Session 3549

Teaching Structural Analysis Using *Mathcad* Software

Nirmal K. Das
Georgia Southern University

Introduction

Students in the ABET accredited 4-year Civil Engineering Technology program at Georgia Southern University are required to take three courses in the structures area - Structural Analysis, Steel Design, and Reinforced Concrete Design. In the Structural Analysis course, for analysis of simple statically indeterminate structures (beams and frames), students learn two classical methods - slope-deflection method and moment distribution method - before they are given an introduction to the matrix method which is used in computer-aided analysis. Since slope-deflection method forms the basis of the computer-aided analysis, the importance of a thorough understanding of this method can not be overemphasized. Through the traditional way of lecturing and having the students work on related assignments, a good number of students do not seem to quite comprehend the underlying principle, even though by following the procedure rather mechanically, they obtain the correct answers to the problems.

An alternative approach to teaching this very important classical method of structural analysis is discussed in this paper. The suggested approach complements the traditional lecturing by another means such that students can actually see how a structure undergoes a series of deformations at various key locations and consequently, how the internal reactions change at those locations, in a step-by-step manner. Through the use of *Mathcad* software, students will perform calculations in a sequence and then synthesize the results of these intermediate steps to obtain the final results, while they will also have a graphical representation of their solution. A *Mathcad* program is developed for this purpose, including graphics computationally linked to the calculations.

Slope-Deflection Method

The slope-deflection method used in analysis of statically indeterminate beams and framed structures, is so called because it relates the unknown slopes and deflections to the moments at the ends of the members of a structure. It is a classical method and is referred to as a displacement method because the unknown joint displacements (slopes and deflections) are determined first by solving the structure's equilibrium equations; then the other response characteristics are evaluated. These unknown joint displacements are referred to as the *degrees of freedom* (also called the *degree of kinematic indeterminacy*). The slope-deflection method takes into account the flexural deformations of structural members (i.e., rotations and settlements), but neglects shear and axial deformations.

By Slope-deflection method, it is possible to deal with beams under any degree of restraints at the ends, and with any settlement of the supports. The moments at the ends of a beam span depend on the applied loading, the angle through which the ends rotate, and the relative movement of the supports. A pair of slope-deflection equations for each span of a continuous beam can be obtained by first considering *separately* the moments developed at the ends of span AB (Figure 1) due to each of the displacements θ_A, θ_B, and Δ, and then the applied loads, and then by using the principle of superposition.

$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - 3\psi_{AB}\right) + FEM_{AB}$$

$$M_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_A - 3\psi_{AB}\right) + FEM_{BA}$$

where

M_{AB} = moment (internal) at end A of span AB
M_{BA} = moment (internal) at end B of span AB
θ_A, = slope or angular displacement (radians) at end A
θ_B, = slope or angular displacement (radians) at end B
ψ_{AB} = chord rotation (radians) in span AB due to linear displacement Δ ($\psi_{AB} = \Delta$/L)
E = modulus of elasticity of material of span AB
I = moment of inertia of span AB
L = length of span AB
FEM_{AB} = fixed-end moment at end A due to applied loading on span AB
FEM_{BA} = fixed-end moment at end B due to applied loading on span AB.

