Teaching Finite Element Method, the controversy between the use of commercial software and the development of mathematical skills

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Abstract

Finite Element Method (FEM) course is a good opportunity to develop superior skills in our students; the controversy today is the use of commercial software in order to develop "Computational mechanical skills" which allow to the students to be a very good users and designers. Instead the use of programming skills in order to build codes which solve specific problems. My idea of an adequate course in FEM includes first of all the mathematical foundations like differential equations, matrix algebra, etc. Then the development of the matrix expression for different kinds of elements which includes spring element, bar element, beam element and plate element. With this intention the students can understand the concept of element, assembly of the rigidity matrix, boundary conditions, etc; the evolution of the complexity of the problems can justify the use of the computer code in order to solve big problems. But, what's going on with complex geometry? The needed of the special code to mesh a complex solid take us in the way to use a commercial one, the most important thing at this moment is the student "have the right to use a commercial software" because already he knows the mathematics secrets of the black box. Obviously the develop of the computer code involves the mathematical aspects such like the weak formulation of the differential equation, the solution of the integrals using numerical procedure. The discussion of this aspect must involve the opinion of the faculty and students in order to clarify the expectative of the curricular plan in which the finite element method is included.

Introduction

In the last ten years the experience of teaching finite element method (FEM) to undergraduate students let us to understand the implications of the use of commercial software in order to give to them the professional experience for the industry challenge, but the first question is ¿How long time take, for a student, to learn and understand a commercial one? Today we know the true, it is not necessary a one semester course time in order to learn and use this kind of commercial software.

Instead of this kind of experience we used to teach the concepts of FEM in a progressive approximation we started with the concept of spring element, the first definition of node, element, elemental matrix, assembly of the elements and boundary conditions. Then the bar element can allow us to go more in deep because the problem solved are related to previous

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course of the undergraduate student like static and mechanics of materials, Hutton [1] and Logan [2] consider this aspects and presents it in their textbooks.
The next level is to solve ordinary differential equation which model engineering problems like heat conduction with convection, bar deformation, electric potential, in this part of the course the Galerkin approach appears and the weak formulation which allow the concept of trial function in order to get an approximate solution, the concept of convergence is understated using a different number of element in the solution of this kind of problem. Here could be the time to use of commercial software in order to compare the solution obtained by hand-write or the use of personal code.

Structural Problem Solve

The spring element could be the first experience in FEM for undergraduate student, after the construction of the elemental matrix the students can assembly all the elements in any spring structure, and then we got the rigidity-assembled-matrix, the determinant of this matrix is zero and the first aspect of this is the necessities of the essential boundary condition. Additionally we can consider the natural boundary conditions which can allow getting the solution of the problem. The second level of the encounter with FEM is the analysis of structures based in truss element, this kind of structure are closed to the students because in they previous courses they learn to solve problems which involves the calculation of the responses of the any member in the structure, the nodal method and the section method can allows to do that.

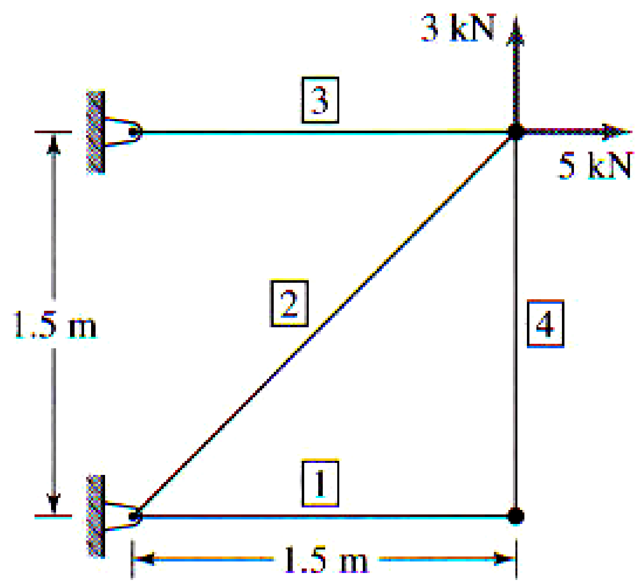

Figure 1, typical 2D structure from [1]

FEM allows calculating these responses and also can calculate the deformation and the stress; at this point the students can build a computational code to simulate the behavior of this kind of structures under the respective boundary conditions, see figure 1.
The computational code gets the problem and solves it given the results in a numerical and a graphical fashion, today authors like Bhatti [3], Kattan [4] and Kwon [5] includes

computational codes for MATLAB and other tools to construct this kind of codes which can allow to solve structural problem. The plane truss shown in the figure 1 is composed of member having a square 15 mm x 15 mm cross section and modulus of elasticity *E*=69 Gpa, we want to calculate the nodal displacement and the axial stress in each element.
Figure 2 shows the deformed structure, the student have already one tool to solve structural problems.

