AC 2008-2885: STUDY OF ACCURACY OF CNC MACHINE TOOLS

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Study of Accuracy of CNC Machine Tools

ABSTRACT

The dimensional accuracies as well as surface finishes of parts produced by a CNC machine tool are strongly dependent on the motion accuracy of each axis of the machine. The overall accuracy of the machine tool is decided by the mechanical characteristics of the machine as well as the characteristics if the control system driving the individual axes. A CNC Machine is programmed to travel along a predetermined or contour, and any deviation from the programmed path is referred as the contouring error. A typical test that is used is a circular test in which the machine is programmed to travel along a circle, and the difference between the programmed path and the measured actual path is compared. The purpose of this study is to identify the various contributors to this contouring error, and in particular estimate the error due to stick slip motion using analytical techniques. It is planned to measure the errors experimentally making use of capacitance probes. The study is intended to develop an appreciation for the sources of different errors produced in a machine tool and as such forms a module in a typical semester long class devoted to manufacturing processes.

INTRODUCTION

Computer controlled machine tools are expected to perform within the specified limits of accuracy, especially for the contouring operations. With the increasing demands placed on high precision combined with high contouring accuracy a careful study of the accuracy degrading mechanisms arising during machine operation needs to be considered. A CNC machine is programmed to travel along a predetermined contour, and any deviation of the actual path as compared to the programmed path is the contouring error. Most contouring tests use a circular path, and the contouring error is the difference between the programmed circle and the actual circular (a distorted one) path described by the machine tool. This is what constitutes the circular test as described by Knapp (1983). The specific focus of this work is to analytically determine the magnitude of such an error in a circular test due primarily to the stick slip friction.

CONTRIBUTORS TO CONTOURING ERROR

If the two axes of a CNC machine are not perpendicular to one another then an oval path will result. In addition backlash in lead screws, axis straightness, pitch and yaw errors can all affect contouring accuracy. These errors constitute geometric errors. In particular, backlash is a positioning error that occurs when the spindle approach direction is reversed and in general it is due to the tolerance of the ball lead screw or the guide way. The other set of errors arise due to control system, and are the interpolation errors, errors due to hysteresis, and those due to servo mismatch and master slave changeover.

Servo mismatch appears due to different velocities of the spindle in the two axes, resulting in different gain values leading to distorted circle on the contouring data. For two dimensional motions there is usually a leading axis (master) and a following axis (slave). In a circular interpolation path, distortion or glitches appear every 45 degrees of the circular path. This is termed “master-slave” changeover, and is due to the change in the feed rates in the different axes at a given position.

STICK SLIP ERRORS

In this work, we investigate the degradation of the contouring accuracy due to dry friction at the sliding interfaces of the CNC machine work tables. This dry friction appears in two different forms: (i) as a resistance against the beginning of motion from equilibrium (stick mode); and (ii) as a resistance against the existing motion (slip mode). The friction resistance is a constraining force in the stick mode and an applied force in the slip mode. When the work tables are in motion, both phenomena are present resulting in a stick-slip motion. In machine tools it is well known that that a work table moving at a slow speed may execute jerky motion instead of a smooth travel. This may give rise to indexing errors. Such responses are also possible in the vent of rapid starts and stops.

As the work tables execute coordinated motions corresponding to the commanded input, the velocities of the individual tables go through different velocities including zero. Such a variation in velocities leads to a variation of friction forces in a nonlinear fashion. The inclusion of these nonlinear effects into the dynamics of the X-Y tables leads to a coupled system of nonlinear differential equations. Numerical solutions to the equations of motion would yield responses typical of a nonlinear dynamical system and has been explored in this study.

ANALYTICAL MODEL

Figure 1 illustrates a typical 3-axis CNC machine with its mathematical idealization. The two tables moving in the X and Y directions are connected to the individual lead screws that provide them with the required motions as programmed in the controller. For each of the two tables masses are assigned which are arranged to move in the X and Y directions. The connections between the masses and the driving elements are never perfectly rigid and therefore springs may be substituted for them.

