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Teaching First-order Systems to Electrical Engineering Students Using Visual and Intuitive Examples

Dr. Daniel Raviv, Florida Atlantic University

Dr. Raviv is a Professor of Computer & Electrical Engineering and Computer Science at Florida Atlantic University. In December 2009 he was named Assistant Provost for Innovation and Entrepreneurship.

With more than 30 years of combined experience in the high-tech industry, government and academia Dr. Raviv developed fundamentally different approaches to "out-of-the-box" thinking and a breakthrough methodology known as "Eight Keys to Innovation." He has been sharing his contributions with professionals in businesses, academia and institutes nationally and internationally. He was a visiting professor at the University of Maryland (at Mtech, Maryland Technology Enterprise Institute) and at Johns Hopkins University (at the Center for Leadership Education) where he researched and delivered processes for creative & innovative problem solving.

For his unique contributions he received the prestigious Distinguished Teacher of the Year Award, the Faculty Talon Award, the University Researcher of the Year AEA Abacus Award, and the President's Leadership Award. Dr. Raviv has published in the areas of vision-based driver-less cars, innovative thinking, and teaching innovatively. He is a co-holder of a Guinness World Record. He is a co-author of five books on innovative thinking and teaching innovatively.

Dr. Daniel Raviv received his Ph.D. degree from Case Western Reserve University in 1987 and M.Sc. and B.Sc. degrees from the Technion, Israel Institute of Technology in 1982 and 1980, respectively.

Mr. Daniel Ryan Barb

Daniel Barb is a Nuclear Engineer for PSEG. He spent six years in the United States Navy working in a nuclear power plant aboard a fast attack submarine and earned his BSME from Florida Atlantic University.

Mr. George Roskovich, Florida Atlantic University

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Daniel Raviv, Daniel Barb and George Roskovich

College of Engineering and Computer Science Florida Atlantic University

Emails: ravivd@fau.edu , danielrbarb@gmail.com, georgeroskovich@gmail.com

Abstract

First Order Differential Equations is a topic that is prevalent in mathematics and is foundational to several engineering disciplines. Electrical engineering is a field where understanding first order systems is crucial. It is a cornerstone of topics such as transient electrical systems including RC and RL circuits. Despite this, many students struggle with conceptual understanding of this subject. The equations and mathematics can be overwhelming and frustrating, in part because it is often hard to visualize the concept.

Today's students have plenty of distractions at their fingertips. In the midst of the COVID-19 pandemic, which has resulted in more online-learning, students will oftentimes browse the internet or pull out their phones if they begin feeling bored or frustrated with a topic. Simply put, today's students learn differently. They learn more intuitively and have shorter attention spans, and lessons should compensate for this with presentation methods that are clear, visual, and intuitive.

The primary focus of this work is to help teachers explain, and learners to understand, the fundamental concepts of First Order Differential Equations through the use of intuitive and example-based approaches as they relate primarily to electrical engineering. This paper seeks to simplify the introduction to the topic of First Order Differential Equations into something that is clear and easy to comprehend. To accomplish this, the paper starts with a visual background of first order systems and an explanation of exponential growth vs. exponential decay. It then moves into (1) electrical examples, including the charging rate of cell phones and the idea of transient response in electrical systems such as RC and RL circuits, (2) electromechanical examples, including DC motors and heat transfer rates of different types of stoves, (3) various topics from other STEM disciplines, such as vehicle accelerations (dynamics), diffusion (physics), and currency depletion (economics). The paper concludes with a related brain teaser.

The goal of this approach is to provide students with examples that translate textbook explanations to real life and help in understanding the material. We believe that when using these intuitive examples students tend to better understand first order systems, especially as they relate to the field of electrical engineering. This paper should be considered a work in progress. The presented information is meant to be supplemental in nature and not to replace existing textbooks or other teaching and learning methodologies. The contents of this work have been shared with students in a remote (Zoom-based) classroom setting and assessed following the lecture using an anonymous questionnaire. The initial results, based on 40 responses, indicate that this teaching method is effective in helping students comprehend the basic idea behind the concept of First Order Differential Equations. This intuitive and engaging approach to teaching and learning has been tested in the past for many topics including Statics (explaining center of gravity), Calculus (explaining integration and explaining derivation by chain, product, and quotient rules), Thermodynamics (explaining entropy), Statistics (explaining normal distribution), Differential Equations, Control Systems, Digital Signal Processing, Newton's Laws of Motion, and Computer Algorithms. In all of these cases, students highly praised the approach and found it to be very effective for learning.

Introduction

Most mathematics textbooks are loaded with mathematical formulas and explanations with little focus on conceptual understanding. Textbooks focusing on differential equations are no different. This method is useful because it is written in a precise manner, but at the same time students may become frustrated with the material as they do not intuitively grasp some of the concepts, and miss the "aha" moment – a moment of sudden insight or understanding. Today's student is looking for a connection to the real world. For this reason, it is important for instructors to modify their approach to teaching by first introducing the concept in a clear way that is easy to understand, and only later focus on the textbook math.

The focus of this paper is to provide a set of supplementary material that can help instructors to teach and students to comprehend, a specific mathematics concept, namely First Order Differential Equations. It is a collection of examples, some known and some unknown, that teachers and learners can choose from. It is not intended for teachers to teach every example in this work, but rather to select the ones they feel would be most effective for their particular learning group. By introducing this concept in visual and intuitive ways, the material becomes more meaningful for the student, particularly when they can relate it to real-life. This paper is a work in progress, as more assessments and results are forthcoming. Our research approach was to collect visual and intuitive examples and present them to students who will then provide feedback regarding the effectiveness of the approach. This feedback was meant to guide future iterations of the work and provide direction for additional assessments that will follow.

The contents of this work have been shared with students in a remote (Zoom-based) classroom setting and assessed following the lecture using an anonymous questionnaire. The initial results, based on 40 responses, are detailed in Appendix A, and indicate that this teaching method is effective in helping students comprehend the basic idea behind the concept of First Order Differential Equations. Students generally felt that understanding the concept of First Order Differential Equations was important, as shown in their responses to question 1. They also felt that learning differential equations through methods such as visual examples (question 2), hands-on activities (question 3), and in-class exercises (question 4) was important, while the general opinions on learning through methods such as traditional presentations (question 7) and reading the relevant textbook material (question 8) were more mixed. In the near future this work will be presented to a larger group of students learning First Order Differential Equations, and their feedback will be assessed using more rigorous formative and summative assessments.

An extra credit assignment was given to the same students to come up with their own real-life examples of First Order Differential Equations, including an explanation of why it qualifies as a First Order Differential Equation. Students were required to describe 15 examples from different disciplines including engineering, mathematics, physics, chemistry, biology and medicine. Students' feedback on this assignment indicated that it helped them comprehend First Order Differential Equations in a more thorough way.

As previously stated, the main contribution of this work is to be a collection of examples, some new and others that are well known, that an instructor can pick and choose from to supplement his or her teaching in the ways they find most appropriate. This paper, which is an expansion of the work in [1], starts with a brief mathematical background of first order systems – explaining exponential equations and the time constant through the use of graphs. It then transitions into real-life examples of first order systems in different STEM areas focusing on electrical and electromechanical engineering, and also covering other topics such as mechanical devices, physics, and even economics. These examples briefly discuss the relevant equations, solutions, graphs, and time constants where applicable without allowing them to become the focus of the work. While this work is focused on teaching concepts to Electrical

Engineering students, some topics are used from other fields in order to further help students understand by relating First Order Differential Equations to real life experiences. It is important for students to gain knowledge through their exposure to different areas of expertise and life experiences, and, in fact, ABET accreditation requires it. For example, one of the ABET outcomes specifically states, "An ability to acquire and apply new knowledge as needed, using appropriate learning strategies." For this reason, we have included more than just mechanical engineering examples. The results from the recently performed survey shows that these examples are helping students learn. Beyond just a collection of examples, this work asserts that learning through visual and intuitive methods is more effective than traditional presentations and book learning, specifically for introductory purposes.

