Teaching Problem-Solving by Storyboard

Dr. Barry Dupen
Indiana University Purdue University Fort Wayne

Abstract

Problem-solving methods taught in Statics, Dynamics, and Strength of Materials courses vary in complexity. Some methods require one or two computations, such as calculating the tensile stress in a rod of a given diameter subject to a known tensile load. Other methods require a series of steps, such as frame analysis (three steps), or calculating the moment of inertia about one or more centroidal axes of a compound shape (ten steps). The standard approach used in textbooks is to first explain the theory, then demonstrate a solution with an example problem. The mathematics are presented in sequential order, beneath a diagram. Linear dimensions, angles, and forces which are calculated in the solution are shown on the diagram for reference. In the classroom, an instructor may follow a similar approach: present the theory, then demonstrate a solution with an example problem. The instructor presents the mathematics in sequential order on a blackboard, adjacent to a diagram. Dimensions, forces, and so on are added to the diagram in the order of their calculation, so the student sees a changing diagram…a step-by-step "movie" of the problem-solving process on the chalkboard. The diagram goes into a student's notes in the same manner, but the finished product is a single, finished diagram, not a movie. Later, when a student attempts to solve homework problems, the diagram becomes a point of confusion.

In order to help students learn to solve multistep problems, a series of handouts were developed for Statics and Strength of Materials classes. Each handout uses a storyboard approach to problem solving, where mathematical calculations are matched with an in-process diagram…a series of movie frames. Each diagram includes a shadow of the previous diagram, so the student can visualize the problem solving sequence.

Introduction

In the first semester of teaching Strength of Materials to Mechanical Engineering Technology students, a trend developed in homework performance. Student success varied inversely with the number of computational steps. For example, five problems requiring a single computation were usually all solved correctly, but a single problem consisting of five computational steps was rarely solved correctly. Students who could manage arithmetic and algebra would get lost in a multistep problem. This trend recurred in subsequent Statics and Strength of Materials classes.

These classes were structured so that students attended a lecture on a topic, then the students completed a series of homework problems which were submitted at the following lecture to be graded. Students observed and took notes on problem-solving techniques in class, but most of the learning really took place outside the classroom, one or two days later, as students attempted to solve homework problems.

Students working on homework the night before it was due had three resources for help in solving multistep problems: the textbook, student notes from the lecture, and class handouts. The textbook described a series of mathematical steps, usually accompanied by a single diagram containing many construction lines, dimensions, and other quantities developed in the mathematical solution. For example, standard textbooks typically use a single diagram to illustrate how to calculate the moment of inertia of a complex shape, as shown in Figure 1.

Student notes consisted of a series of equations accompanied by a single diagram…a mirror of the textbook format. The diagram in the notes was created in a several sequential steps, copied from the chalkboard during class, but only the finished diagram appears. The method for constructing the diagram is not evident from the finished product. When the student attempted to solve a similar homework problem, neither the textbook nor the student's notes were much help. Therefore, there was a need for class handouts to help students solve multistep problems.

