I took and broke this interval $[0, \infty]$ up across the τ 's that it would change. So it's from . . . $-\infty$ to 1 was just 0. And this interval . . . 0 to 1 is 0. From 1 to 2 it was 1 [. . .] And then from 2 to 4 it was just -1 . So that's what I got.

.....

S01: And for this problem I said [. . .] I'm going to break it up into two integrals, and this integration for the first part, where $u(\tau)$ is from 1 to 2, it would be $e^{-2(t-\tau)} d\tau$ because [. . .] $u(\tau)$ is equal to 1, plus the integral from [. . .] 4 to 2, because the next part, where [. . .] $u(\tau)$ is negative, and it's going to be still $e^{-2(t-\tau)} d\tau$.

Participant S05 not only matched the limits of integration of the convolution integral with the limits of the time domains over which the input $u(t)$ and the impulse response $g(t)$ are non-zero, but also he matched the time intervals over which the output occurs with the time intervals over which both $u(t)$ and $g(t)$ are non-zero. (See Equation (2).) This incorrect response is similar to the error participants committed in Oral Problem 6. He invoked the reasoning primitive of overlap in the sense that over the time domains in which the occurrence of $u(t)$ and $g(t)$ do not overlap, that is, either $u(t)$ or $g(t)$ is zero, the output is zero. Hence, the participant deduced that for $t < 1$ and $t \geq 4$, the output is zero, and it is non-zero only over the time intervals $1 \leq t < 2$ and $2 \leq t < 4$.

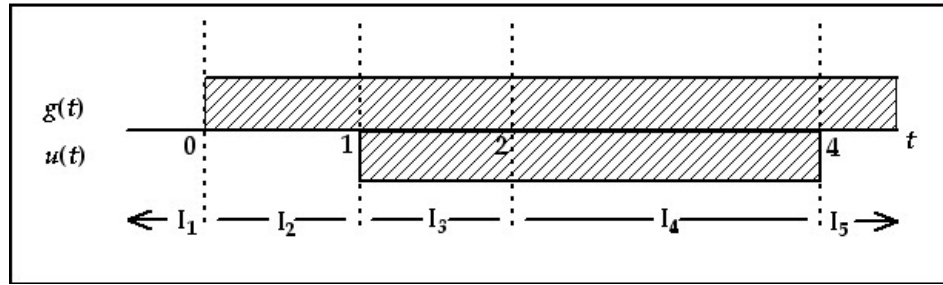


Figure 5: Interval matching readout strategy applied to Oral Problem 7.

Oral Problem 8

Problem Statement.

1. Show that if a signal $h(t)$ has Laplace transform $H(s)$, then the same signal delayed by T seconds, $h(t - T)$, has Laplace transform $e^{-sT}H(s)$. Hint: Write out the Laplace transform integral, and perform a change of variables. What is the region of convergence?
2. For the signal

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & 0 \leq t < 1 \\ -1, & 1 \leq t < 3 \\ 0, & t \geq 3 \end{cases}$$

what is the Laplace transform, $U(s)$, of $u(t)$.

3. A linear, time-invariant system is described by the differential equation

$$y'(t) + 2y(t) = u(t)$$

The initial condition is that $y(0) = 0$. For the $u(t)$ given in part (2), what is the Laplace transform, $Y(s)$, of $y(t)$?

4. Find $y(t)$.

Description of the Problem. Oral Problem 8 was used to test student understanding of the Laplace transform and their ability to apply Laplace transform methods in solving a linear differential equation. In the first part of this problem, students were required to derive the time-shift property of the Laplace transform. In the other three parts of the problem, students had to use the time-shift property and Laplace transform methods to compute the output of a linear, time-invariant (LTI) system modeled by a linear differential equation.

Participants. All nine students who sat for Oral Problem 8 participated in this study. Five were male (S38, S40, S71, S84, and S92) and four were female (S26, S32, S64, and S86).

Oral Problem 9

Problem Statement.

1. For the Laplace transform

$$G(s) = \frac{1}{(s+1)^2(s+2)}, \quad \text{Re}[s] > -1$$

find the inverse Laplace transform using partial fraction expansions and the cover-up method.

2. It can be shown that if a signal $h(t)$ has Laplace transform $H(s)$, then the same signal delayed by T seconds, $h(t - T)$, has Laplace transform $e^{-sT}H(s)$, with r.o.c. the same as for $H(s)$. Thus, the pulse signal

$$u(t) = \sigma(t) - \sigma(t - 2)$$

has Laplace transform

$$U(s) = \frac{1 - e^{-2s}}{s}$$

If $u(t)$ is the input to a causal system with transfer function

$$G(s) = \frac{1}{s+2}$$

then the resulting output has Laplace transform

$$Y(s) = \frac{1 - e^{-2s}}{s} \frac{1}{s+2}$$

Find and sketch $y(t)$.