The stiffnesses of the springs are typically those of the lead screws, the most compliant elements of the drive systems. Damping also occurs in the connections and the damping force is assumed to be proportional to the speed. The X-Y tables can therefore be modeled as a system consisting of two masses M_x and M_y attached to two springs K_x and K_y and two dampers C_x and C_y in an arrangement as shown in Figure 1. Thus we have a kinematically coupled system of two orthogonal oscillators. The velocities of the X and Y tables, V_{0x} and V_{0y} are coordinated such that the commanded trajectory is that of a circle drawn at a constant velocity V_0.

In the discrete model of an X-Y table, the self excitation is caused by the friction force acting between the guide way and the slider. Self excitation is only possible when the forces have a decreasing characteristic with increasing speed. Under practical conditions, the coefficient of friction μ varies with sliding speed in a fashion as indicated in Figure 2. Typically at low speeds, the value falls off from its static value to a minimum and then rises again, as described by Tobias (1965). Figure 2 indicates the coefficient of friction as a function of the relative velocity of sliding for the idealized situation as depicted by the signum function, as well as those employed in the investigations by Popp and Stelter (1990), and Feeny and Moon (1989). The function used in this study is a modified version of that of Feeny and Moon (1989) and reflects a falling and rising characteristic typical of machine tools.

The mathematical expression of the function used in this work is given by,

$$\mu(v) = [\mu_2 v^2 + (\mu_1\gamma - \mu_2)\operatorname{sec}h\beta v]\tanh\alpha v \quad (1)$$

Here μ_2 is the dynamic coefficient of friction and μ_1 is the static coefficient of friction. v is the non-dimensional relative velocity between the slider and the guide way. α, β and γ are fitting parameters.

The equation of motion of the X table and that of the Y table are given by equations (2) and (3) respectively as,

$$M_x \ddot{x} + C_x \dot{x} + K_x x = M_x g\mu(V_{0x} - \dot{x}) - M_x \dot{V}_{0x} \qquad (2)$$

$$M_y \ddot{y} + C_y \dot{y} + K_y y = M_y g\mu(V_{0y} - \dot{y}) - M_y \dot{V}_{0y} \qquad (3)$$

The equations (2) and (3) represent a coupled set of nonlinear ordinary differential equations with nonlinearities from the expression of the friction force. The coefficient of friction is of course dependent on the relative velocity. If the commanded trajectory is a circle of radius R generated with a constant angular velocity Ω, then the programmed circle is given by equations (4) and (5) as,

$$X_0 = R\cos\Omega t \qquad (4)$$
$$Y_0 = R\sin\Omega t \qquad (5)$$

From equations (4) and (5) we have differentiating with respect to time,

$$V_{0x} = \dot{X}_0 = -\Omega R\sin\Omega t \qquad (6)$$
$$V_{0y} = \dot{Y}_0 = \Omega R\cos\Omega t \qquad (7)$$
$$\dot{V}_{0x} = \ddot{X}_0 = -\Omega^2 R\cos\Omega t \qquad (8)$$
$$\dot{V}_{0y} = \ddot{Y}_0 = -\Omega^2 R\sin\Omega t \qquad (9)$$

Substituting the values from equations (6) through (9) in equations (2) and (3) we have,

$$M_x\ddot{x} + C_x\dot{x} + K_x x = M_x g\mu(-\Omega R\sin\Omega t - \dot{x}) - M_x\Omega^2 R\cos\Omega t \qquad (10)$$
$$M_y\ddot{y} + C_y\dot{y} + K_y y = M_y g\mu(\Omega R\cos\Omega t - \dot{y}) + M_y\Omega^2 R\sin\Omega t \qquad (11)$$

NUMERICAL SOLUTION

For the numerical solution it will be advantageous if the system of second order equations represented by equations (10) and (11) be transformed into a system of four first order equations.