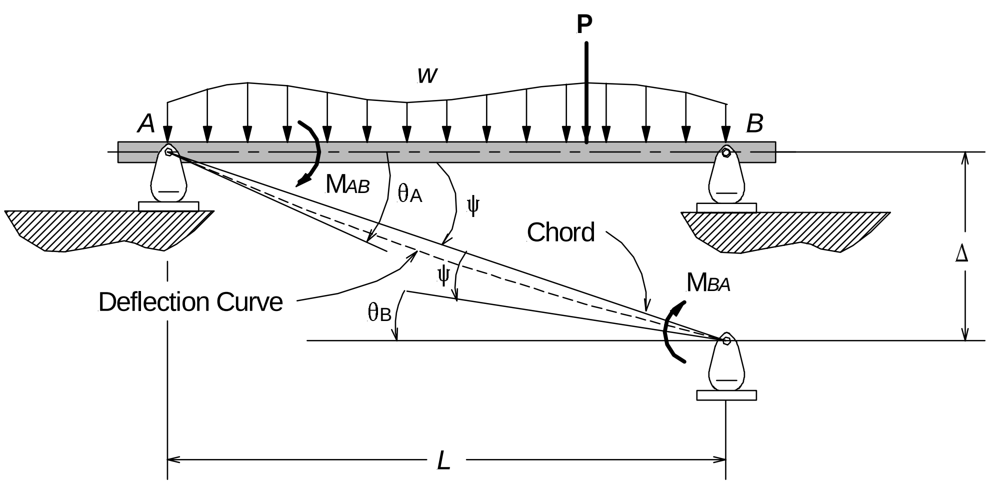


Figure 1

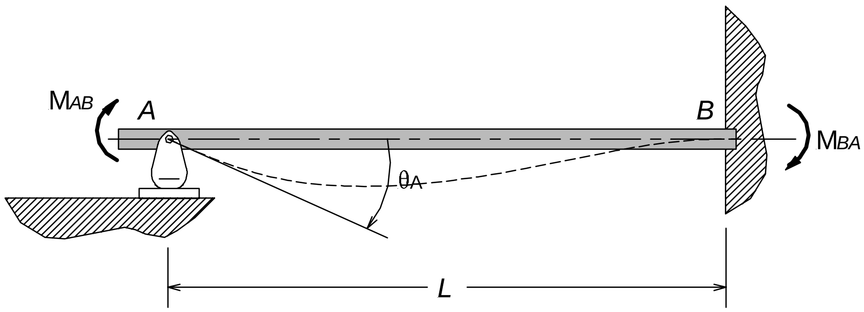


Figure 2

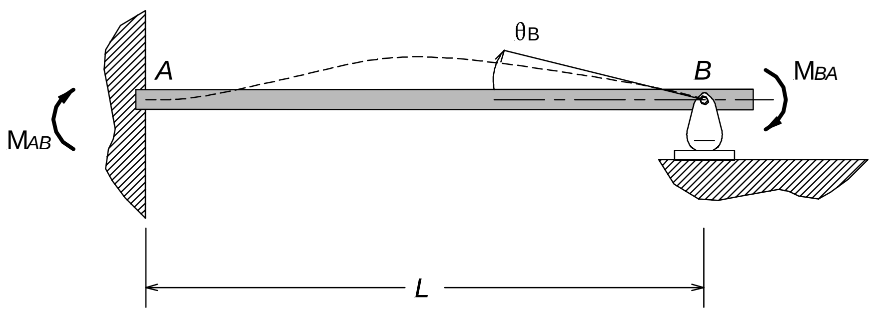


Figure 3

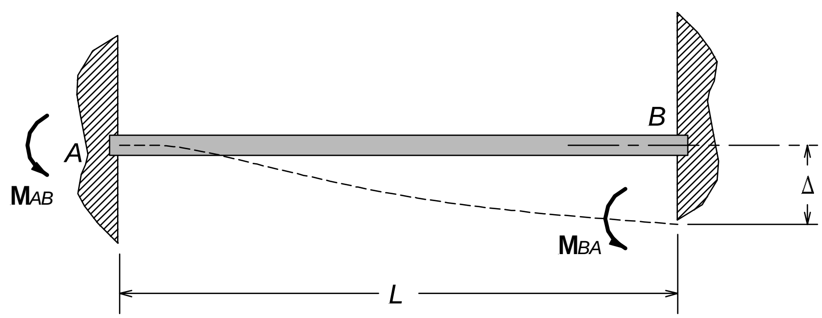


Figure 4

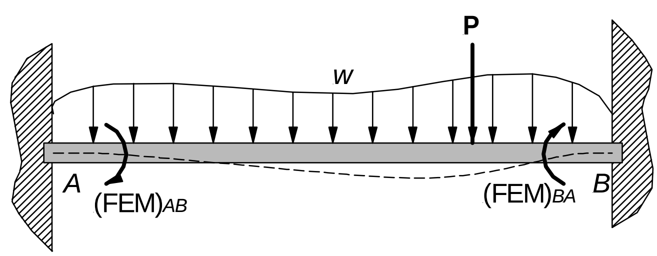


Figure 5

The four terms of the slope-deflection equations are shown in Figures 2, 3, 4 and 5, which represent the moments corresponding to θ_A, $\theta_{B,}$ Δ and the applied load, respectively. The derivations of the first three terms of the slope deflection equations, as well as the fixed-end moments for various loads can be found in any standard text book[1] on Structural Analysis. (Note: Clockwise moments and rotations are considered positive).