Storyboard Handouts

A series of handouts were developed for Statics and Strength of Materials students, covering such topics as beam design, truss analysis by the method of joints, truss analysis by the method of sections, frame analysis, and the calculation of the moment of inertia for a compound shape. Each handout uses a storyboard format, where step-by-step mathematical calculations are matched with step-by-step diagrams. Text and equations appear on the left, with diagrams and tables on the right. Changes to the diagrams are emphasized at each step.
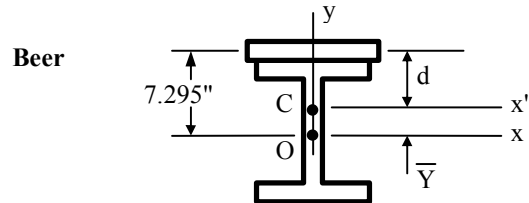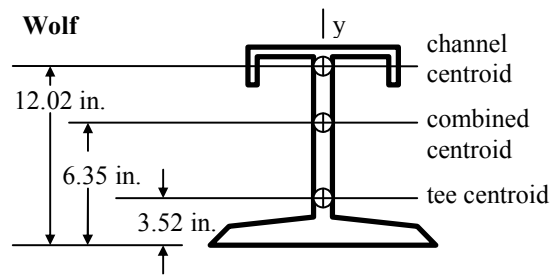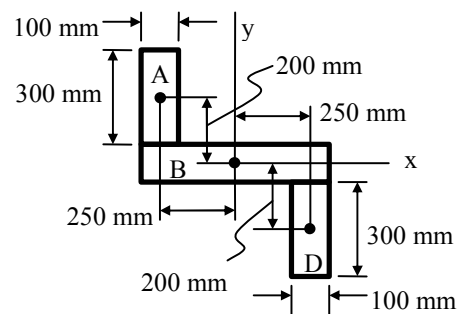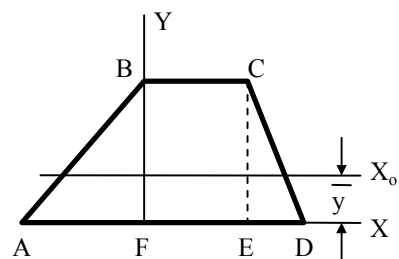

Figure 1: Recent textbooks by Wolf[1], Beer et al.[2], and Hibbeler[3] each use a single diagram to illustrate how to calculate the moment of inertia of a complex shape. This approach is not new; Poorman[4] used the same method in the 1940 edition of *Applied Mechanics*, originally published in 1917.

Example #1: Method of Sections

A storyboard handout for solving truss problems uses six diagrams to illustrate the Method of Sections. The first diagram shows the geometry, dimensions, and applied loads. Accompanying text explains the reason for using the Method of Sections.

Method of Sections

The Method of Joints is the best way to calculate the tensile or compressive forces in every member of a truss. However, if you only need the forces in a few members, the Method of Sections will save you time.

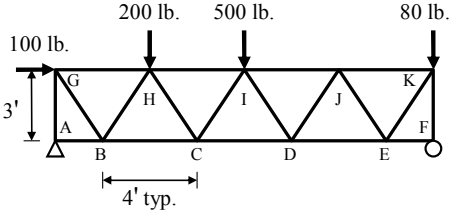**Problem** Find the forces in members ID and CD.

Figure 2: The first part of the Method of Sections storyboard introduces the problem in text and in a diagram.

The second diagram illustrates how to find the reaction forces at the two supports. The entire truss is treated (and drawn) as a solid block, because reaction force calculations are independent of truss geometry. The dimensions shown in this diagram are the specific dimensions required in the calculations at the left.

Step 1 Calculate reaction forces at the supports, using three equations: $\Sigma M=0$, $\Sigma F_x=0$, and $\Sigma F_y=0$. Treat the truss as a solid body, since reaction forces depend only on external forces and dimensions.

$$\Sigma M_A = 0 = -3ft.(100lb.) - 4ft.(200lb.) - 8ft.(500lb.) - 16ft.(80lb.) + 20ft.R_{Fy}$$

$$R_{Fy} = \frac{3ft.(100lb.) + 4ft.(200lb.) + 8ft.(500lb.) + 16ft.(80lb.)}{16ft.} = 398.75lb.$$

$$\Sigma F_y = 0 = R_{Ay} + R_{Fy} - 200lb. - 500lb. - 80lb.$$

$$R_{Ay} = -398.75lb. + 200lb. + 500lb. + 80lb. = 381.25lb.$$

$$\Sigma F_x = 0 = 100lb. + R_{Ax}$$

$$R_{Ax} = -100lb.$$

Since R_{Ax} is negative, the arrow is drawn backwards. Leave the arrow in this direction in all subsequent diagrams, and use the value of $-100lb$.
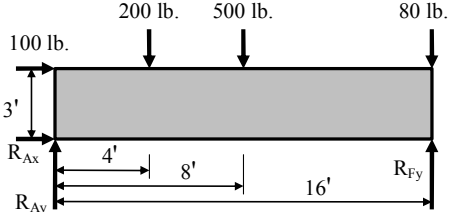

Figure 3: The second part of the Method of Sections storyboard shows how to calculate reaction forces. Equations are listed on the left, interleaved with explanatory text.