3. A causal system has transfer function

$$G(s) = \frac{s+2}{s^2+4}$$

What is the region of convergence of the Laplace transform $G(s)$? Find the inverse Laplace transform using partial fraction expansions and the cover-up method.

Description of the problem. Oral Problem 9 was used to test student ability to find the inverse transform of a function using partial fraction expansion and the cover-up method. This problem consists of three parts: Part (1) involves the case in which the Laplace transform has a double pole. In part (2), the Laplace transform has a numerator that is an exponential and is not a ratio of polynomials. In part (3), students were given the transfer function of a causal system with imaginary poles and were required to determine the inverse Laplace transform of the function and to define the function's region of convergence.

Participants. Out of the ten students who sat for Oral Problem 9, only four students participated in this study. Three were male (S34, S75, and S94) and one was female (S27).

Results from Oral Problems 8 and 9. Four of the participants who responded to Oral Problem 8 and three of the participants who responded to Oral Problem 9 put forth faulty reasonings that could be explained as partly due to the inappropriate invocation of the interval matching readout strategy. Participants S32, S38, S40, S75, S92, and S94 expressed responses that are based on the incorrect reasoning that the region of convergence of the Laplace transform of a function should match the time interval over which the function is non-zero. Also, participant S71 incorrectly expressed the Laplace transform as a function of both the complex angular frequency s and the time variable t .

Oral Problem 8 — Part (1). In their response to part (1) of Oral Problem 8, three participants — S38, S40, and S92 — expressed the faulty reasoning that the region of convergence of the Laplace transform of the delayed signal $h(t - T)$ is the region of convergence of the Laplace transform of the signal $h(t)$ shifted to the right by an amount T . They argued that the region of convergence of the Laplace transform of a function should match the time interval over which the function is non-zero, and hence if the function is shifted by an amount T , then the region of convergence of the Laplace transform of the function should be shifted by the same amount. Particularly, these participants conjectured that if the region of convergence of $\mathcal{L}[h(t)]$ is $\text{Re}[s] > 0$, then the region of convergence of $\mathcal{L}[h(t - T)]$ is $\text{Re}[s] > T$. The excerpts below reflect these participants' responses when asked to determine the region of convergence of $\mathcal{L}[h(t - T)]$.

I: What do think the region of convergence is?

S92: Whatever the region convergence was, that the whole would be shifted to the right, like T on the real axis.

I: So if the original region of convergence were $\text{Re}[s] > 0$, let's say, the new region of convergence would be what?

S92: Real part of s greater than T [$\text{Re}[s] > T$].

.....

I: I want to know what the region of convergence for the Laplace transform of $h(t - T)$ is. [...]

S38: [...] So if you translate the h function, then it's just gonna shift the region of convergence. Cause I mean you're translating this argument too.

.....

I: If I take the Laplace transform of either of those where I multiply by e^{-sT} and integrate, can you just kind of look at those and argue what the region of convergence would be?

S40: [Pause.] It looks like the region of convergence here [$H(s)$] is $s > 0$ and the region of convergence here [$e^{-sT}H(s)$] is $s > T$.

Oral Problem 8 — Part (2). In part (2) of Oral Problem 8, participant S32 incorrectly determined the region of convergence of $U(s)$ based on the same incorrect reasoning that the region of convergence has to match the time interval over which the function is defined. She correctly determined the Laplace transform of $u(t)$ to be

$$U(s) = \frac{1}{s} - \frac{2e^{-s}}{s} + \frac{e^{-3s}}{s} \quad (3)$$

However, the participant provided an incorrect answer for the region of convergence of $U(s)$. She stated that the region of convergence for each of the three terms in the expression for $U(s)$ is $s > 0$, and hence deduced that the region of convergence of $U(s)$ is $s > 0$. The following exchange reflects the faulty reasoning underlying the participant response:

I: And tell me why the region of convergence is $s > 0$.

S32: Okay for $\frac{1}{s}$ it's because there's nothing interesting, because the Laplace transform of σ is only defined for $s > 0$.

I: Okay.

S32: Because you don't know what happens on the other side of it.