Extensive work has been done attempting to find more effective ways of teaching foundational STEM courses. Trying to find out why so many students struggle with mathematics and what can be done about it shows that there is no one single approach [2]. One study of visualization in a freshman Chemistry course showed results that, "suggest that visualization skills do facilitate concept learning, but they do not generalize to higher education in the sciences" [3], which shows that visual approaches are helpful for the introduction of new concepts. Studies have also found that visualization improves student motivation in computer science [4], mathematics [5] and in collaborative online learning environments [6]. Intuitive and engaging approaches have been used to introduce topics such as center of gravity [7], the normal distribution [8], control systems [9], and entropy [10] with positive results.

Mathematics and a Visual Background of First Order Systems

Though the mathematics of these concepts are well known, we felt that it was important to include a brief summary for the purpose of a more complete work. A differential equation includes both a function and at least one derivative of that function. If only the first derivative is included in the equation, it is considered a First Order Differential Equation. Simply put, the solution to a this type of equation depends on how fast the value of that solution is changing. An example of a First Order Different Equation is:

$$ax(t) = \frac{dx(t)}{dt}$$

The solution to a linear first order DE is an exponential function of the form:

$$x(t) = ce^{at}$$

Where: c – constant
a - constant

This solution, the exponential function, has two general shapes, depending on whether "a" has a positive or negative value. The general shape of an exponential function is shown in Figure 1. The left figure visualizes a case where the constant "a" is positive, and the right figure shows the case where "a" is negative.

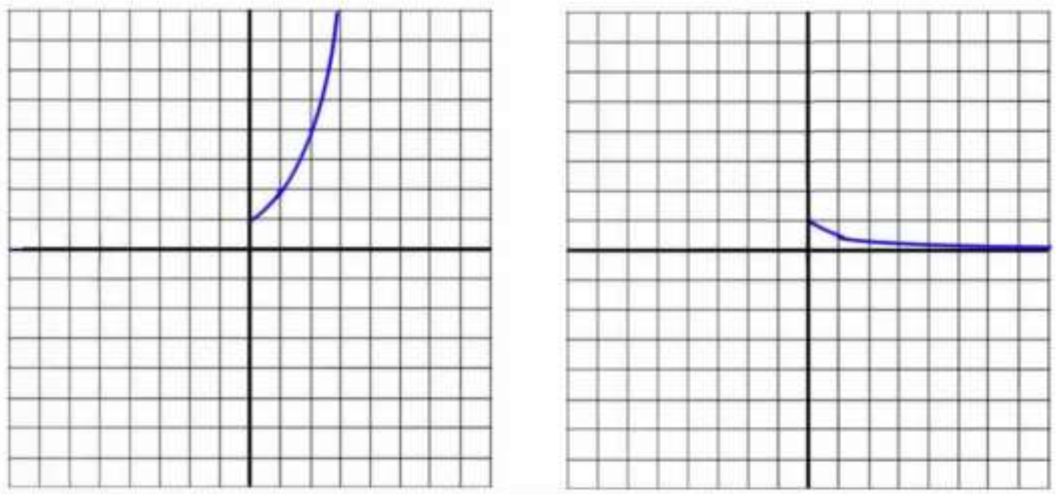

Figure 1: General exponential function with positive exponent (left) and negative exponent (right)

The time constant, generally given with the symbol τ, is a characteristic of the response of first order systems. After one time constant, an exponential response has reached 63.2% of the way from the initial condition to its final value.

<u>Electrical Examples</u>
Cell Phone Charging / RC and RL Circuits

Resistor-capacitor (RC) and resistor-inductor (RL) circuits are both analyzed through the use of differential equations. Capacitors store energy, and while this energy is being built up, the capacitor resists current flow. The equation for voltage across a capacitor is given as:

$$i(t) = C\frac{dv}{dt}$$

Where: i(t) – current as a function of time
C – capacitance
v – voltage
t – time

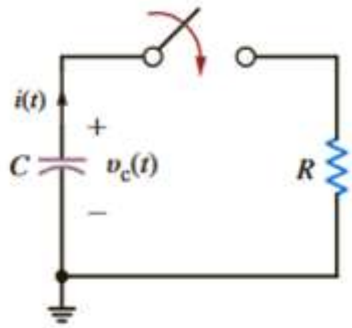

Figure 2: An RC circuit where the switch is closed at time t=0. Courtesy of [11]

For an RC circuit like the one in Figure 2, when the switch is closed Kirchoff's current law can be written as:

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

Where: R - resistance

Since the voltage through the circuit depends on the rate of change of the voltage, it is a first order system. The time constant for an RC circuit is given as R*C.

Inductors store energy in a magnetic field. When a current initially flows in an RL circuit, the magnetic field builds up, crossing the coiled wire and creating current flow which opposes the current from the power source. This limits the rate at which current flow builds up in the entire circuit. Similarly, when current stops flowing through the circuit, the magnetic field is diminished, and the magnetic field's contraction crosses the coiled wire, creates some residual current in the circuit. The equation for voltage across an inductor is given as:

$$v(t) = L\frac{di(t)}{dt}$$

Where: L – inductance

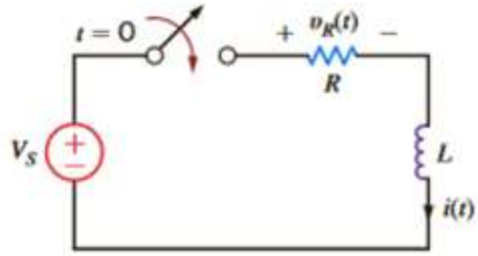

Figure 3: An RL circuit where the switch is closed at time t=0. Courtesy of [11]

For an RL circuit like the one in Figure 3, when the switch is closed Kirchoff's voltage law can be written as:

$$i(t)R + L\frac{di(t)}{dt} = V_s$$

Where: V_s – source voltage

Since the current through the circuit depends on the rate of change of the current, it is also a first order system. The time constant for an RL circuit is given as L/R.

A real-life example of RC and RL circuits is in a cell phone. Have you ever noticed that when your cell phone's battery is low, the phone seems to charge faster? As the battery gets more and more charged, the charging rate also seems to slow. It's not just an illusion or a sense of impatience – the rate at which your phone charges depends on how charged it already is.

As the phone charges, the battery's voltage increases. As we have all experienced, this means the difference in voltage between the outlet and the battery decreases. Since the difference in voltage goes down, the current flow also decreases which results in a slower change in the voltage difference going forward. Since the rate of charging changes based on the amount charged, this charging phenomenon represents a real world first order system. Let's take a look at a graphical representation of this process.

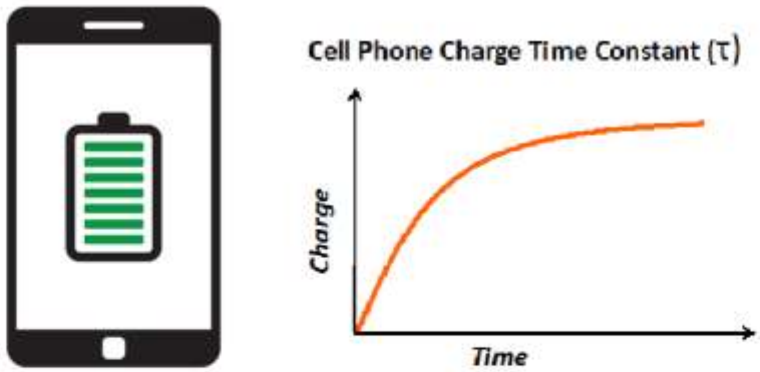

Figure XX: Charging a cell phone – a first order system

In Figure XX, we can see that the slope of the line is initially very steep – that is, voltage initially increases very quickly. As time goes on, the slope of the line gradually decreases – the line starts to level off.

This is an easy experiment to perform at home. Plug your phone in when the battery is at 5% and time how long it takes to charge to 20%. Then, when your phone is at 85%, plug it in and time how long it takes your phone to charge to 100%. Which one takes longer?

Let's observe different initial charging values. Say someone wants to charge his or her phone in three phases, unplugging the charger and quickly plugging it back in, stopping at a certain time. We can divide these time periods into the time constants, τ and its multiples, as depicted in Figure YY.