The third diagram shows the location of the imaginary cut required by this method of solution. The truss geometry appears again, but no dimensions or load values are shown, because the location of the cut is independent of these values.

Step 2 Cut the truss. The cut must pass through a member you are solving for. The cut does not have to be vertical, or even straight, but it should pass through the fewest possible members. In this problem, a single cut passes through members ID and CD. In some problems, you may need to make more than one cut to calculate all the required forces.
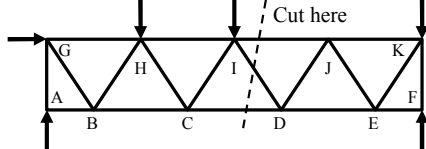

Figure 4: The third part of the Method of Sections storyboard discusses the imaginary cut required by this solution method, and shows the location on a diagram.

The fourth diagram shows a free-body diagram of the cut truss, with arrows representing the longitudinal forces in the cut members. The text discusses sign conventions and how to perform the calculations. The fifth diagram illustrates how to use the ratio of the hypotenuse to the leg of a triangle, instead of using trigonometric functions, to calculate horizontal and vertical components of force.

Step 3 Draw one portion of the cut truss. Draw cut members as if they were external forces acting on the cut portion.

Step 4 Use $\Sigma M=0$, $\Sigma F_x=0$, and $\Sigma F_y=0$ to calculate the forces in the cut members.

$$\Sigma M_I = 0 = 3ft.CD + 4ft.(200lb.) + 3ft.R_{Ax} - 8ftR_{Ay}$$

$$CD = \frac{-4ft.(200lb.) - 3ft.(-100lb.) + 8ft.(381.25lb.)}{3ft.} = 850lb.$$

CD is positive, so the arrow is drawn correctly. CD = 850lb. tension.

The vertical component of ID is $(3/\sqrt{13})ID$.

$$\Sigma F_y = 0 = R_{Ay} - 200lb. - 500lb. - (3/\sqrt{13})ID$$

$$ID = \frac{381.25lb. - 200lb. - 500lb.}{3/\sqrt{13}} = -383.09lb.$$

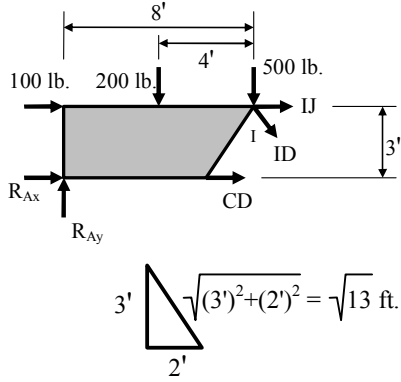ID is negative, so the arrow is drawn backwards.
ID = 383.09 lb. compression.

Figure 5: The fourth part of the Method of Sections storyboard shows how to calculate forces in the cut members.

The sixth diagram shows students how to reduce the complexity of a solution by selecting a different portion to analyze.

Alternate Solution Use the other portion of the cut truss. This solution looks simpler…instead of three applied loads and two reaction forces, there is only one applied load and one reaction force. Use $\Sigma F_y=0$ to find ID, then use $\Sigma M_J=0$ to find CD.
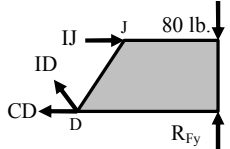

Figure 6: The fifth part of the Method of Sections storyboard shows an alternate solution.

Example #2: Frame Analysis

Students in a summer Statics course struggled with frame analysis because they found it difficult to keep track of the directions of the forces. Although there are only three steps, the second step may need to be repeated many times. The first diagram shows all dimensions and external loads. In the second diagram, the frame is considered to be (and drawn as) a monolithic block, which helps students understand how to calculate reaction forces. The third diagram uses a free-body diagram of member D-E to illustrate how to solve for pin reactions.