The analysis procedure consists of the following steps:

1. Determine the number of unknown joint displacements (degrees of freedom).
2. Write slope-deflection equations for each span.
3. Write as many joint equilibrium equations as the degrees of freedom, in terms of the unknown internal moments specified by the slope-deflection equations. For beams and frames, write a moment equilibrium equation at each support, and for frames also write moment equilibrium equations for joints. For frames having sidesway, write a force equilibrium equation in addition, for each story, expressing column shears in terms of column-end moments
4. Solve the equilibrium equations to obtain the unknown joint displacements.
5. Substitute the results of step 4 into the slope-deflection equations to determine the internal moments at the ends of each member.

Why use *Mathcad*[2]?

Mathcad, which is an industry standard calculation software, is used because it is as versatile and powerful as programming languages, yet it is as easy to learn as a spreadsheet. Additionally, it is linked to the Internet and other applications one uses everyday.

In *Mathcad*, an equation looks the same way as one would see it in a textbook, and there is no difficult syntax to learn. Aside from looking the usual way, the equations can be used to solve just about any mathematics problem one can think of. Text can be placed anywhere around the equations to document one's work. *Mathcad*'s two- and three-dimensional plots can be used to represent equations graphically. In addition, graphics taken from another Windows application can also be used for illustration purpose. *Mathcad* incorporates Microsoft's OLE 2 object linking and embedding standard to work with other applications. Through a combination of equations, text, and graphics in a single worksheet, keeping track of the most complex calculations becomes easy . An actual record of one's work is obtained by printing the worksheet exactly as it appears on the screen.

Program Features

The program developed will require input data pertaining to the geometry of the problem, material property and the loading, More specifically, the following information is required as input data: number of joints, number of members, type of end supports, lengths of members, moments of inertia of members, modulus of elasticity, support settlements (for beams), magnitudes of distributed loads on members and magnitudes and locations of concentrated loads. The computer solves a set of linear equations in a matrix format, i.e., given the relationship **[A]{x}={b}** and given the matrix **[A]** as well as the vector **{b}**, the computer solves for the unknown vector **{x}**. Hence the equilibrium equations need to be rearranged to transform them into a matrix format.

Based on the input data, calculations are carried out in th following steps:

1. Determine number of unknown displacements (slopes for beams and frames without sidesway/ slopes and deflections for frames with sidesway).
2. Calculate the fixed-end moments.
3. Calculate the member stiffness elements.
4. Calculate the structure stiffness elements through assembly of member stiffness elements to obtain the matrix **[A]**.
5. Evaluate the vector **{b}** comprised of fixed end moments and member stiffness elements.
6. Solve for the unknown displacements (vector **{x}**) using the built-in function *Isolve*.
7. Calculate the member-end moments using the results of step 6 into slope-deflection equations.

Results

The solutions obtained through use of this program for some example problems are shown in the following pages. For any of these problems, one or more input data change would translate to change in the member-end slopes, and hence the member-end moments. Any number of combinations of input data is possible and students can see the effects of these changes instantaneously. Furthermore, with further additions to the program, it would be feasible to use computationally linked plots of shear and moment diagrams to provide additional graphical representation of results.

Summary

The suggested approach to complement the traditional lecturing would likely provide a better insight in the subject matter, in addition to making a convenient checking procedure readily available.

Mathcad PROGRAM FOR SLOPE-DEFLECTION METHOD

by Nirmal K. Das, Ph.D., P.E.

Sign Convention: Clockwise member-end moments and rotations are positive
NOTE: All applied loads are considered to be acting vertically downward.