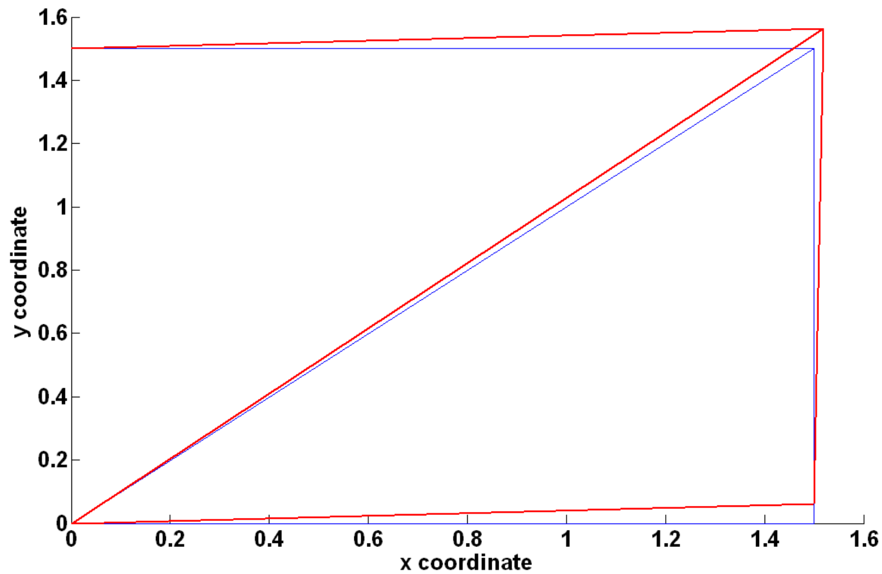

Figure 2, the deformed structure

Once the computational code simulates the structure the next recommended step is to solve the problem using commercial software, that permit to compare the solution coming from our computational code.
Thinking about the motivation for our students when they can compare the solution coming up from the commercial software and they own computational code that is very significations and they understand the importance of the knowledge of the mathematical concept behind the black box.

Ordinary Differential Equation

Solve engineering problem modeled by ordinary differential equation could be the real first encounter with FEM for undergraduate student, Reddy [6] presents the FEM formulation for the heat conduction with convection in a bar of variable transversal section is a very motivate and interesting problem to solve, this kind of problem is modeled by (1)

$$-\frac{d}{dx}\left(k\frac{dT}{dx}\right)+\left[\frac{h\,p(x)}{A(x)}\right]\left(T-T_{\infty}\right)-Q(x)=0 \qquad a\le x\le b \tag{1}$$

Where $A(x)$ and $p(x)$ are the section transversal area and the perimeter of the bar, respectively, at any point x; $Q(x)$ is the internal heat source of the bar.
The geometry of the problem and the boundary conditions can not allow us to calculate an analytical solution for (1), instead of it we propose an approximate solution $\tilde{T}$, but it can not satisfy the equation (1), but we can propose the following

$$\int_a^b \left[-\frac{d}{d\,x}\left(k\,\frac{d\,\tilde{T}}{d\,x} \right) + \left[\frac{h\,p(x)}{A(x)} \right] (\tilde{T} - T_\infty) - Q(x) \right] \phi_i(x)\,dx = 0 \qquad i = 1,2,\ldots,N \tag{2}$$

After part integration we reach the weak formulation for equation 2 and then following the Galerkin approach, we propose an expression for $\tilde{T}$ in each element of the mesh, see figure 3.