Frame Analysis

The object of Frame Analysis is to calculate the shear force on each pin. The solution takes three steps, although you may have to repeat the second step several times.
Only three equations are required: $\Sigma M=0$, $\Sigma F_x=0$, and $\Sigma F_y=0$.

Step 1 Calculate the external reactions on the frame *if possible*, using $\Sigma M=0$, $\Sigma F_x=0$, and $\Sigma F_y=0$. Treat the entire frame as a single unit. In some cases, there are too many unknowns, and it is not possible to calculate all of the reactions.
In this frame, pin B is on a roller, so there is no horizontal reaction at B.
$\Sigma F_x = 0 = R_{Ax}$
$\Sigma M_A = 0 = -500N \times 30cm + R_{By} \times 80cm$
$R_{By} = (500N \times 30cm)/80cm = 187\frac{1}{2}N$
$\Sigma F_y = 0 = R_{Ay} + R_{By} - 500N$
$R_{Ay} = -R_{By} + 500N = -187\frac{1}{2}N + 500N = 312\frac{1}{2}N$

Step 2 Analyze each frame member separately. Calculate the pin reactions using $\Sigma M=0$, $\Sigma F_x=0$, and $\Sigma F_y=0$.
$\Sigma F_x = 0 = D_x - E_x$
$E_x = D_x$…two unknowns, one equation. Solve for these later, using a different frame member.
$\Sigma M_D = 0 = -500N \times 30cm + E_y \times 80cm$
$E_y = (500N \times 30cm)/80cm = 187\frac{1}{2}N$
$\Sigma F_y = 0 = D_y + E_y - 500N$
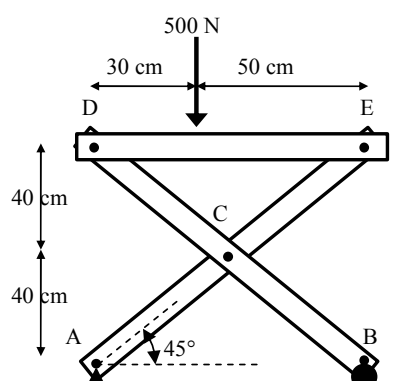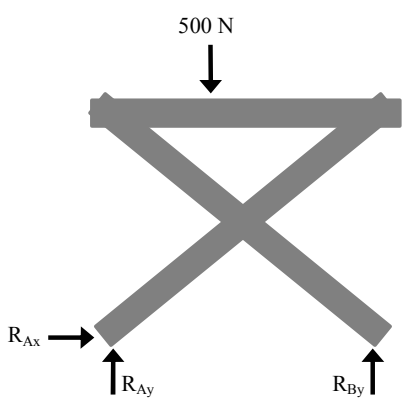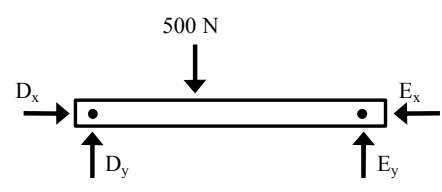$D_y = -E_y + 500N = -187\frac{1}{2}N + 500N = 312\frac{1}{2}N$

Figure 7: The first half of the Frame Analysis storyboard handout introduces the problem statement, shows how to find the reaction forces, and introduces analysis of one frame member.

The fourth diagram shows how to analyze pin forces on a second frame member. The text explains why the force vectors appear in the opposite direction to the previous figure. Finally, pin forces are calculated and listed in a table for comparison. The text explains the practical purpose for calculating loads on frame pins.

Continue with **Step 2** for the remaining frame members, until all pin reactions are found.

Notice that the forces on pin D are drawn in the opposite direction to their orientation on the previous member. In static equilibrium, forces applied by one member of the frame onto a pin must be equal and opposite to the forces applied by another member of the frame onto that pin (otherwise the structure will start to move).