Participant S32 reasoned that because the unit step function is non-zero only for $t \geq 0$, then the values of s over which the Laplace transform of $u(t)$ is defined are $s > 0$. This faulty reasoning was further verified when the participant was asked to determine the region of convergence of the Laplace transform of $u(t)$ in the case that $u(t)$ has non-zero values only in the time interval $0 < t < 1$:

I: Okay, now let's for a moment just look at ... suppose that u had been this function, that's 1 between 0 and 1, and didn't have the negative part, okay [...] what would the transform of this be?

S32: The integral from 0 to 1 of $1e^{st}e^{-st}$ [$\int_0^1 e^{st}e^{-st} dt$].

I: Okay and for what values of s do you think that integral converges?

S32: Greater than 0 and less than 1.

Oral Problem 9 — Part (3). In part (3) of Oral Problem 9, two participants — S75 and S94 — put forth a response that is also consistent with the faulty reasoning that the region of convergence of the Laplace transform of a function should match the time interval over which the function is non-zero. Particularly, these participants claimed that because the system G is causal then the region of convergence of the Laplace transform $G(s)$ can only span a region to the right of the imaginary axis. (See Figure 6.) In fact, since the system is causal, and hence the impulse response is a right-sided signal, then the region of convergence of $G(s)$ is right-sided and to the right of the right-most pole. Since in this problem the right-most pole(s) is on the imaginary axis, the region of convergence is to the right of the imaginary axis.

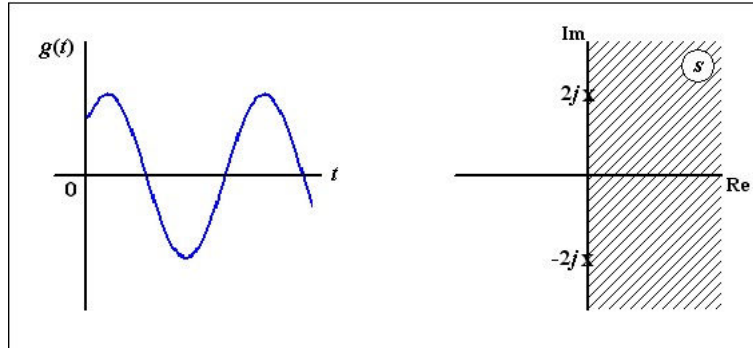


Figure 6: Figure illustrates the faulty reasoning that because $g(t)$ spans the domain $t > 0$, the region of convergence spans the domain to the right of the imaginary axis.

However, even though the participant response that the region of convergence of $G(s)$ is $\text{Re}[s] > 0$ is correct, the reasoning underlying their response was faulty. Their response was based on the invalid argument that since $G(s)$ is the transfer function of a causal system, and hence the inverse Laplace transform of $G(s)$ spans the domain $t > 0$, then the region of convergence of $G(s)$ should span only the region to the right of the imaginary axis; that is, the region of convergence should be $\text{Re}[s] > 0$, as participant S94 explained:

S94: It's a causal system, so it has to be from zero to something. It has to be greater than zero.

Also, participant S75 invoked the same reasoning when determining the region of convergence of $G(s)$. Participant S75 first inferred that the region of convergence of $G(s)$ is the horizontal strip between the two imaginary poles $-2j$ and $2j$. Then he decided that his inference is incorrect because the region of convergence $-2j < \text{Im}[s] < 2j$ overlaps the second and third quadrants in the complex plane and hence is not consistent with his answer that $g(t)$ is the causal function $(\cos 2t + \sin 2t)\sigma(t)$. His argument is based on the reasoning that because $\sigma(t)$ nulls out the value of a function for $t < 0$, then the region of convergence of $G(s)$ should not span the region in the second and third quadrants in the complex plane. This argument is evident in the following excerpt:

I: And why is that $[(\cos 2t + \sin 2t)\sigma(t)]$ inconsistent with the way you've chosen your region of convergence?

S75: Because, well, if you're saying [...] that convergence region has area that $\sigma(t)$ will just wipe out. Because I am saying [...] if $\sigma(t)$ will wipe out all of this over here [in the second and third quadrants].

I: Yeah.

S75: Meaning that [...] I shouldn't say converges there or I shouldn't say $\sigma(t)$.

I: Oh, I see. And when you say wipes out everything there, that is because ...

S75: Well, this is just 0.

I: Because σ is 0 for negative . . .

S75: Right.