Introducing variables,

$$
\begin{aligned}
X_1 &= x \\
X_2 &= \dot{x} \\
Y_1 &= y \\
Y_2 &= \dot{y} \qquad (12)
\end{aligned}
$$

We then have the following equations;

$$\dot{X}_1 = X_2 \qquad (13)$$

$$\dot{X}_2 = g\mu(-\Omega R \sin\Omega t - X_2) + \Omega^2 R \cos\Omega t - \frac{C_x X_2}{M_x} - \frac{K_x X_1}{M_x} \qquad (14)$$

$$\dot{Y}_1 = Y_2 \qquad (15)$$

$$\dot{Y}_2 = g\mu(\Omega R \cos\Omega t - Y_2) + \Omega^2 R \sin\Omega t - \frac{C_y Y_2}{M_y} - \frac{K_y Y_1}{M_y} \qquad (16)$$

The functional relationship of the coefficient of friction, μ is of course given by equation (1). The system of first order differential equations (13) through (16) is solved by a fifth order Runge –Kutta procedure with variable time step size adjuster. The time step size is varied between 10^{-4} second and 10^{-6} second, such that convergence is achieved.

The following numerical values are used in the numerical computations

M_x = 90kg, M_y = 120kg,
$K_x = 10^8$ N/m, $K_y = 10^8$ N/m,
C_x = 1900 N-s/m, and C_y = 2200N-s/m

The above damping values are based on a damping ratio of 1% of the critical damping calculated from the stiffness data. The mass and stiffness values correspond to that of a typical CNC machine. The stiffness values correspond to those of the lead screws driving the two tables. These values may vary somewhat from the nominal values, but for this analysis this variation has been ignored. The radius of the circle is taken as 0.1 meter. The angular velocities in the computations, Ω are 0.5 rad/s and 1.5 rad/s. The numerical values in the expression of the coefficient of friction, μ in equation (1) are:

μ_1=0.15, μ_2=0.08, $\alpha = 50\ s^2 / m^2$, β = 5s/m, and γ = 1.5.

Time history of the two displacement components have been obtained for Ω = 0.5 rad/sec and Ω = 0.5 rad/sec and are shown in Figure 3. For the lower speed the errors are of the order of 2-3 μm and are a little higher for the higher value of the speed chosen.

We have demonstrated analytically using a simplified dynamic model of the X-Y tables of a typical CNC machine that stick-slip friction can potentially result in contouring errors. The distortion is produced due to the nature of the stick-slip friction at speeds equal or close to zero.

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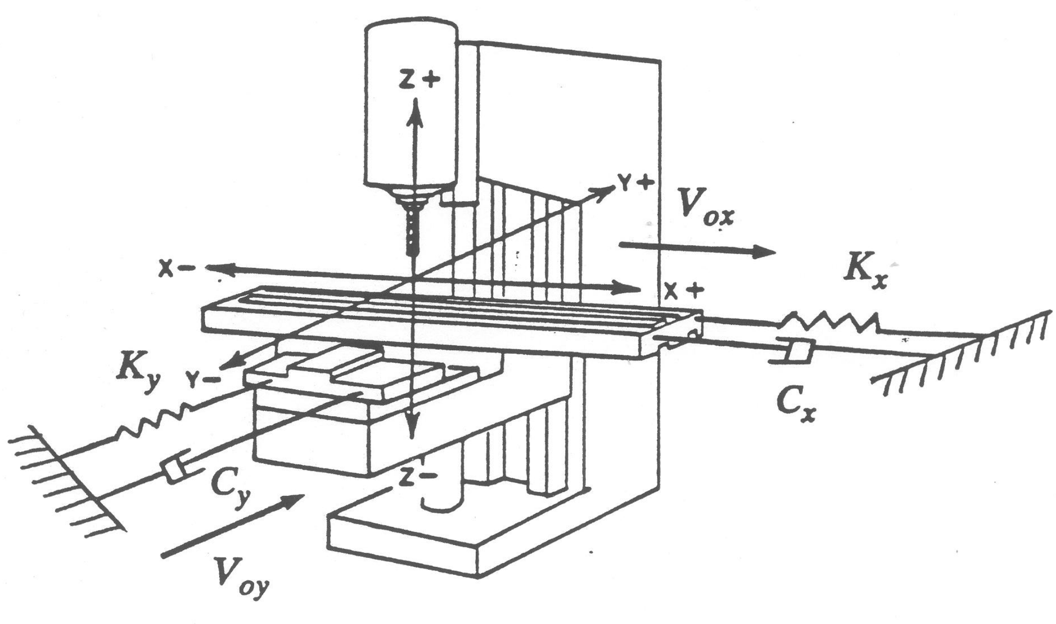

