AC 2012-4138: TEACHING PYTHAGORAS'S THEOREM USING SOFTWARE

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Teaching Pythagoras's Theorem Using Software

The Pythagorean Theorem is one of the most famous equations in mathematics. Since Pythagoras derived the theorem on the shores of Greece the world has used it to build the strong structures such as the Parthenon and the Acropolis.

My generation learned about the Pythagorean Theorem in high school. A teacher wrote the equation on the board and then explained the equation by showing examples of right triangles. The class was fascinated by the ability to solve geometrical problems using right triangles and applying the Pythagorean Theorem. However, today's Sesame Street generation requires more motivation to become enthused with basic mathematics. Students who have grown up with Sesame Street have become passive listeners with short attention spans. They require visual stimuli to grasp mathematical concepts.

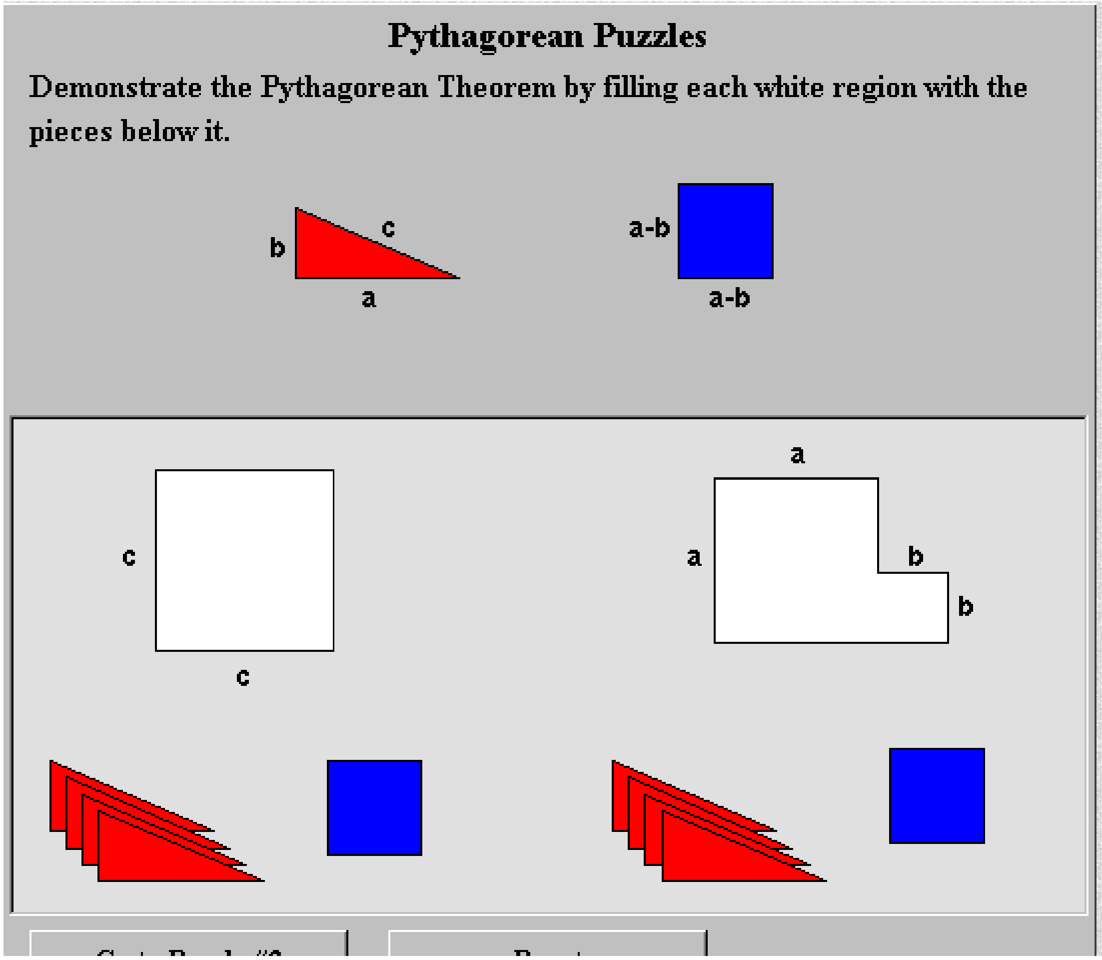
The challenge of teaching mathematics to today's students is formidable. The technology now exists to stimulate the creative side of students. We now have students who play games on the internet the entire day rather than devoting time to open their math book to learn the required material. Even with the math books of today, with colored triangles and fancy fonts, we are unable to reach a large segment of our students who find math boring.