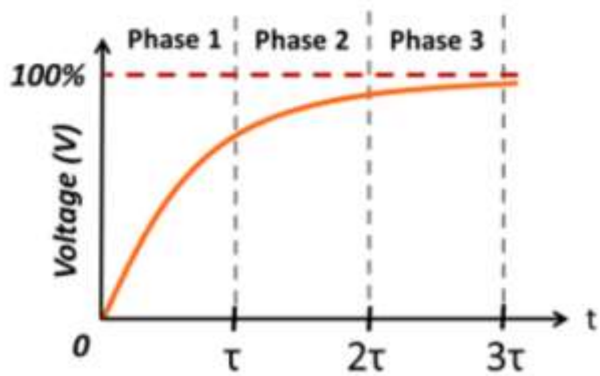

Figure YY: Charging of a cell phone, broken up into three parts.

The three phases of charging can be broken up, as shown in Figure 4. Note that the slope of each phase is different, with phase 1 being the steepest and phase 3 being the flattest. The bottom right graph of Figure ZZ shows the tangent lines of each phase.

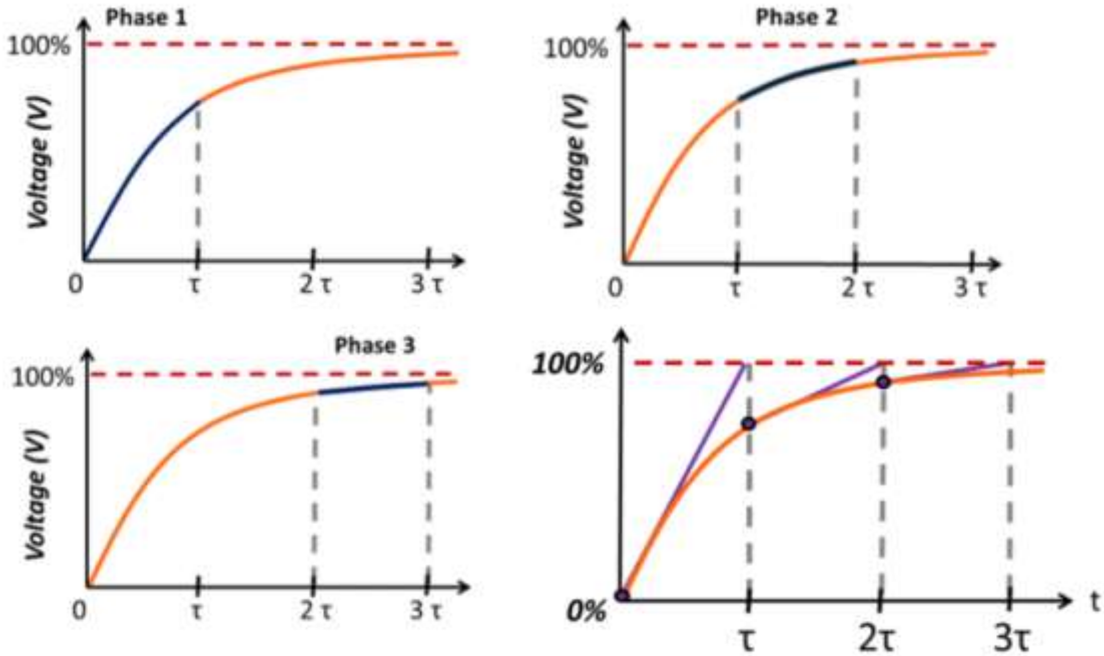

Figure 4: The three phases of charging.

The tangent lines in the bottom right graph of Figure 4 show the initial charging rate for each charging phase. It's clear that the rate of charging decreases as the amount of charge increases.

<u>Electromechanical Examples</u>

DC Motor

DC motors, like the one shown in Figure 5, are common engineering examples of first order systems. DC motors have an electrical and a mechanical time constant.

Figure 5: DC motor with attached wiring.

Since the mechanical time constant is longer, it is dominant and the system can be approximated to have only one time constant. The following equation approximates the transfer function while ignoring the electrical time constant:

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_m}{T_m s + 1}$$

If V_a (voltage) is a step function, the integration of this in the Laplace domain becomes V_a/s. Omega represents $V_a(s)$ multiplied by the transfer function, and this equation can be solved to determine the response to a step function (such as a flipped switch which allows current to flow).

The graph shown in Figure 6 shows a system with a step input V_a and a DC motor's resulting output (omega). Eventually, omega becomes a constant as steady state operation is reached, but in transient responses it is a first order system.

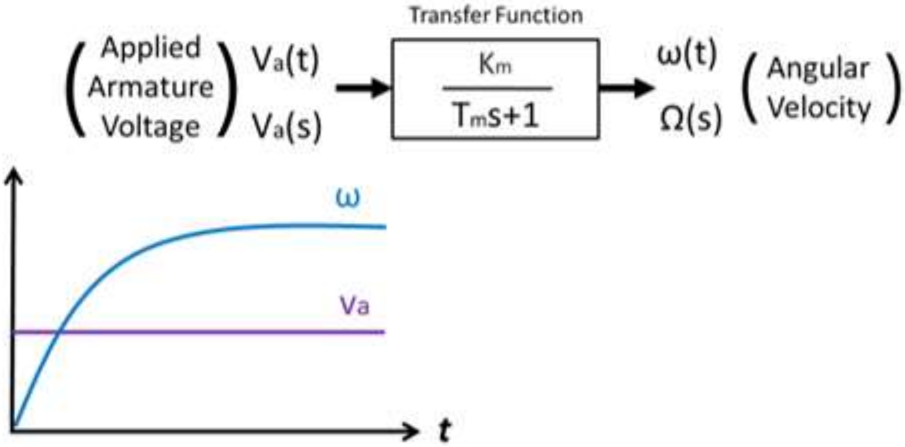

Figure 6: Voltage represents a step function, while omega is a DC Motor's response to that step.

Stove Types

Figure 7: Gas (left) and electric (right) stoves

Consider two different stove types – gas and electric – as shown in Figure 7. Gas stoves operate by using a spark to ignite the gas, which creates a flame. When this occurs, the gas stove is immediately giving off the full extent of its heat energy. This can be plotted as a theoretical step function, similar to the left side of Figure 8. In real life, it takes a few milliseconds for this to take place, but for our purposes it can be considered effectively instantaneous. On the other hand, electric stoves operate through the use of electrical current which heats up coils. This heating of the coils takes time, which means that unlike the gas stove, electric stoves do not immediately give off the full extent of heat energy. This is shown on the right side of Figure YY, and this curve approximates a first order function.

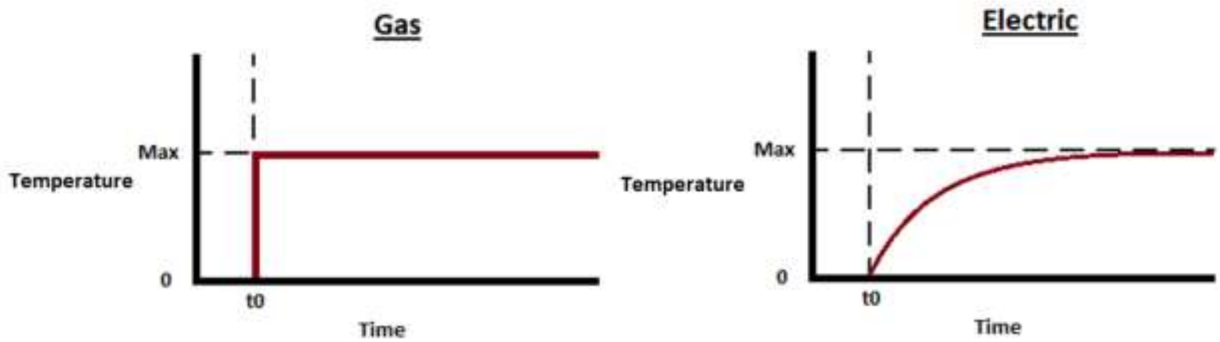

Figure 8: Gas stoves act as a step function, while electric stoves act as a first order function.