Figure 1 Basis for Mathematical Model of CNC Machine X-Y Tables

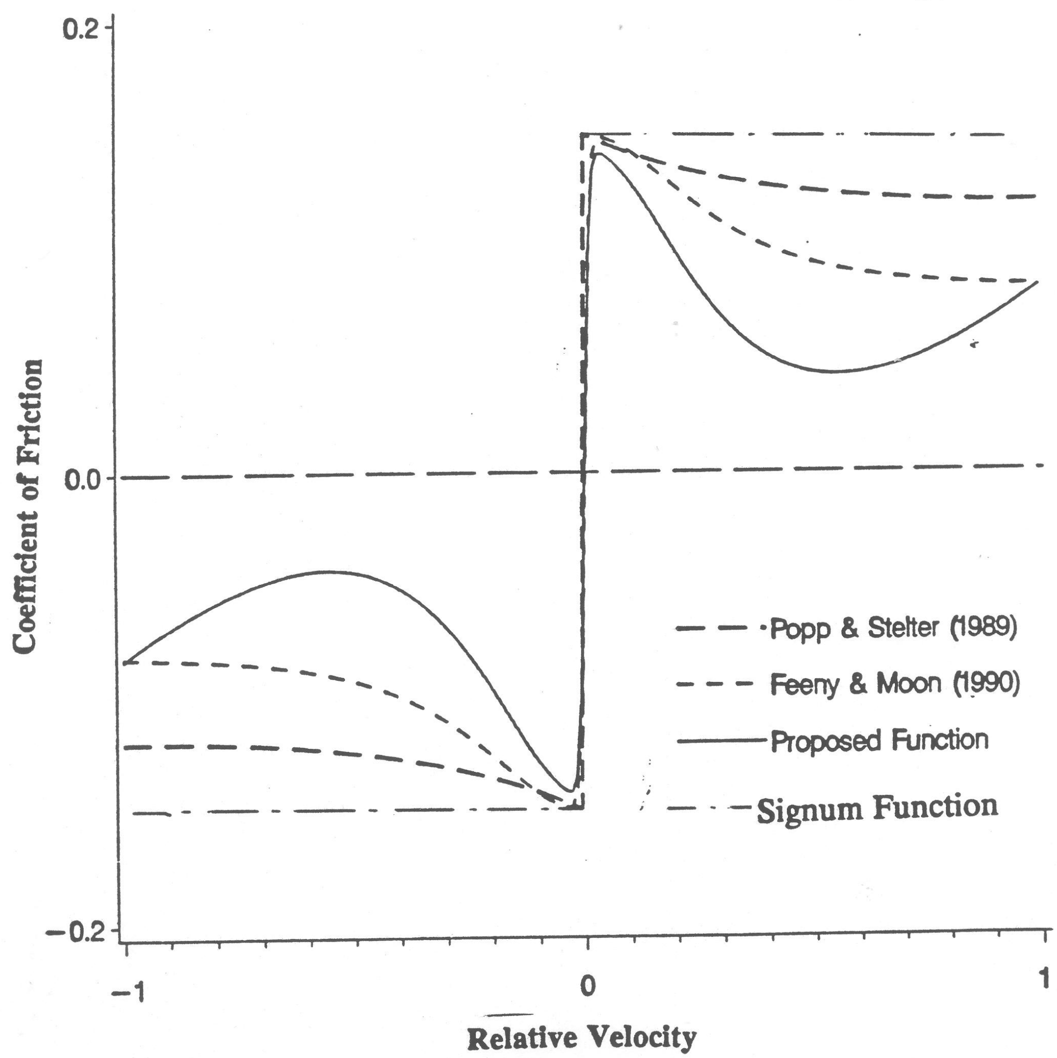

Figure 2 Coefficient of Friction