$\Sigma M_C = 0 = D_y \times 40cm + D_x \times 40cm + R_{By} \times 40cm$

$0 = D_y + D_x + R_{By}$

$D_x = -D_y - R_{By} = -312\frac{1}{2}N - 187\frac{1}{2}N = -500N$, therefore the arrowhead is drawn the wrong way.

$\Sigma F_x = 0 = C_x + D_x$

$C_x = -D_x = -(-500N) = 500N$
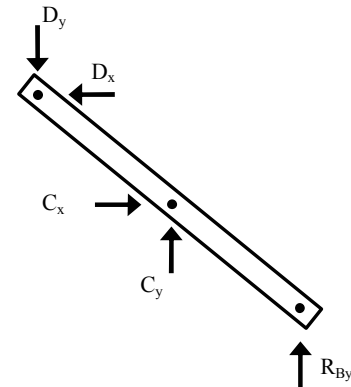
$\Sigma F_y = 0 = -D_y + C_y + R_{By}$

$C_y = D_y - R_{By} = 312\frac{1}{2}N - 187\frac{1}{2}N = 125N$

Going back to the horizontal member,

$E_x = D_x = 500N$

Step 3 Enter the pin forces into a table, and calculate the resultant force on each pin per $R = \sqrt{F_x + F_y}$. If you were building a frame, you would use these results to select pins. Instead of using the strongest pin at every joint, you can save money and size the pins according to the load they must carry. For example, the pin used at joint B can be one third as strong as the pin required for joint D.

Pin	F_x	F_y	R
A	0N	312½N	312½N
B	0N	187½N	187½N
C	500N	125N	575N
D	500N	312½N	590N
E	500N	187½N	534N

Figure 8: The second half of the Frame Analysis storyboard handout continues with a free-body diagram of a second member, and tabulation of the final results.

Example #3: Moment of Inertia of a compound shape

Calculating the moment of inertia of this T-shaped section with respect to the x-x centroidal axis is a ten-step process. The diagram in a textbook or in a student's notes is similar to Figure 1 or Figure 9, with all calculated dimensions shown. A careful student will have a ten-step mathematical list to follow in solving the problem. However, the order of calculation is not immediately obvious from the diagram, so a less-careful student may struggle to solve the problem.
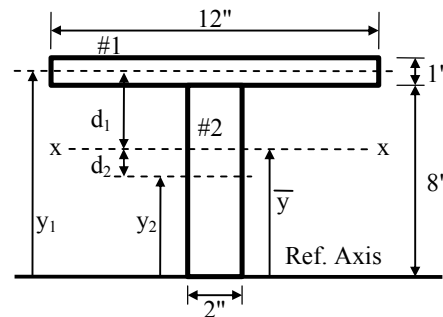
Figure 9: Diagram for calculating the moment of inertia of a T-shaped section with respect to the x-x centroidal axis, as it might appear in a student's notes or in a textbook.

The storyboard handout for Moment of Inertia uses six diagrams which evolve during the problem-solving process. Numerical results are shown in a series of tables which also build as the solution progresses.

Calculating Moment of Inertia of a Complex Shape

You can calculate the moment of inertia with respect to the y-y axis (I_y) of this complex shape by breaking it up into two rectangles…a horizontal one on top, and a vertical one below. Since the y-y centroidal axes of the complex shape and both of its components coincide, you can calculate the moment of inertia of each component with respect to the y-y axis, then add the results.

Calculating I_x is a little more involved because the shape is not symmetrical about the x-x axis. However, since the complex shape is a combination of simple shapes, you can still calculate I_x, using the following ten steps.

Step 1 Separate the complex shape into simple shapes, and label the component areas. This complex shape can be split into two rectangles, but this method also works for complex shapes made up of many simple shapes.

Step 2 Calculate the area (a) of each component. Enter the areas and their sum into a table. Be sure to list the units, because in some problems, you may need to use conversion factors in the calculation.