Utah State University has developed interesting software in an attempt to meet the challenge of stimulating students. Utah State University has a National Library of Virtual Manipulates. The web address is http://nlvm.usu.edu/en/nav/topic_t_3.html.

A group of High School Teachers who were taking a course in teaching mathematics using Utah State University Software were having difficulty. They asked me to explain how it was possible to use this software to actually teach the Pythagorean Theorem. In other words "How does the solution of the puzzle prove the Pythagorean Theorem?" After my discussion with the teachers, it was clear to me that the concept of using manipulates motivated them to ask questions. This led me to believe that students would be similarly motivated by these manipulates.

The puzzle is set up as shown in Figure 1

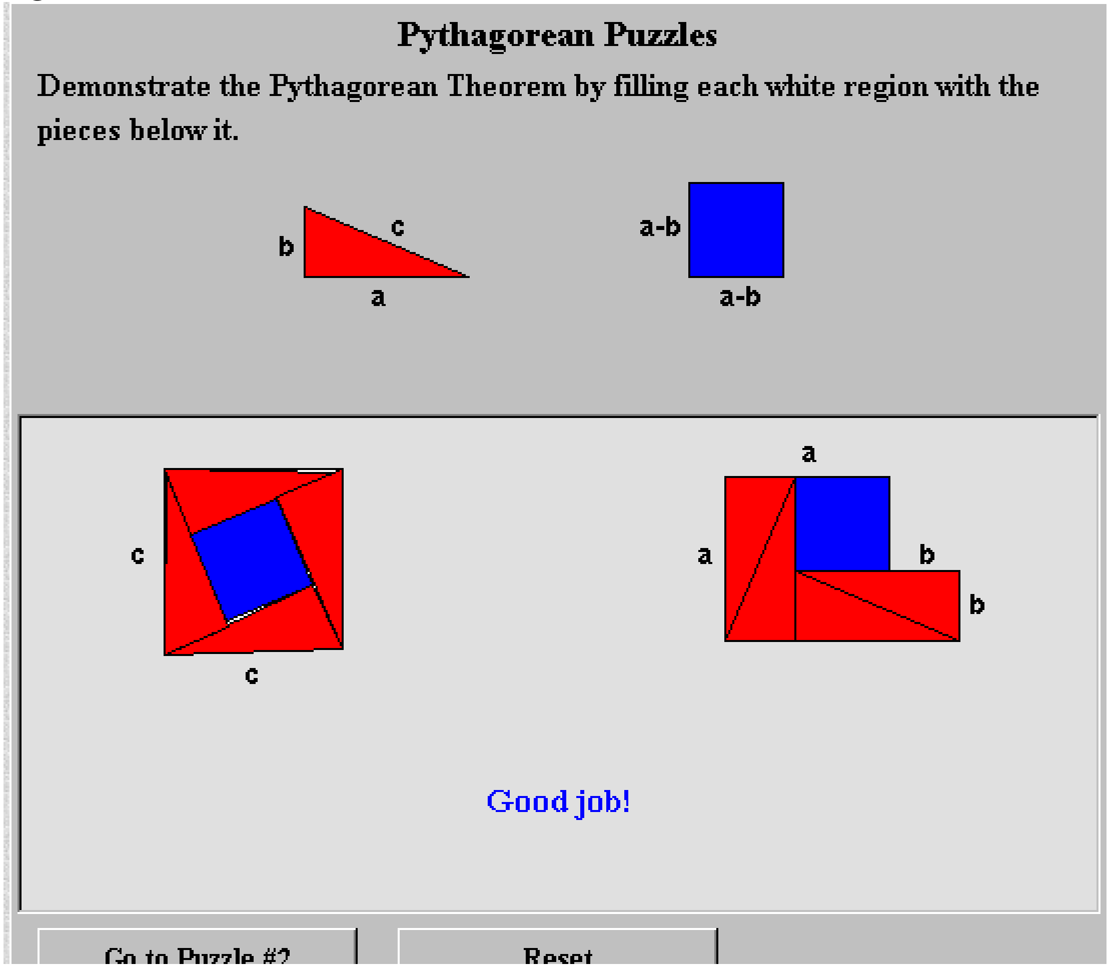
Figure 1



Viewing figure 1 the title is" pythagorean puzzles."
In the top part of figure 1 there is a triangle and a square. The red triangle is a right triangle. The base is b the height is a and the hypotenuse is c. The area of the red triangles is equal to ½ (ab) The little blue square has both base and height a-b. The area of the blue square is $(a-b)^2$ which is equal to $a^2 -2ab + b^2$.

In the lower half of the figure1 there are two puzzles. The puzzle on the left is a large square whose side is c and whose area is c^2. The puzzle on the right is a structure which is made with two squares. A large square with a side a with an area a^2 is joined with a small square. The smail square has a side b and an area b^2. The object of each puzzles is to use 4 red right triangles and the blue square to fill both areas.

The Figure 2 shows that both puzzles have been solved.
Figure 2