versus Relative Velocity

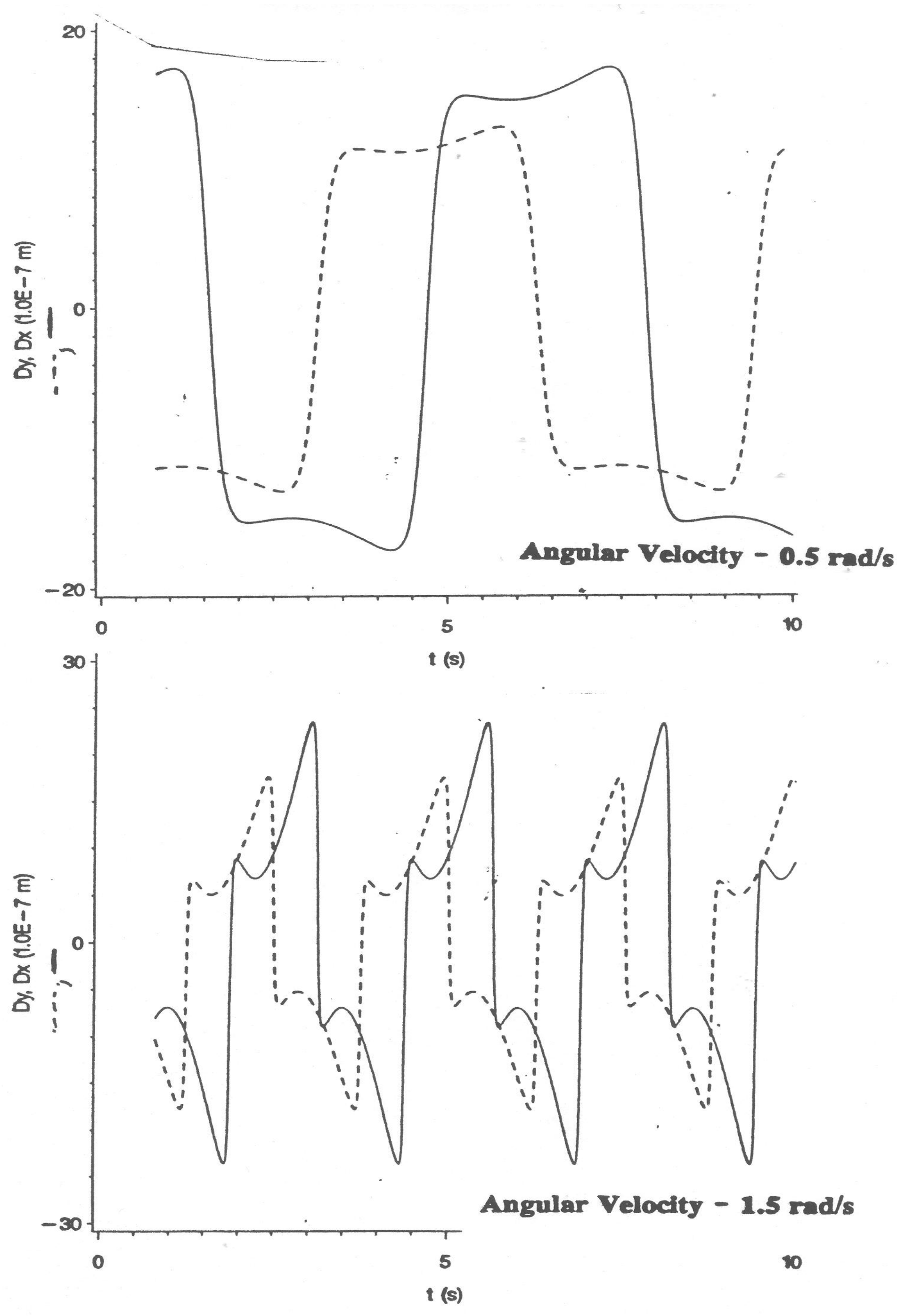

Figure 3 Displacement Deviations in X and Y Directions Versus Time