Teaching Students about the Value of Diversification - A Retirement Portfolio's Efficient Frontier

Dr. Ted Eschenbach P.E., University of Alaska Anchorage

Dr. Ted Eschenbach, P.E. is the principal of TGE Consulting, an emeritus professor of engineering management at the University of Alaska Anchorage, and the founding editor emeritus of the Engineering Management Journal. He is the author or coauthor of over 250 publications and presentations, including 19 books. With his coauthors he has won best paper awards at ASEE, ASEM, ASCE, & IIE conferences, and the 2009 Grant award for the best article in The Engineering Economist. He earned his B.S. from Purdue in 1971, his doctorate in industrial engineering from Stanford University in 1975, and his masters in civil engineering from UAA in 1999.

Dr. Neal Lewis, University of Bridgeport

Neal Lewis received his Ph.D. in engineering management in 2004 and B.S. in chemical engineering in 1974 from the University of Missouri – Rolla (now the Missouri University of Science and Technology), and his MBA in 2000 from the University of New Haven. He is an associate professor in the School of Engineering at the University of Bridgeport. He has over 25 years of industrial experience, having worked at Procter & Gamble and Bayer. Prior to UB, he has taught at UMR, UNH, and Marshall University. Neal is a member of ASEE, ASEM, and IIE.

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Abstract

The contents of most engineering economy texts imply that relatively few engineering economy courses include coverage of investing. However, most current and future students must save and manage their investments for retirement, buying a house, sending children to college, etc. We believe that engineering economy—the only course about money that most engineers take—should cover this. Fortunately, a short discussion of a powerful real-world model for investing also helps support understanding of (1) the relationship between risk and return, and (2) the value of diversification. Increasing a course's relevancy to the student's life has been shown to increase both motivation and understanding.

Keywords: risk, return, diversification, investing

Introduction

Engineering economy and finance courses and texts overlap but they focus on different topics. Risk coverage in finance focuses on the value of diversification in reducing risk, the Capital Asset Pricing Model, and the relationship between risk and return. In contrast, risk coverage in engineering economy typically focuses on calculating the standard deviation of the project's PW or IRR.

Most investments including retirement plans are made up of 'portfolios', a mixture of different kinds of investments put together in order to balance risk and return. Individuals are increasingly being given the responsibility of managing their own retirement portfolios. They need to understand the fundamentals of risk and return so that they can effectively manage their portfolios instead of blindly following the advice of others (who may not have the individual's best interest at heart).

Many finance courses use a simple two-stock portfolio model to explain the value of diversification in achieving better combinations of risk and return. The math would be easy for engineering students, and the lessons learned can be qualitatively extended to the selection of engineering projects. Nevertheless, such a presentation fails a common engineering test, "How am I going to use this?"

We suggest that the important role of diversification in reducing risk merits coverage in engineering economy courses. Students should consider this in planning their investments for retirement, home purchases, and educating their children. Firms should consider this in selecting projects for investments. Governments should consider this when promoting economic development.

The material presented here was developed to achieve better results in both our engineering and business classrooms. This paper is a text version of what we presented to students for the first time in the spring 2015 semester (shortly after midterm). Much of this introduction, most of the literature review and references, the discussion of classroom results, and some of the conclusions are not part of our class presentation.

After the literature review, we present some social security information and discuss the availability of defined benefit (which few will get) and defined contribution plans (which most will have). We then discuss long-term return and risk data from 10-year government bonds, the stock market, and treasury bills. The presentation includes inflation data so that we can also discuss realistic returns on savings and investments to buy a home, pay for a future college education, or retire. This also introduces students to the very important reality of what expected returns are realistic and what risks are likely to accompany those returns. It also introduces a second very important reality in economic decision-making—neither maximizing expected returns nor minimizing risks may be the best approach.

The next step is presenting the model of risk and return from a portfolio based on two securities—bonds and stocks. This clearly shows the value of diversification, and the theory as represented in the underlying equations. This simple diagram is easy to present and understand, and an easy way to say why your investments should not be in one type of security—or even worse in one stock, such as your employer's.

When time permits we find that this is a good place to include SOLVER in our course—so we calculate the minimum variance portfolio and discuss dominated portfolios. Next, a brief discussion and an optional figure show how this curve depends on the correlation coefficient. We also explain why the risk reducing feature of diversification means that prices for stocks and bonds assume that buyers are fully diversified. There is no price adjustment for being undiversified. The market price is set to match the return and risk of diversified investors; undiversified investors are accepting more risk for the same return. However, our data and low-cost stock or bond index funds already represent widely diversified portfolios of stocks and bonds.