$$\tilde{T} = \sum_{j=1}^{N} T_j\,\phi_j(x) \tag{3}$$

Where T_j is the approximate value of the temperature at each node of the mesh, and $\phi_j(x)$ are called the trial function, these functions can be linear, quadratic, cubic, etc. Substituting (3) into equation (2) after part integration we have the elemental matrix form.

$$\begin{bmatrix} K_{11} & K_{12} & \ldots & K_{1N} \\ K_{21} & K_{22} & \ldots & K_{2N} \\ & & & \\ K_{N1} & K_{N2} & \ldots & K_{NN} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ \\ T_N \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ \\ f_N \end{Bmatrix} \tag{4}$$

The members of the elemental matrix K_{ij} will be calculated using numerical integration, in order to solve a problem is necessary to create the one-dimensional mesh, assemble the elemental matrix and then imposes the essential and natural boundary conditions; the convergence procedure is used here to decide how many elements we will used.

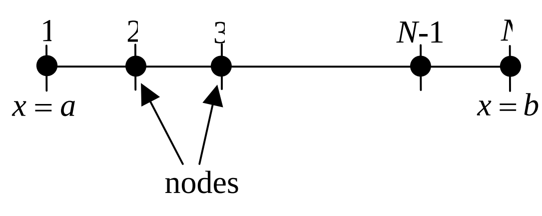

Figure 3, one dimension FEM mesh

Is important to understand all the mathematical concept cover from start to here, the students knows how a FEM program is developed, they understand all the basis and all the mathematical procedure, we can do the next step in order to do the FEM formulation for beams with a distributed load, also the formulation for two dimension formulation for elasticity problems and plates. In parallel to these formulations the students can start to use commercial software in order to solve engineering-big-industrial problems, we can say the students have the right and the permission to use it because already they know how the "black box" works.

Research for Undergraduate Students

Additional to mentioned topics we can present challenges for our student in order to motive them to research, stabilized method could be an interesting topic for student's basic research, Akin [7] propose us start with the equation

$$u\frac{d\varphi}{dx}-k\frac{d^2\varphi}{dx^2}+Q(x)=0 \qquad 0\le x\le L \tag{5}$$

With the boundary conditions

$$\varphi(0)=0 \qquad \varphi(L)=1 \tag{6}$$

For this specific problem the exact solution is knowledge

$$\varphi(x)=\frac{1-e^{px/L}}{1-e^{p}} \qquad p=\frac{uL}{k} \tag{7}$$

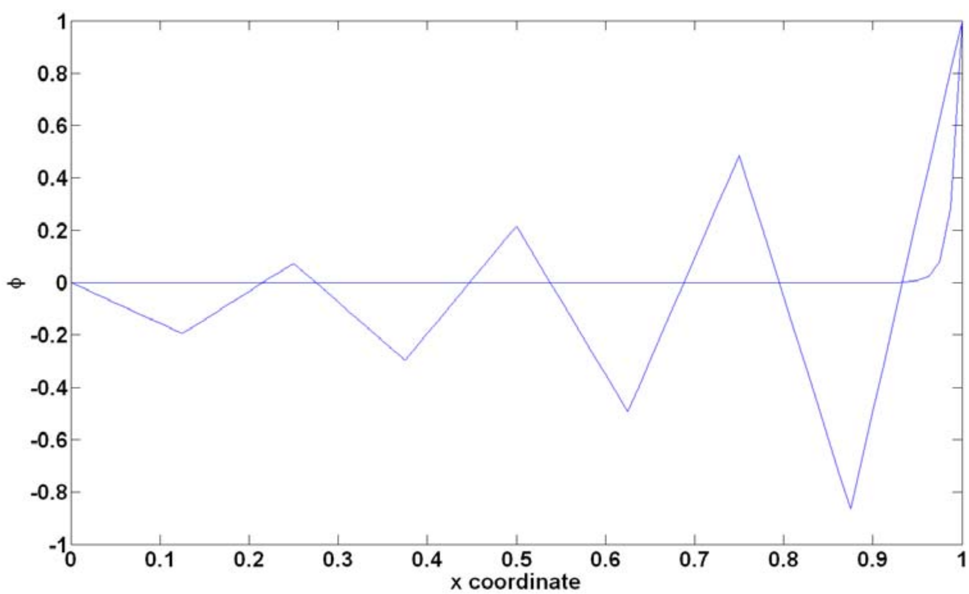

Figure 4, a first grade approximation

Where p is the Peclet number, after the FEM formulation of the equation (5) we got the elemental matrix

$$\begin{bmatrix} (1-p) & -(1-p) \\ -(1+p) & (1+p) \end{bmatrix} \begin{Bmatrix} \varphi_1 \\ \varphi_2 \end{Bmatrix} = \begin{Bmatrix} -\lambda_1 \\ \lambda_2 \end{Bmatrix} \tag{8}$$

Using 10 elements with a first order trial function and $p = 100$, we found a very bad approximate solution because it presents oscillations, that's due because the trial function are first grade.