The same concept holds true when the stove is turned off as well. For the gas burner, the flame extinguishes immediately as the gas is cut off. The energy given off by the burner itself drops to zero as a step function, as shown on the left side of Figure 9. On the other hand, when the electric stove is turned off the current flow stops but the coils still have energy that must dissipate. Thus, the electric stove still exhibits the characteristics of a first order function, as shown on the right side of Figure 9.

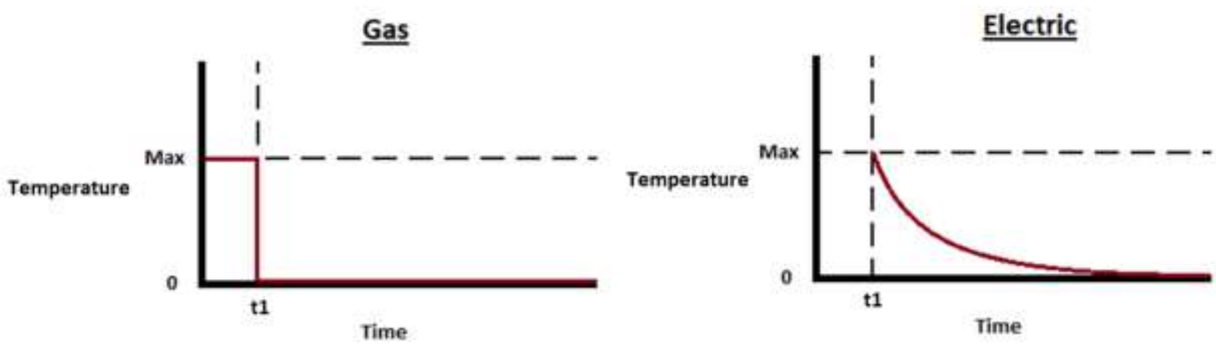

Figure 9: Gas stoves act as a step function, while electric stoves act as a first order function.

<u>Non-Electrical Engineering Examples – Mechanical Engineering, Physics, and Economics</u>

Mechanical – Three Vehicles

This example focuses on the time it takes different types of vehicles to accelerate to a specified speed. First, consider a typical sedan. If accelerating from a stop with constant pressure on the gas pedal, is the speed linear, exponential, or a step function?

The answer is exponential. As the vehicle's speed increases, the acceleration goes down. If this doesn't seem immediately intuitive, consider the force felt when accelerating from a stop. If you "floor it," you are immediately thrown back into your seat. As speed increases, this force holding you in your seat becomes less and less. After a few seconds you can once again lean forward.

Now consider three different vehicles: a sports car, a minivan, and a moving truck. Intuitively, it makes sense that these vehicles accelerate at different rates. As shown in the graph of Figure 10, the sports car reaches top speed much faster than the minivan. Likewise, the minivan reaches top speed faster than the moving truck.

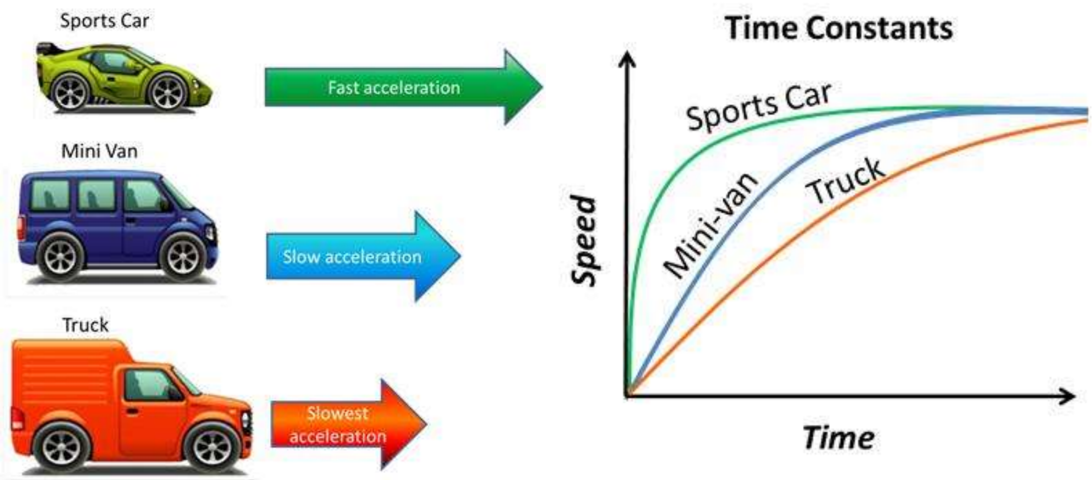

Figure 10: Different types of vehicles have different acceleration rates

The speed of these vehicles all represent First Order Differential Equations.

Physics – Diffusion

Diffusion is essentially the movement of molecules from a region of higher concentration to a region of lower concentration. One example, shown in Figure 11, is a drop of dye or food coloring into a cup of water. Initially, the dye is dropped into one specific spot, but as time passes it spreads until it is uniformly distributed throughout the water.

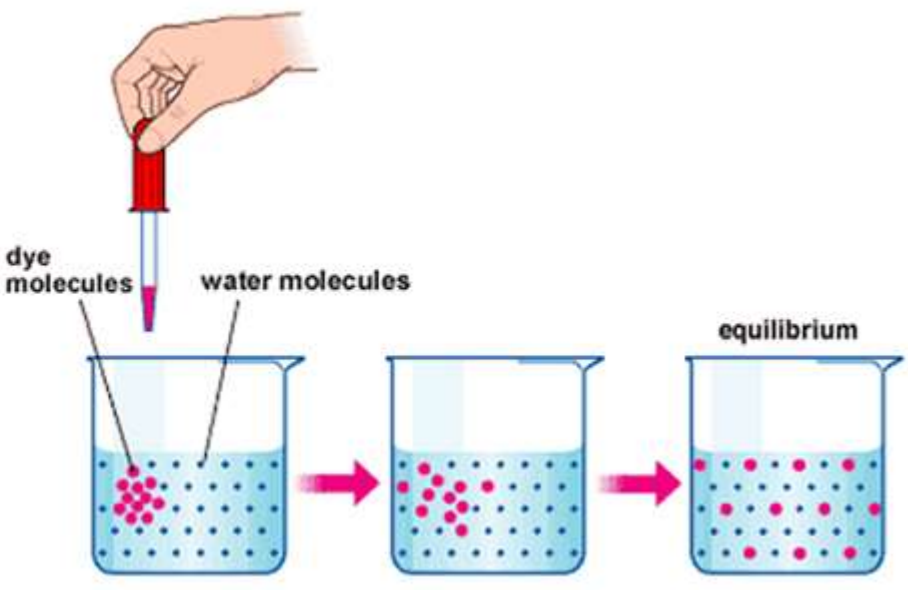

Figure 11: Adding dye to a cup of water

Another example of diffusion is the movement of gas between regions of different pressure. Imagine two sides of a tank separated by a divider. Side “A” is filled with a specific gas. Side “B” is a theoretically perfect vacuum. Once a portion of the divider is removed, the molecules on side A will tend to move towards the area of lower concentration (side B). Over a period of time, the concentration difference between the two sides gets smaller and smaller, and eventually both sides reach equilibrium. This process is shown in Figure 12.