Part	a (in²)
1	12
2	16
Sum	28

Step 3 Pick a Reference Axis, and label it on the diagram. In theory, you can select any axis, but in practice, the math is usually easier if you pick an axis along the top or bottom of the complex shape (left or right for calculating I_y), or along the centroidal axis of one of the component areas.

Step 4 Draw the distance from the Reference Axis to the centroidal axis of each area. Label these distances y_1, y_2, etc.
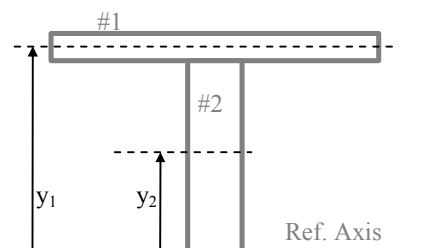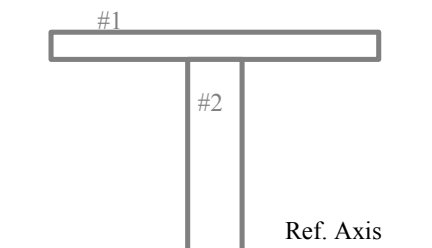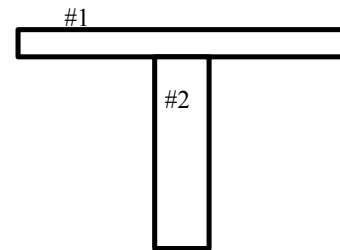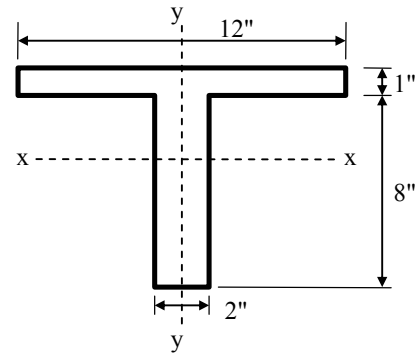
Figure 10: The Moment of Inertia storyboard starts by introducing the problem, and uses a series of diagrams to show the progression of the solution. Black lines show the current step; gray lines indicate previous steps.
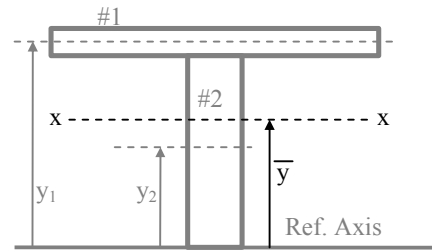
Enter these results into the table.

Part	a (in²)	y (in.)
1	12	8.5
2	16	4
Sum	28	

Step 5 Calculate the value of ay for each component area. Enter these values and their sum into the table.

Part	a (in²)	y (in.)	ay (in.³)
1	12	8.5	102
2	16	4	64
Sum	28		166

Step 6 Draw the distance from the Reference Axis to the x-x centroidal axis of the complex shape.

Calculate this distance as $\bar{y} = \dfrac{\Sigma ay}{\Sigma a} = \dfrac{166 \text{ in.}^3}{28 \text{ in.}^2} = 5.93$ in.

Step 7 Draw the Transfer Distance (d) for each area. This is the distance from the centroidal axis of the area to the centroidal axis of the complex shape.
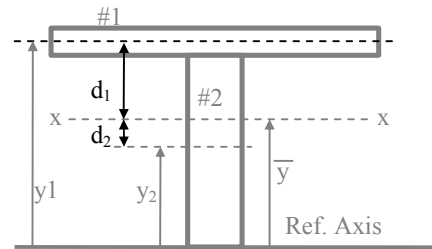
Enter the results into the table.

Part	a (in.²)	y (in.)	ay (in.³)	d (in.)
1	12	8.5	102	2.57
2	16	4	64	1.93
Sum	28		166	

Step 8 Calculate I for each component area about its centroidal axis. For rectangles, $I_o = (b\ h^3)/12$, where b is the width of the base, and h is the height. For other simple shapes, I_o formulas are listed in the inside back cover of the textbook. Enter these values and their sum into the table.