The lower part of Figure 2 shows that the areas of both objects are now full. The puzzel on the left shows that 4 red triangles and the blue square have filled up the large square whose side is c. The puzzel on the right is also filled with 4 red triangles and the blue square. Notice that the bottom of figure 2 has the words "Good Job!"

Good Job! This message indicates that the puzzle is solved. This means that the Pythagorean Theorem is proved. Here is where the teachers needed further explanation on how the puzzles prove the Pythagorean Theorem. The following is the method I used to help them comprehend that proof.

The square on the left whose side is c has an area of c^2 .The square is filled with 4 triangles and one little square. This means that the area of the 4 triangles and the small square must equal the area of the large square. The area of a red triangle is ½ ab. Therefore the 4 red triangles have an area of 4 * (1/2 ab) or 2ab. The area of the blue square is $(a-b)^2$ which is equal to $a^2 -2ab +b^2$ When we add the area of the 4 red triangles and the blue square the sum is $a^2 +b^2$ which is equal to the area of the big square c. Therefore $a^2 +b^2 = c^2$, The Pythagorean Theorem!

Now look at the puzzle on the right of figure 2. The puzzle is made up of 2 squares. The red square of side a has an area of a^2 . The blue square of side b has an area of b^2. Since the same 4 red triangles and the big blue square (at the top of figure 2) have a total area of $a^2 + b^2$ from the puzzle on the left, the puzzle on the right must have the same combined area because it is made up of the same parts. Therefore the equation $a^2 + b^2 = c^2$ once again proves the Pythagorean Theorem.

There are hundreds of proofs of the Pythagorean Theorem. In 1999 at the ASEE convention I presented a Geometrical Proof of the Pythagorean Theorem[1]. Figure 3 is taken from that paper. Figure 3 was constructed using a compass and straight edge ruler.