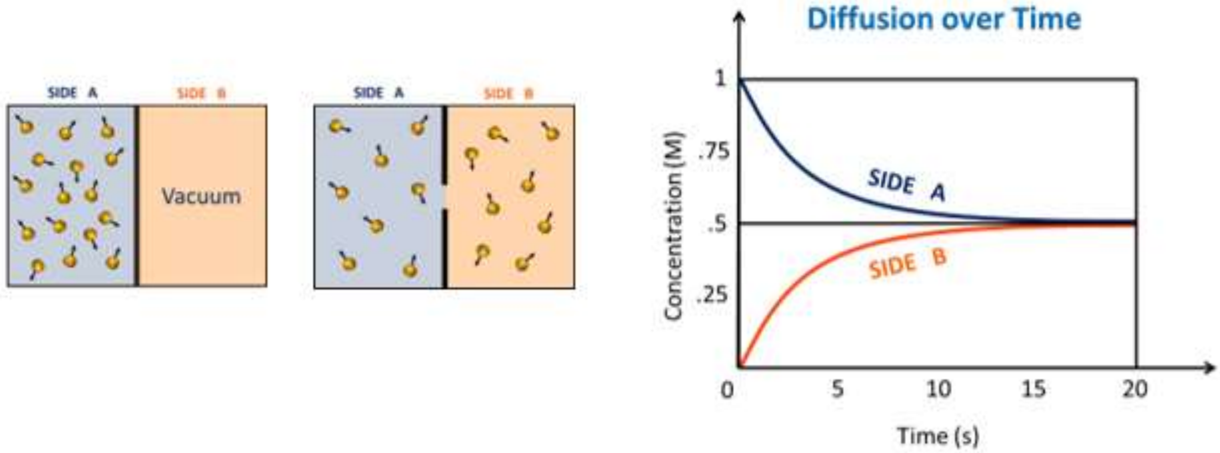

Figure 12: Diffusion of molecules in a tank

In all diffusion situations, the rate of diffusion goes down as the system gets closer and closer to equilibrium (see Figure 12, right). For this reason, diffusion approximates a first order system.

Economics – Savings Account

Savings accounts are a good example of exponential growth. Consider a savings account with an initial balance of $100 and a compounding interest rate of 5%. If the money is left untouched at a set interest rate, it will grow exponentially like in Figure 13.

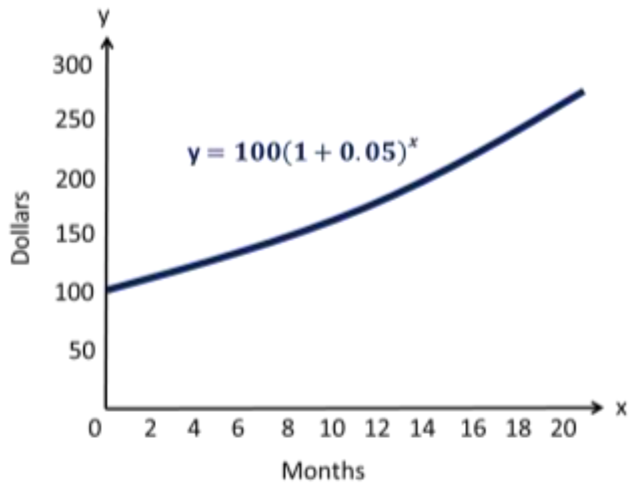

Figure 13: The dollar value grows exponentially as a result of the interest accumulation

On the other hand, picture an account with an initial balance of $100 that accumulates zero interest. The account owner decides to withdraw 5% of the remaining balance each month. This is an example of exponential decay, and is shown in Figure 14.

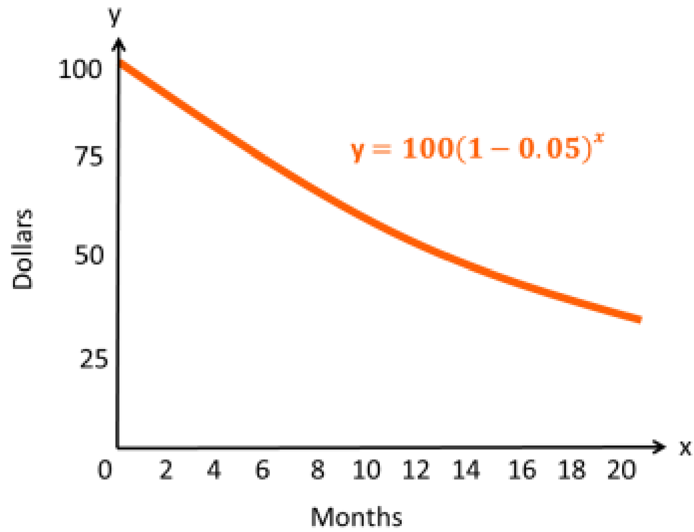

Figure 14: The dollar value decays exponentially as a result of the withdrawals

With this continuous rate of withdrawal, how long will it take for the account to reach a balance of zero? This is a trick question. Theoretically, never, since the graph will never achieve a zero value. Even after a long time the account will always have some fraction of a cent remaining. The account balance will asymptotically approach zero but never quite get there.

Brain Teaser – Moe vs. Joe Coffee

Puzzle: One day, two brothers had a dispute. The first brother, Joe, claimed that coffee stays hotter if one pours cold creamer ten minutes after initially pouring the coffee. The second brother, Moe, claimed that it would stay hotter after 10 minutes if cold creamer is added right away. Which brother is correct?

Figure 15: Joe and Moe's dispute, visualized.

Moe is correct – the coffee stays hotter if cold creamer is added right away. In the beginning, Joe's coffee is hotter than Moe's because it has no creamer. This means that Joe's coffee will cool more rapidly. Since Moe puts creamer in his coffee right away, his coffee starts closer to room temperature which results in a lower rate of cooling. After 10 minutes, when Joe adds creamer to his coffee, his drink's temperature will drop to a lower temperature than Moe's. The graph in Figure 16 shows how both coffees cool, and zooms in to emphasize the temperature drop after 10 minutes [12].

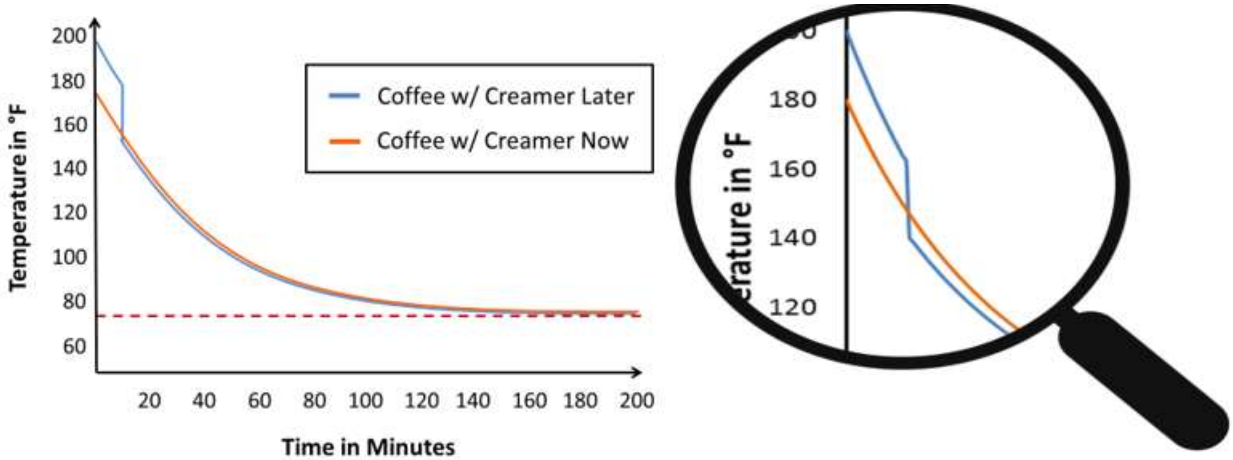

Figure 16: Visualization of coffee cooling based on when creamer is added.

Acknowledgements

Special thanks to Professor Moshe Barak for great discussions regarding teaching and learning First Order Differential Equations. We also thank Mr. Juan D. Yepes for his help with the anonymous questionnaire.

Conclusion

The explanations in this paper were chosen to introduce the concept of First Order Differential Equations in visual and intuitive ways. The paper is not comprehensive and does not attempt to replace current teachings and textbooks. The explanations intentionally do not focus on numerical solutions or heavy mathematics in order to avoid intimidating students, but rather focus on the basic understanding of the concept itself. We hope that those who teach differential equations will use some of these examples as a supplement to their teaching, and that students find this resource helpful in understanding a concept that is widely regarded as difficult. The initial assessment of this work as described in this paper is an indicator that visualization of the concept of First Order Differential Equations is highly desired by students.