Part	a (in.²)	y (in.)	ay (in.³)	d (in.)	I_o (in.⁴)
1	12	8.5	102	2.57	1
2	16	4	64	1.93	85.3
Sum	28		166		86.3

Step 9 Calculate ad² for each component area. Enter these results and their sum into the table.

Part	a (in.²)	y (in.)	ay (in.³)	d (in.)	I_o (in.⁴)	ad² (in.⁴)
1	12	8.5	102	2.57	1	79.3
2	16	4	64	1.93	85.3	59.5
Sum	28		166		86.3	139

Step 10 Use the Transfer Formula to calculate I for the complex shape.
Note: if a complex shape is not symmetrical about the y-y axis, you can use this 10-step method to find I_y using a vertical Reference Axis.

$I = \Sigma(I_o + ad^2) = \Sigma I_o + \Sigma(ad^2)$
$= 86.3 \text{ in}^4 + 138.9 \text{ in}^4 = \mathbf{225.2 \text{ in}^4}$.

Figure 11: The Moment of Inertia storyboard shows how the diagram and table changes as the solution develops.

Each diagram in the Moment of Inertia storyboard shows a shadow of the previous diagram in gray, with new additions in black, so students can visualize the problem-solving sequence. Since this method includes a tabular solution, the table evolves with the diagram. An additional diagram explains how to calculate the moment of inertia of a section which has one or more holes.

Some complex shapes have holes. To solve this type of problem, let part #1 be the entire rectangle (with no hole), and part #2 be the square hole. In all calculations, the area of the hole and the moment of inertia of the hole are negative numbers. Thus a_1, a_1y_1, I_{o1}, and $a_1d_1^2$ are positive numbers, and a_2, a_2y_2, I_{o2}, and $a_2d_2^2$ are negative numbers. Follow the same ten steps as in the previous problem.

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{54\ \text{in.}^3}{28\ \text{in.}^2} = 1.93\ \text{in.}$$

$I_x = 41.33\ \text{in}^4 - 1.14\ \text{in}^4 = \textbf{40.19 in}^4$.
Without the hole, $I_x = 42.83\ \text{in}^4$, so the hole reduced I_x by about 6%.

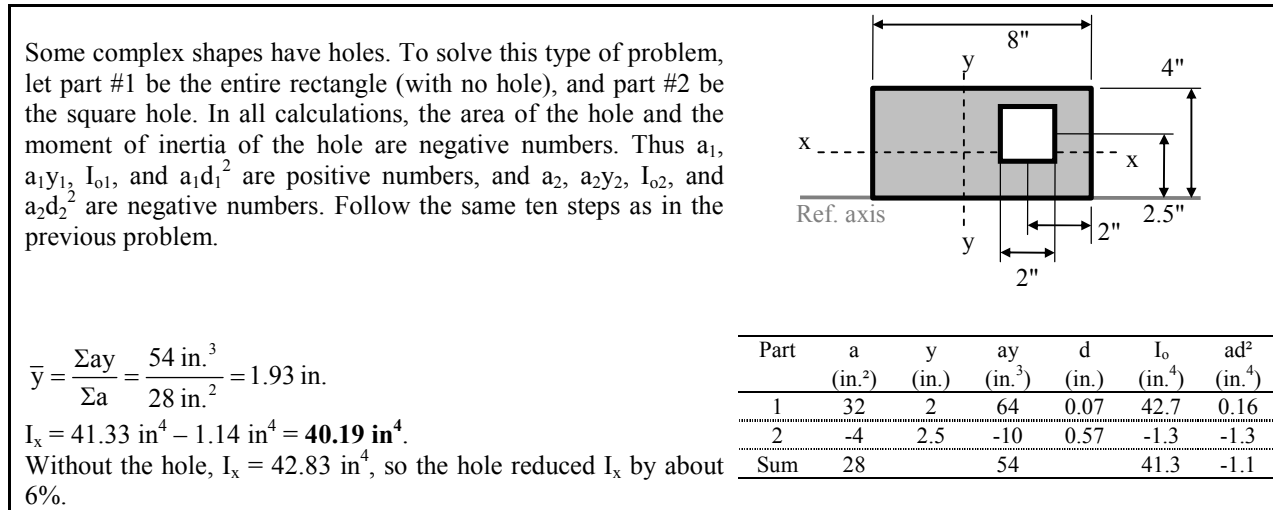| Part	a (in.²)	y (in.)	ay (in.³)	d (in.)	I_o (in.⁴)	ad² (in.⁴)
1	32	2	64	0.07	42.7	0.16
2	-4	2.5	-10	0.57	-1.3	-1.3
Sum	28		54		41.3	-1.1