The next section adds the T-bill rate to our model after explaining why it is simply plotted at its historical expected return and standard deviation. The new efficient frontier of weighted portfolios includes choices that mix T-bills with an approximately 50/50 split between stocks and bonds. This model is elaborated by a table of different portfolios with their expected return and standard deviation. This linear envelope implies that heavier weighted bond portfolios are dominated. More importantly, it demonstrates the inadequacies and errors contained in common rules of thumb that link recommended bond/stock weights to an individual's age. These apply both to young students and to faculty nearing retirement. We also use this step to illustrate how GOAL SEEK can be used to improve on the minimum variance portfolio.

This material is meant to help prepare instructors for the inevitable student questions and to answer questions that the instructors may have regarding investment portfolios. We believe this material can be covered in a standard class of 50 to 80 minutes.

The paper concludes with a summary of our classroom results and a call to include this material in engineering economy courses. It is our hope that some version of the material presented here, such as Table 2 and Figure 2 or 5, will be added to texts as we believe that the value of diversification in project selection and in personal investing has long-term importance to engineering students and the firms that hire them.

Literature Review

Harry Markowitz won the 1990 Nobel Prize in Economic Sciences for his work on portfolio selection.[1] That work is presented in introductory corporate finance texts in coverage of diversification for stocks and the capital asset pricing model for risk and return relationships. It is also included in more advanced engineering economy texts,[2 – 4] but not in introductory engineering economy texts.[5 – 14]

In considering the application of portfolio theory to the specifics of an individual investing in stocks and bonds, we have not found introductory textbook presentations. Even texts focused on investments, a more advanced course for finance majors, may or may not include material on the efficient frontier for debt and equity funds. One leading investments text does;[15] however, it is more common to include results from different combinations of investment allocations and to discuss that much of the variation between mutual funds is due to differing allocations between debt and equity funds.

This paper adds the use of treasury bill (T-bill) data to generate a more realistic "risk free" rate of return for the capital asset pricing model and for retirement planning. In contrast, finance texts generally ignore the standard deviation of returns on T-bills and assume instead that the standard deviation is zero. Since our focus is pedagogical, we have chosen not to try and sort through the over 9000 Google Scholar citations for "risk free rate of return."

Behavioral finance[16, 17] is one pedagogical area that is linked to some of our discussion points about investment performance by individuals. It also provides an incredibly apropos example of the 1/N heuristic in investing. Markowitz himself picked a 50/50 stock/bond portfolio based on an even split between two options offered to him by TIAA-CREF in mid-1980s. He did not analyze the efficient frontier for stock and bond portfolios. "I visualized my grief if the stock market went way up and I wasn't in it—or if it went down and I was completely in it. My intention was to minimize my future regret."[18]

While, introductory engineering economy texts[5 - 14] do not discuss security portfolios, virtually every one includes examples and/or problems on saving for retirement. The sole exception[8] chose the path of drawing examples and problems from real world corporations. The link between minimum attractive rates of return or interest rates and risk in some form is universal, but only the advanced texts[2 – 4] discuss the capital asset pricing model. It is common to cover the weighted average cost of capital, but to omit coverage of where the values come from. Between the submission of this paper to the proceedings and its presentation at the conference, another paper[20] has analyzed some of the technical issues and details that are presented here as relying on typical values.

An individual with a 401(k) or other form of defined contribution plan needs to realize that there are several forms of investments and that each carries a unique set of risks and returns. In general, investment assets that have higher risks (larger standard deviations) must have a corresponding higher average return in order to retain investors. Stocks typically have the highest risks and returns. Long term bonds (with maturities greater than one year) have a lower risk and return than stocks. Short term Treasury bills (T-bills) have maturities of one year or less and have the lowest risks and returns. Well balanced portfolios often have a mixture of all three.

Many people who have defined contribution retirement plans also have wages that are subject to social security withholding. Social security is a significant part of many retirement portfolios, and is a good place to begin understanding the different aspects of a broad-based portfolio.

Data on Investment Needs and Investment Options

Social Security. For simplicity and clarity, we will focus on an individual who is single. When looking at retirement, one possible option is to live on social security. Thus a logical starting point is "How large can an individual's monthly benefit be?" For 2015 the maximum benefit at normal retirement age (66 in 2015) is $2685.50/month. This primary insurance amount (PIA) totals $32,226 annually. This is significantly less than most starting engineers earn.

This maximum social security benefit is available only to those who have earned the maximum taxable social security earnings (or more) for the last *35 years*. Details of the calculations are not important here but the 2015 maximum average indexed monthly earnings (AIME) is $9066. However, social security taxes are imposed in 2015 up to an upper limit of $118,500 which is more than 12 times the AIME. The AIME can be thought of as averaging those upper limits over 35 years. Note that AIME indexing adjusts for wage inflation, but not for a salary trajectory from entry-level to higher-paying positions.