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<u>Appendix A – Survey Results</u>

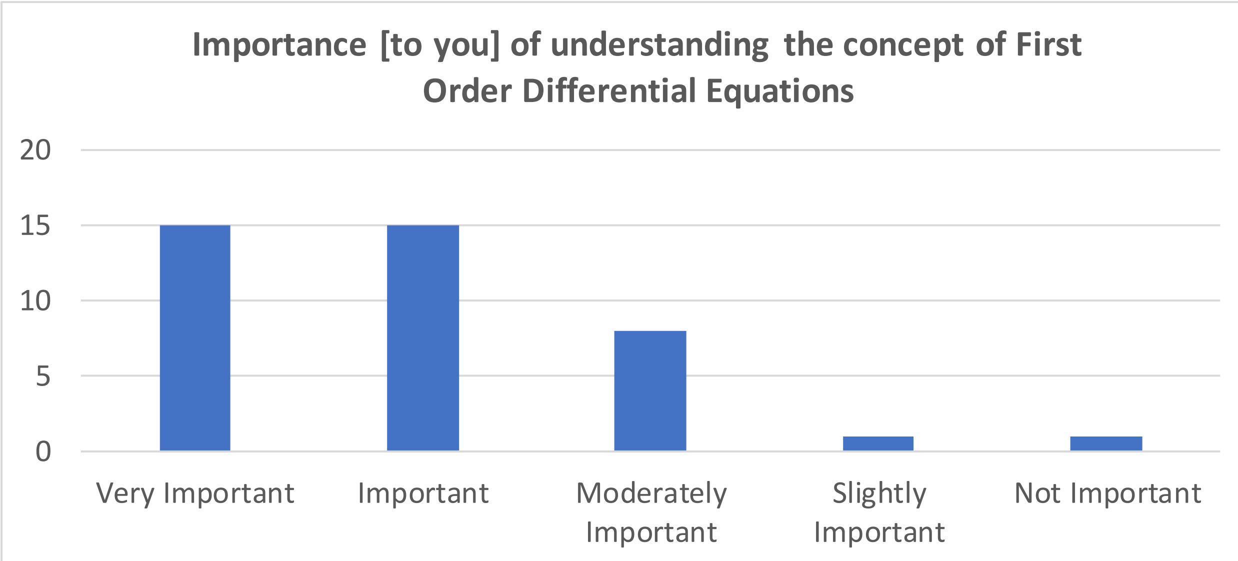

Figure A.1: Student feedback on importance of understanding First Order DE concepts

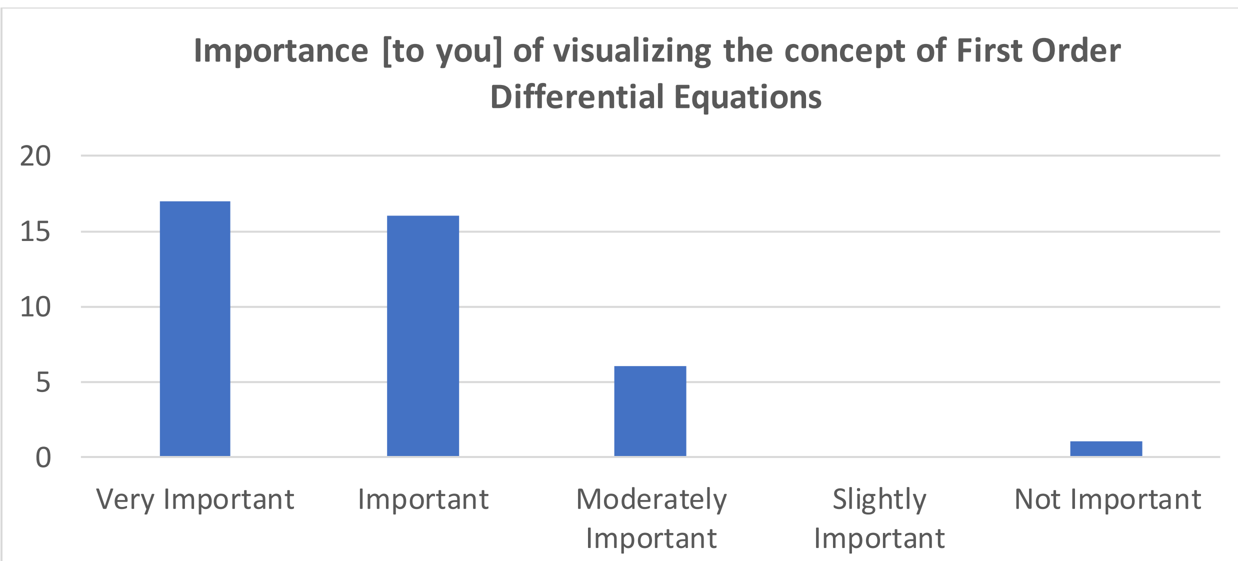

Figure A.2: Student feedback on importance of visualizing the concept of First Order DE's

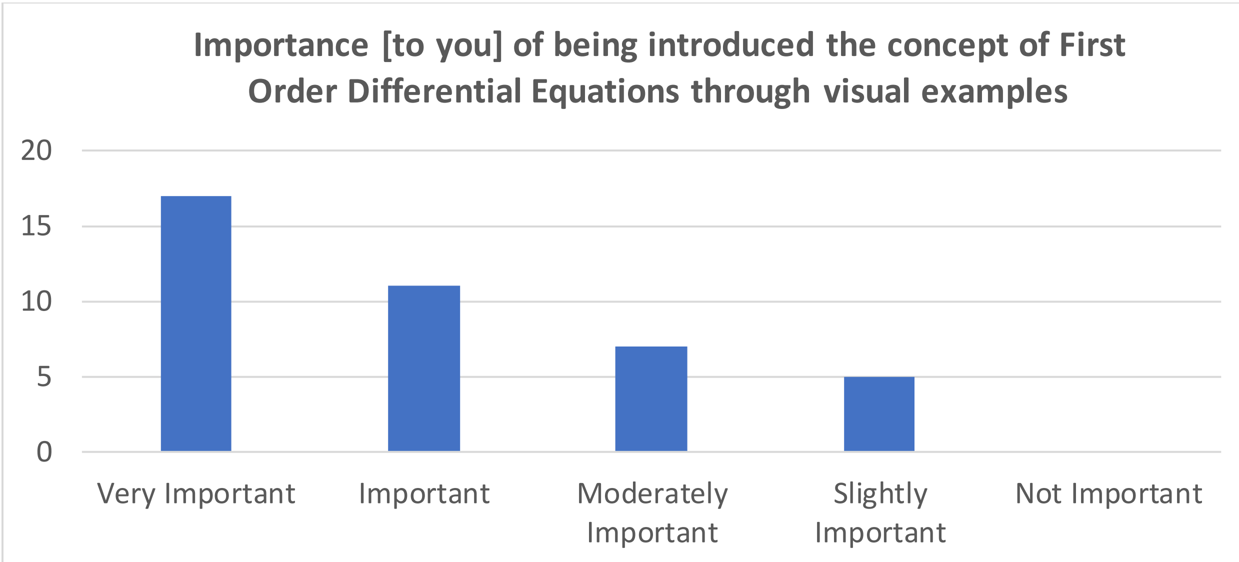

Figure A.3: Student feedback on importance of being introduced to First Order DE's through visual examples