Input Variables:

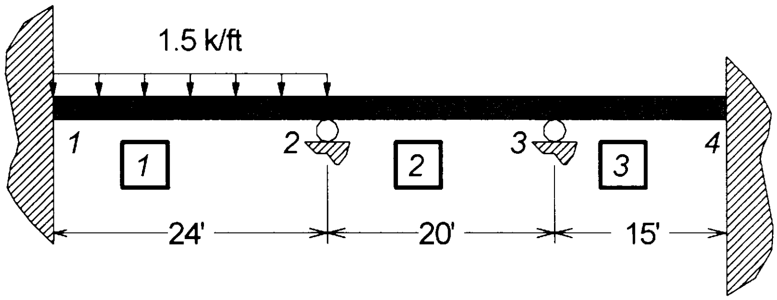
Case	represents support types at beam ends (1 = fixed support at one end, 2 = fixed support at both ends, 3 = no fixed support at either end)
n	number of nodes (supports for beams; supports and joints for frames)
m	number of beam spans or frame members
L_i	span length of i-th member
E	modulus of elasticity of the beam material
I_i	moment of inertia of i-th member
Δ	settlement of the i-th support
w_i	uniformly distributed load on the i-th member
$P_{i1,i2}$	concentrated loads (maximum of two in each span) on i-th span
$a_{i1,i2}$	distances to concentrated loads from the left end of i-th span

Other variables:

u	number of unknown displacements (slopes for beams; slopes/deflection for frames)
c_i	member stiffness coefficient (4EI/L) for i-th member
d_i	member stiffness coefficient ($6EI\Delta/L^2$) for i-th member
$S_{i,j}$	structure stiffness coefficient
$FEM_{i,j}$	fixed-end moment for the i-th member at j-th end
$Theta_j$	slope at the i-th node

OUTPUT DATA: $\mathbf{Mom_{i,j}}$ represents the end-moment for i-th span at j-th end (j = 1 or 2).

:**Example 1:** Determine the internal moments at the supports of the beam shown below. The supports at 1 and 4 are fixed, and the supports at 2 and 3 are rollers. The support at 3 settles by 1 in. Assume E = 29,000,000 psi and I = 1200 in^4.



Input Data:

Number of support: $\mathbf{n} = 4$

Number of members: $\mathbf{m} = 3$

Type of beam-end supports: $\mathbf{Case} = 2$

Modulus of elasticity: $\mathbf{E} = 4.176 \times 10^{9}\ \frac{\text{lb}}{\text{ft}^2}$

Member spans: $\mathbf{L} = \begin{pmatrix} 24 \\ 20 \\ 15 \\ 0 \end{pmatrix} \text{ft}$

Moments of inertia: $\mathbf{I} = \begin{pmatrix} 0.058 \\ 0.058 \\ 0.058 \\ 0 \end{pmatrix} \text{ft}^4$

Support settlements: $\Delta = \begin{pmatrix} 0 \\ 0.083 \\ -0.083 \\ 0 \\ 0 \end{pmatrix} \text{ft}$

Distributed loads: $\mathbf{w} = \begin{pmatrix} 1.5 \times 10^3 \\ 0 \\ 0 \\ 0 \end{pmatrix} \frac{\text{lb}}{\text{ft}}$

Concentrated loads: $\mathbf{P} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \text{lb}$

Location of concentrated loads:

$\mathbf{a} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \text{ft}$

Solution

Fixed end moments:

$$\mathbf{FEM} = \begin{pmatrix} -7.2 \times 10^{4} & 7.2 \times 10^{4} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \text{lb ft}$$

Member stiffness elements:

$$\mathbf{c} = \begin{pmatrix} 4.028 \times 10^{7} \\ 4.833 \times 10^{7} \\ 6.444 \times 10^{7} \end{pmatrix} \text{lb ft} \qquad \mathbf{d} = \begin{pmatrix} 0 \\ 3.021 \times 10^{5} \\ -5.37 \times 10^{5} \end{pmatrix} \text{lb ft}$$

Structure stiffness elements:

$$\mathbf{S} = \begin{pmatrix} 8.861 \times 10^{7} & 2.417 \times 10^{7} \\ 2.417 \times 10^{7} & 1.128 \times 10^{8} \end{pmatrix} \text{lb ft}$$

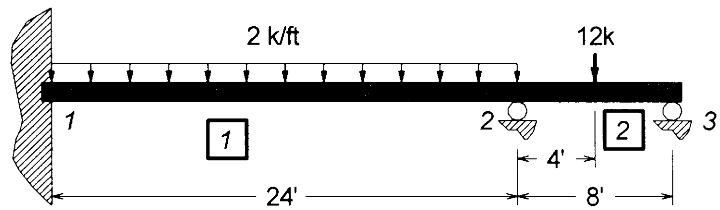
Vector on the other side: $\mathbf{b} = \begin{pmatrix} 2.301 \times 10^{5} \\ -2.35 \times 10^{5} \end{pmatrix} \text{lb ft}$