> Is it likely that someone used to earning $118,500 or more annually will enjoy living on $32,226 in retirement? No.

So who can live on social security in retirement? Equation 1 shows how the PIA is calculated based on the AIME for 2015. The bend points in the PIA calculation are adjusted for inflation each year. Only in the first 90% bracket will the PIA will be close to the AIME, so the maximum level of near replacement annual income is only 12×0.9×$826 = $8921/year. This corresponds to a bit more than ½ time at the minimum wage—much less than a good retirement income for an engineer.

$$\begin{aligned}
&\text{If AIME} < \$826, && \text{PIA} = (0.90)(\text{AIME}) \\
&\text{If } \$826 < \text{AIME} < \$4980, && \text{PIA} = (0.90)(826) + (0.32)(\text{AIME} - 826) \qquad (1) \\
&\text{If } \$9066 > \text{AIME} > \$4980, && \text{PIA} = (0.90)(826) + (0.32)(4980 - 826) + (0.15)(\text{AIME} - 4980)
\end{aligned}$$

Thus social security will likely be part of a retirement strategy, but there must be other sources of retirement income. The retirement strategy for social security must address the question of when to start collecting social security. This can be used as a case study in economic analysis.[21]

Increasing Dominance of Defined Contribution Pensions. Since 1980, there has been a strong trend away from employers offering defined benefit (DB) pensions, moving instead toward defined contribution (DC) programs, including 401(k) plans. Defined benefit programs provide guaranteed lifetime annuities to employees upon retirement, usually based on salary and the number of years of service. In DC plans, both the employer and the employee may contribute a set amount (usually a fixed percentage or match) into a retirement account in the employee's name. At retirement, the employee generally receives a lump sum payment.

From 1980 through 2008, the proportion of workers in private business participating in DB pension plans fell from 38% to 20%.[22] The shift in defined benefit and defined contribution plans among the Fortune 500 companies is shown in Figure 1.[23] While the DC plans now far outnumber DB plans, the trend may be leveling off.

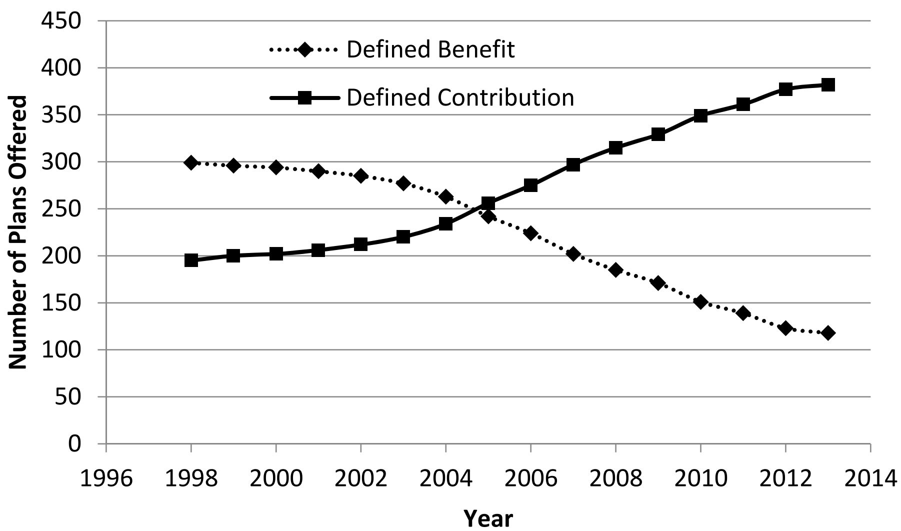

Figure 1. Retirement plans among the Fortune 500 companies[22]

This shift is why we believe engineering economy students should have at least a short introduction to saving for retirement. Defined benefit plans can work well for employees with little attention from the employee—if the employee stays with the employer for an extended period of time. Because most DB plans require a minimum length of employment for vesting (generally 5 years), many younger workers have preferred the portable 401(k) plans—even when they had a choice.[24]

One critique of 401(k) plans is that the decisions (and risks) are given to under-educated employees, who may not know what they are investing in. According to Zvi Bodie[25] "401(k) plans really place the burden on the individual participant to have an adequate retirement. And the vast majority of ordinary people don't know how to do that. It's a very complex task." Where people do not understand investing, 401(k) plans can be closer to gambling than investing.

As an example of the complexity, the DB plans manage the risk of income maintenance over long life spans, by pooling groups of employees with statistically distributed life spans. With individual 401(k) plans, the risk of outliving their money must be managed by every individual.

Thus when we look at the question of investing's pedagogical coverage, we are left with the conclusion from the literature review.