Figure 3 Geometrical Proof of the Pythagorean Theorem[1]

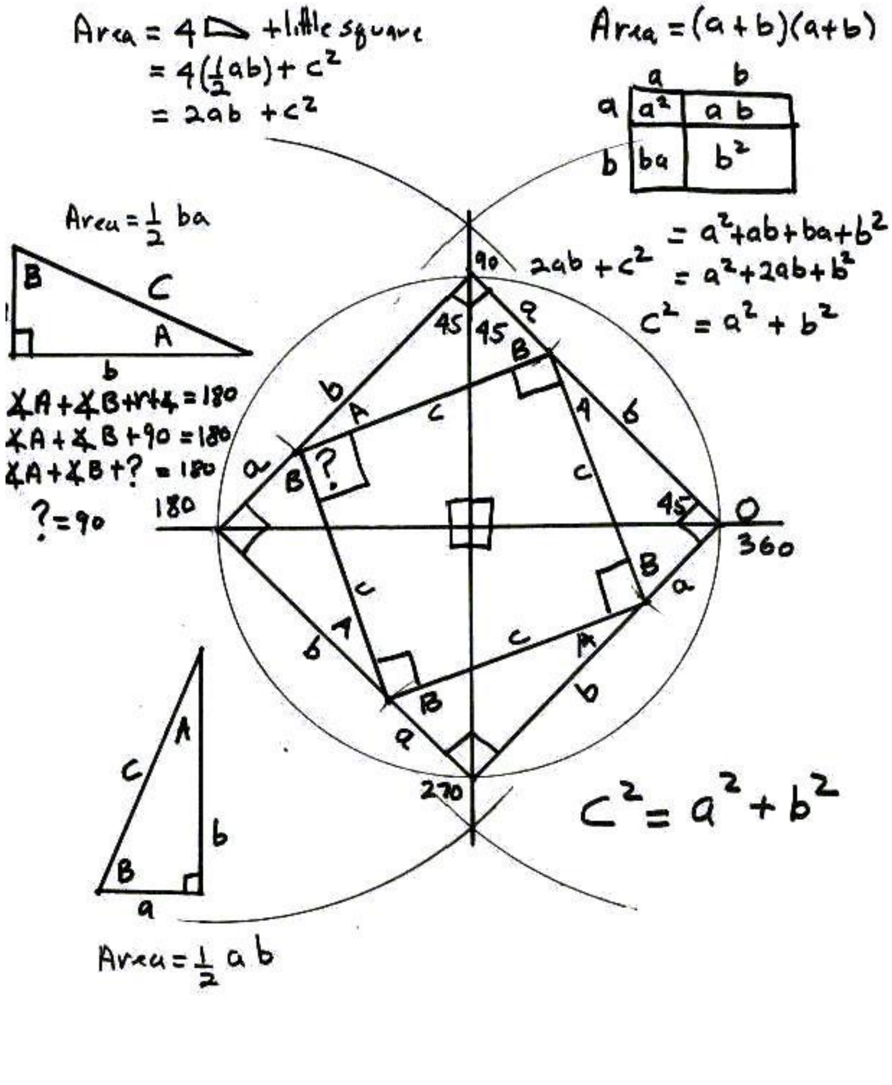


Figure 3 Geometrical Proof of the Pythagorean Theorem[1]

To help my college students learn how to construct Figure 3, I made a video that explains the construction. The web address is http://www.tcicampus.net/userfolder/bpariser.[2] The video is listed under video lectures. It takes time for motivated students to learn how to do this proof and when they get it, they feel empowered.

This software produced by the Utah State University stimulates the creative game urge of some students. Hopefully it will interest them to proceed with the goal of learning mathematics. It is a useful approach to motivate students. By using this software some students may be inclined to ask why the program ends with Good Job! When they ask what does "Good Job!" mean, they are on their way to learn the mathematical steps to prove the Pythagorean Theorem. This process should help build their confidence and motivate them to proceed to discover the beauty of mathematics.

Bibliography

1 B Pariser, "A GEOMETRICAL PROOF OF PYTHAGORAS'S THEORM", ASEE 1999, SESSION 3365
2 B Pariser, http://www.tcicampus.net/userfolder/bpariser