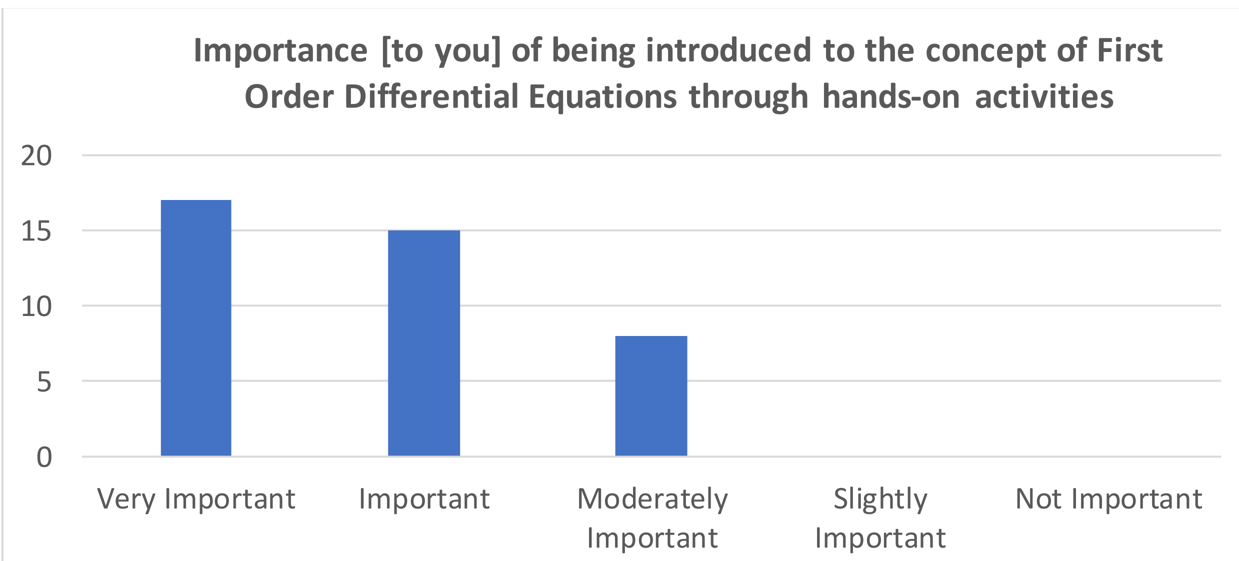

Figure A.4: Student feedback on importance of being introduced to First Order DE's through hands-on activities

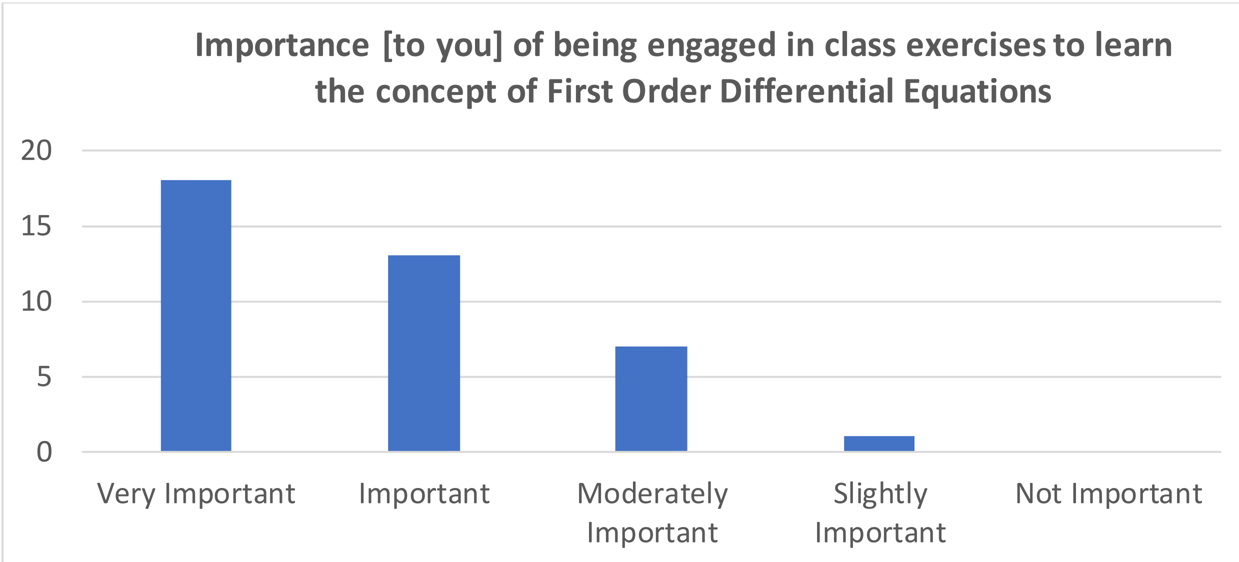

Figure A.5: Student feedback on importance of being engaged in class exercises when learning First Order DE's

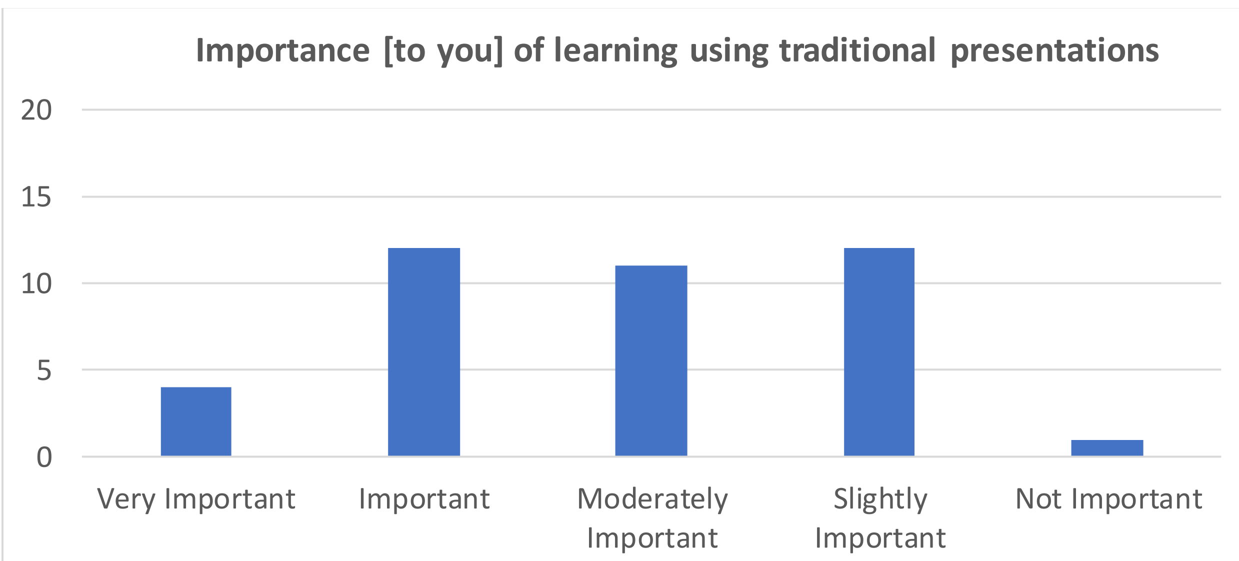

Figure A.6: Student feedback on importance of learning through traditional presentations

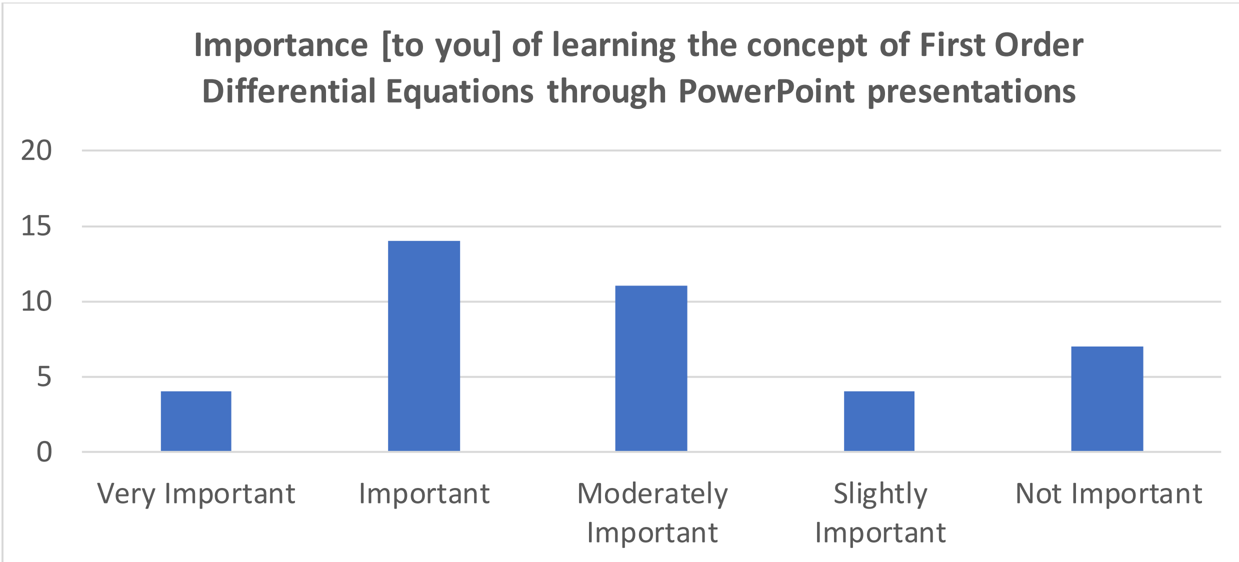

Figure A.7: Student feedback on importance of learning First Order DE's through PowerPoint presentations