Generally, the incorrect reasoning that the region of convergence of the Laplace transform of a function $x(t)$ should match the time interval over which the function $x(t)$ is non-zero could be explained as the product of the invocation of the interval matching readout. Participants identified the time intervals that correspond to those over which the function has non-zero values, and utilized those to determine the interval or the frequency domain that defines the region of convergence. Based on this readout, participants reasoned that the time intervals over which the function is non-zero should match the frequency intervals that define the region of convergence. Also, they concluded that if a function is shifted by a certain amount, then the region of convergence of the Laplace transform of the function should be shifted by the same amount.

Oral Problem 8 — Part (2). An interesting error that one participant — participant S71 — committed in his response to part (2) of Oral Problem 8 was expressing the Laplace transform of the signal $u(t)$ as a function of both the complex frequency s and the time variable t . Participant S71 expressed $U(s)$ as

$$U(s) = \begin{cases} 0, & t < 0 \\ 1/s, & 0 \leq t < 1 \\ -1/s, & 1 \leq t < 3 \\ 0, & t \geq 3 \end{cases}$$

Participant S71 consistently committed this error throughout his solution to the problem. The expressions he wrote for $Y(s)$ in part (3) and for $y(t)$ in part (4) are piecewise over the same time intervals.

From a reasoning resource perspective, this error originates from an inappropriate invocation of the interval matching readout strategy. The participant reasoned that the intervals over which the Laplace transform is defined should match the time intervals over which the time function is defined. This interpretation established a structure for the Laplace transform $X(s)$ of a function $x(t)$ that is described by different expressions over different time intervals to parallel the time intervals over which $x(t)$ is defined, so that

$$x(t) = \begin{cases} x_1(t), & t \in I_1 \\ x_2(t), & t \in I_2 \\ \vdots & \vdots \\ x_n(t), & t \in I_n \end{cases} \longrightarrow X(s) = \begin{cases} X_1(s), & t \in I_1 \\ X_2(s), & t \in I_2 \\ \vdots & \vdots \\ X_n(s), & t \in I_n \end{cases}$$

where the I_n are the different time intervals, and $X_n(s)$ is the Laplace transform of $x_n(t)$. That is, if $x_1(t)$ is the expression for $x(t)$ that occurs over the interval I_1 , then its corresponding Laplace transform $X_1(s)$ should be defined over the same time interval I_1 .

Conclusion

In this paper, we report on the role of the interval matching readout strategy in student reasoning in signals and systems, and the explanatory power of this resource in accounting for student responses to different problem situations. Students inappropriately invoked interval matching when: (1) determining the step response of an LTI system, given an input and the corresponding output of the system; (2) determining the output of an LTI system using convolution; and (3) determining the Laplace transform of a function and the region of convergence of a Laplace transform. Students' difficulties with the superposition principle, convolution, and the Laplace transform led students to attend to an external feature of the problems — the intervals over which the given functions are defined. This feature seems to particularly evoke student attention when solving a problem, and they superficially draw upon it this feature in an attempt to interpret the problem situation hence inappropriately bringing to bear the interval matching reasoning resource.

The high cuing priority and the cognitive propensity to invoke the reasoning resource of interval matching could be attributed to the parallel configuration this resource imposes on a problem situation. This configuration is defined by the piecewise temporal structure of the time functions given in a problem. The function to be determined is then easily expressed as a piecewise function having different expressions over the different intervals that match the defined configuration. Also, the origin of the interval matching readout strategy could derive from students' formal learning of kinematics. In problems in which the motion of an object changes over time, students learn to analyze the motion of the body based on interval matching. They dissect the problem into distinct time intervals, and then analyze the motion of the body separately over these time intervals.

This study offers course instructors an in-depth understanding of the nature of student faulty reasonings in signals and systems. Particularly, this paper reports on the power of the interval matching reasoning resource to explain various conceptual problems that students encounter in their learning of the course. With this improved understanding, instructors can better predict the faulty reasonings students could express in other problem situations, and design more effective instructional material and approaches tailored to support students in overcoming their conceptual difficulties. Moreover, the findings of this study could inform future developments of the Signals and Systems Concept Inventory. The identified reasoning resources that students invoke when solving a signals and systems problem, and the consistency with which they inappropriately apply certain resources, such as interval matching, could support the design of distractors that more accurately represent and capture student faulty responses.

With regards to the implications of this study for engineering education research, this investigation suggests the need for conducting further in-depth qualitative research on student understanding in other engineering disciplines. While the science education literature is replete with studies that have addressed the nature of student conceptions in the various fields of science,¹ similar research is needed that could inform and support the development of effective instructional environments in engineering. Without understanding the subtleties and peculiarities that characterize student understanding in a particular engineering domain, efforts to design effective instruction may be undermined.

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