Figure 12: Moment of Inertia of a compound shape with a hole. The solution includes only one figure and a completed table, similar to a classic textbook presentation.

Discussion

Storyboard handouts are designed so that students can visualize the intermediate steps in a multistep solution. In examples #1 and #2, intermediate steps consist of distinct diagrams. In example #3, these steps consist of a single diagram which becomes more complex as construction lines, dimensions, and values are added. Changes to diagrams are emphasized best with color, but gray tones and black are cheaper to print. In examples #1 and #2, shading obliterates parts of the diagrams which are irrelevant to the solution. Atherton[5] remarks that "relevant graphics are useful" in class handouts. In storyboard handouts, relevant graphics are *essential*. Anecdotal evidence suggests that students view storyboard handouts as useful tools for solving homework assignments and open-book exam problems. Towards the middle of one semester, students asked for additional storyboard handouts on new topics.

The field of Mechanics instruction is evolving, with distance learning,[6] experiential learning,[7] and simulation software[8,9] supplementing and supplanting traditional teaching approaches. Storyboard handouts are supplements, not substitutes, for textbooks and classroom lecture presentations. It is not practical to use storyboards in textbooks because of space considerations. However, a large number of storyboard problem-solving guides could be stored on websites maintained by professors, textbook authors, or textbook publishers.

Conclusions

Storyboard handouts can serve as a useful teaching supplement in problem-solving courses such as Statics, Dynamics, and Strength of Materials. They can help a student make sense of problem-solving methods in the textbook and lecture notes, when the student is solving homework problems away from campus.

Storyboard handouts need not be limited to classical mechanics courses. Within engineering, they can be applied to a range of multistep engineering problem-solving methods, from applying Kirchoff's Law to an electrical circuit, to applying Bernoulli's Equation in a fluid power circuit. Beyond engineering, they can be applied to any multistep process involving visual components, such as the procedure for adding error bars to an x-y graph in spreadsheet computer program. The greatest benefit of these handouts may arise after graduation, when the engineer attempts to solve a similar problem at work, several years after completing the relevant course.

In the future, students will be invited to critique these problem-solving handouts, and recommend improvements. The experience of editing other people's work may help the students in their future careers in engineering and management.

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7. Anna Dollár & Paul Steif, "Reinventing the Teaching of Statics". *Proceedings of the 2004 American Society for Engineering Education Annual Conference & Exposition*, Session 1368.

8. Shahin Vassigh, "Teaching Statics and Strength of Materials Using Digital Technology". Presented at the ARCC Spring Research Conference at Virginia Polytechnic Institute and State University, April 2001, p. 92-96.

9. *West Point Bridge Designer 2005*, available from The U.S. Military Academy at West Point.

BARRY DUPEN
Dr. Dupen earned his B.S. in Mechanical Engineering, and his M.S. and Ph.D. in Metallurgy, all at the University of Connecticut. After working for nine years in the automotive industry as a metallurgist, materials engineer, and materials laboratory manager, he joined Indiana University Purdue University Fort Wayne (IPFW) as an Assistant Professor of Mechanical Engineering Technology. His primary interests lie in materials engineering, mechanics, and engineering technology education.