Figure 4 shows the oscillations in the first grade approximation, the next step is to use a second grade order for trial functions, the approximate solution is improved but not yet is satisfactory, and figure 5 shows it.

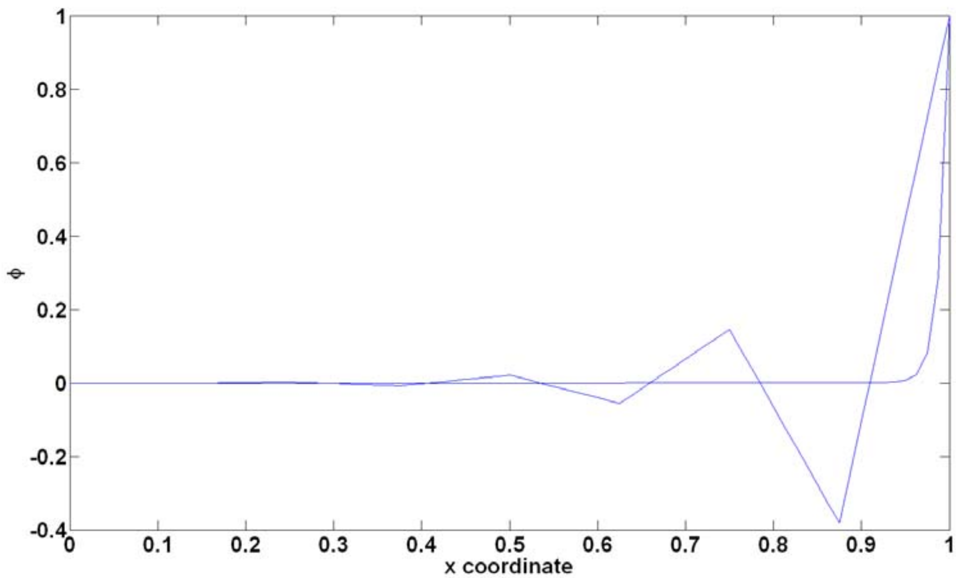

Figure 5, a second grade approximation

The elemental matrix for a second grade approach for the trial function is

$$\begin{bmatrix} \left(\frac{4}{3}-p\right) & -\frac{1}{3}(2-p) \\ -\frac{1}{3}(2+p) & \left(\frac{4}{3}+p\right) \end{bmatrix} \begin{Bmatrix} \varphi_1 \\ \varphi_2 \end{Bmatrix} = \begin{Bmatrix} -\lambda_1 \\ \lambda_2 \end{Bmatrix} \tag{9}$$

The student can continue improving the solution but a different idea must be used, the Petrov-Galerkin method, it implies a different philosophy, additional to the classic Galerkin approach must used the stabilization terms, it implies high mathematical challenges for the students. In our opinion is the way to give the students the opportunity to start the journey to research.

Conclusions

The Finite Element Method is a real powerful tool to solve engineering problem, but in our experience and opinion the course for undergraduate student will be includes a very strong mathematical concept in order to develop skills and allows the student understand all the concepts in which is based the develop of a commercial software for FEM, obviously the student needs to develop computational mechanical skills in order to be ready for the challenges of the professional life. The best combination of mathematical consideration for FEM formulation and the understanding of all the concepts give the students the right to use any kind of commercial software. In addition of them this kind of course could be the gate for research and industrial project for the undergraduate students.

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Dr. Cruz-Bañuelos currently serves as Professor and Head of the Mechanical and Industrial Engineering Department at the Universidad de Monterrey in México. His research interests are in the Finite Element Method and Boundary element method, also he is working in the design of new blades for wind mills for water pumping proposes.