Slopes: $\mathbf{Theta} = \begin{pmatrix} 0 \\ 3.361 \times 10^{-3} \\ -2.804 \times 10^{-3} \\ 0 \end{pmatrix} \mathbf{rad}$

Member-end moments:

$$\mathbf{Mom} = \begin{pmatrix} -4.31 \times 10^{3} & 2.074 \times 10^{5} \\ -2.074 \times 10^{5} & -3.564 \times 10^{5} \\ 3.564 \times 10^{5} & 4.467 \times 10^{5} \end{pmatrix} \text{lb ft}$$

:Example 2: Determine the internal moments at the supports of the beam shown below. The support at 1 is fixed, and the supports at 2 and 3 are rollers. Assume E = 29,000,000 psi and I = 1200 in^4.



Input Data:

Number of support: $\mathbf{n} = 3$

Number of members: $\mathbf{m} = 2$

Type of beam-end supports: $\mathbf{Case} = 1$

Modulus of elasticity: $\mathbf{E} = 4.176 \times 10^9 \frac{\text{lb}}{\text{ft}^2}$

Member spans: $\mathbf{L} = \begin{pmatrix} 24 \\ 8 \\ 0 \\ 0 \end{pmatrix} \text{ft}$

Moments of inertia: $\mathbf{I} = \begin{pmatrix} 0.058 \\ 0.058 \\ 0 \\ 0 \end{pmatrix} \text{ft}^4$

Support settlements: $\Delta = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ft}$

Distributed loads: $\mathbf{w} = \begin{pmatrix} 2 \times 10^3 \\ 0 \\ 0 \\ 0 \end{pmatrix} \frac{\text{lb}}{\text{ft}}$

Concentrated loads: $\mathbf{P} = \begin{pmatrix} 0 & 0 \\ 1.2 \times 10^4 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \text{lb}$

Location of concentrated loads:

$\mathbf{a} = \begin{pmatrix} 0 & 0 \\ 4 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \text{ft}$

Solution

Fixed end moments:

$$\mathbf{FEM} = \begin{pmatrix} -9.6\times 10^{4} & 9.6\times 10^{4} \\ -1.2\times 10^{4} & 1.2\times 10^{4} \end{pmatrix} \mathrm{lb\,ft}$$

Member stiffness elements:

$$\mathbf{c} = \begin{pmatrix} 4.028\times 10^{7} \\ 1.208\times 10^{8} \end{pmatrix} \mathrm{lb\,ft} \qquad \mathbf{d} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \mathrm{lb\,ft}$$

Structure stiffness elements:

$$\mathbf{S} = \begin{pmatrix} 1.611\times 10^{8} & 6.042\times 10^{7} \\ 6.042\times 10^{7} & 1.208\times 10^{8} \end{pmatrix} \mathrm{lb\,ft}$$

Vector on the other side: $\mathbf{b} = \begin{pmatrix} -8.4\times 10^{4} \\ -1.2\times 10^{4} \end{pmatrix} \mathrm{lb\,ft}$

Slopes: $\mathbf{Theta} = \begin{pmatrix} 0 \\ -5.959\times 10^{-4} \\ 1.986\times 10^{-4} \end{pmatrix} \mathbf{rad}$

Member-end moments:

$$\mathbf{Mom} = \begin{pmatrix} -1.08\times 10^{5} & 7.2\times 10^{4} \\ -7.2\times 10^{4} & 0 \end{pmatrix} \mathrm{lb\,ft}$$

Bibliography

1. Hibbeler, R.C., *Structural Analysis,* 4th ed., Prentice Hall, 1999
2. Mathsoft, Inc.,*Mathcad 2000 User's Guide,* Mathsoft, Inc., Cambridge, Massachusetts, 1999

NIRMAL K. DAS
Nirmal K. Das is an associate professor of Civil Engineering Technology at Georgia Southern University. He received his Bachelor of Civil Engineering degree from Jadavpur University, India, and M.S. and Ph.D. degrees in Civil Engineering (structures) from Texas Tech University. His areas of interest include structural analysis, structural design and wind engineering. Dr. Das is a registered professional engineer in Ohio and Georgia