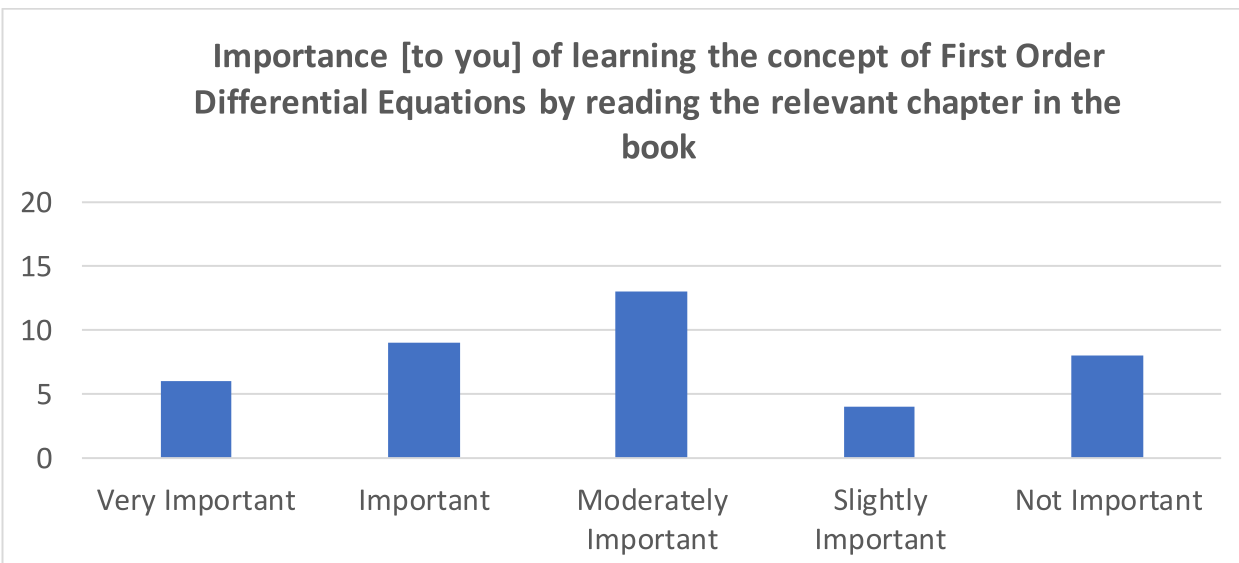

Figure A.8: Student feedback on importance of learning First Order DE's by reading the textbook

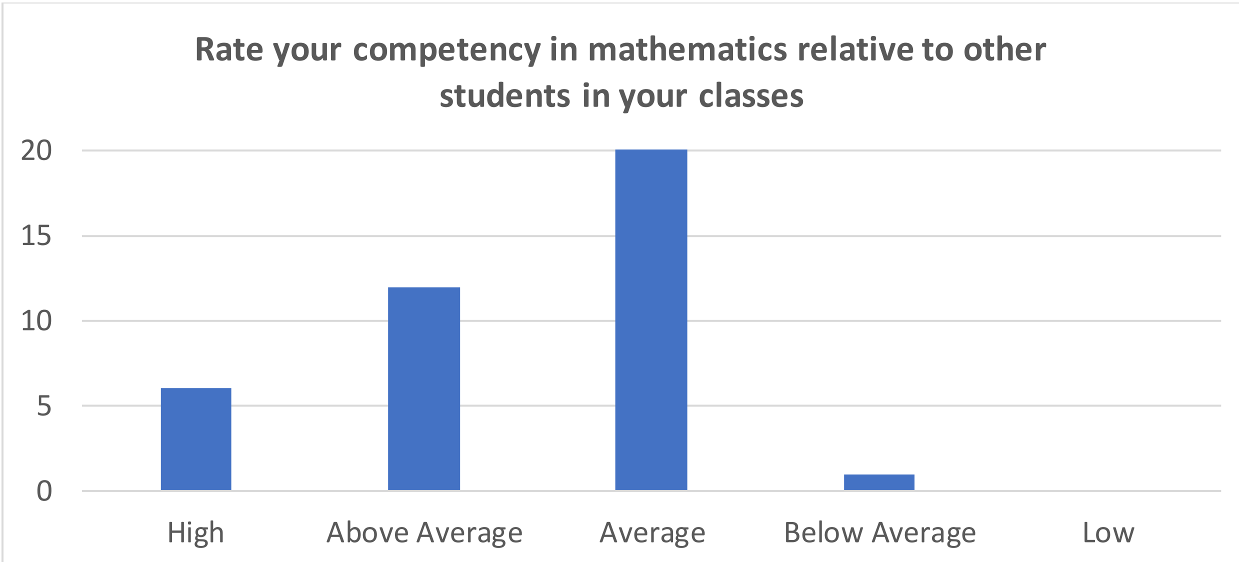

Figure A.9: Student self-assessment of mathematics competency

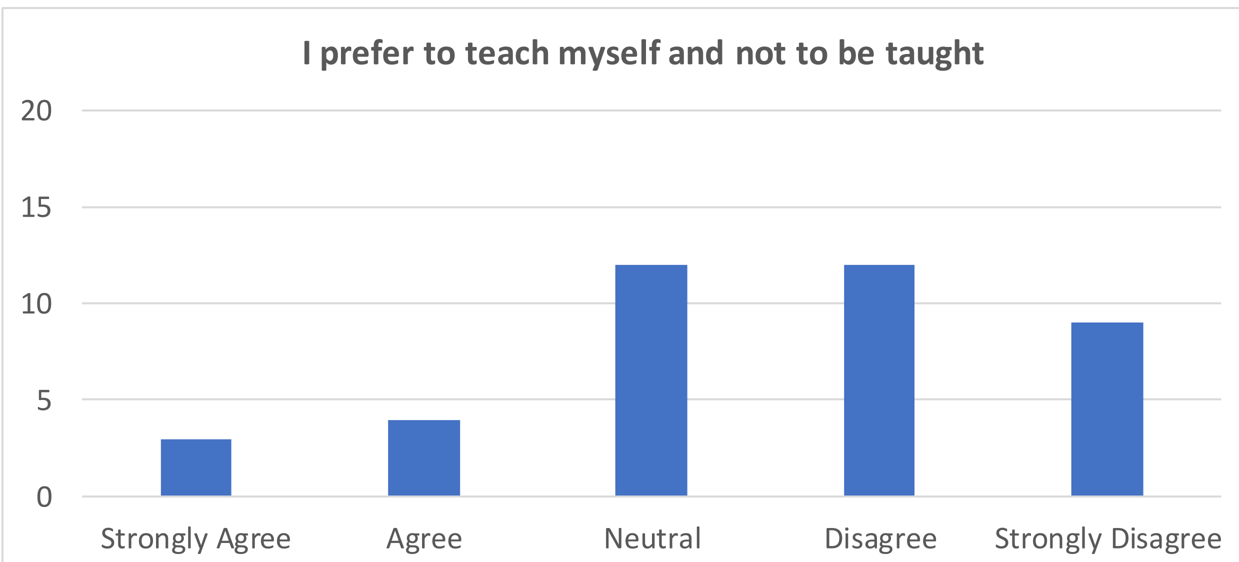

Figure A.10: Student opinions on self-learning