> Virtually all engineering economy texts include examples of saving for retirement that are designed to demonstrate the importance of starting early. However, how to save and invest for retirement receives little coverage.

Data on Investment Returns and Risks. We begin our discussion of risk and return by explaining that investors normally prefer higher returns and lower risks. As a noteworthy exception, we point out that gambling is linked to the pleasure of taking risks. We also note that the opulence of Las Vegas is linked to the house's expected average return and the gambler's probability of ruin. The normal state of the world is that the higher returns are linked with higher risks, and lower risks are linked with lower returns.

Table 1 presents detailed annual data on inflation and on the average returns and risks of stocks, 10-year government bonds, and treasury bills (T-bills). To emphasize the variability of the data, some of the years have been chosen to represent highs and lows. One choice when gathering data is choosing a starting point. As shown in Table 1, we have chosen to start with 1950. This is obviously a "round number" choice, but it could be justified as the approximate starting point for knowledge of Markowitz's results to influence investing behavior of *rational* investors. In our case, we would like to thank John Nofsinger[26] for providing us a copy of his updated data set. When selecting a starting date we suggest picking a year and not "fifty" years of data. It is easier for the instructor and is probably educational to add years for future classes as time progresses.

Table 1 provides representative values for early years of our data set and complete values since 2000. Of more use is Table 2 which summarizes the values that are used for further analysis. This table is a good starting point for discussions about realistic assumptions about achievable rates of returns. This is when we explain that the geometric rate of return is correct and that the arithmetic average overstates expected returns (especially over long intervals with positive and negative annual values). A quick example is gaining 50% one year and losing 50% the next year; you are not back where you started as an arithmetic average of 0% indicates. Instead, as the geometric average indicates, you have 75% of what you started with, and you have lost 25%. Thus from Table 2 over the past sixty years, stocks have returned about 7.3% over inflation, long-term treasury bonds about 2.5% above inflation, and T-bills about 0.9% above inflation.

For the most part the results in Table 2 are simple computations from our data set with two exceptions. The first is that computing the geometric mean with 0 or negative values fails in Excel. The easiest fix is to add 1 to each year's return. For example, the −35.5% return on stocks in 2008 is recoded as having 64.5% of your starting year value as your total value at the end of the year, and year 2009 becomes a year-end total of 123.45%. Then find the geometric mean for the set of each year's total return, and finally subtract 100% to find the geometric mean of the annual returns.

Table 1. Selected annual rates for stocks, bonds, T-bills, and inflation (1950 – 2012).

*Years prior to 2000 shown if highlighted values among column's 3 largest or smallest

	Common Stocks	Treasury Bonds	Treasury Bills	Inflation Rate
1954	52.6%	7.2%	0.9%	-0.5%
1955	31.6%	-1.3%	1.6%	0.4%
1958	43.4%	-6.1%	1.5%	1.8%
1967	24.0%	-9.2%	4.2%	3.0%
1974	-26.5%	4.4%	8.0%	12.2%
1979	18.4%	-1.2%	10.4%	13.3%
1980	32.4%	-4.0%	11.2%	12.4%
1982	21.4%	40.4%	11.6%	3.9%
1985	32.2%	31.0%	7.9%	3.8%
1995	37.4%	31.7%	5.8%	2.5%
1999	21.0%	-8.7%	4.9%	2.7%
2000	-9.1%	20.1%	6.0%	3.4%
2001	-11.9%	4.6%	3.5%	1.5%
2002	-22.1%	17.2%	1.8%	2.5%
2003	28.7%	2.1%	1.0%	1.8%
2004	10.9%	7.7%	2.2%	3.0%
2005	4.9%	3.1%	3.1%	3.4%
2006	15.8%	1.9%	4.7%	2.5%
2007	3.5%	9.8%	4.4%	4.2%
2008	-35.5%	22.7%	1.5%	0.1%
2009	23.5%	-12.2%	0.2%	2.7%
2010	15.1%	9.4%	0.0%	1.5%
2011	2.1%	29.9%	0.0%	3.0%
2012	16.0%	3.6%	0.0%	1.7%

Table 2. Returns and standard deviations for stocks, bonds, T-bills, and inflation.

	Common Stocks	Treasury Bonds	Treasury Bills	Inflation Rate
Arithmetic average	12.44%	6.71%	4.54%	3.71%
Median	15.06%	3.65%	4.39%	3.02%
Geometric mean	11.00%	6.21%	4.53%	3.67%
Standard deviation	17.52%	10.84%	2.84%	2.89%
Coefficient of variation	1.41	1.61	0.63	0.78
Correlation stocks/bonds	0.11			

The second exception is that we have chosen to list the correlation between stocks and bonds as 11% (Data up to 2007[26]) rather than the −0.012 value calculated with our data set. That near zero value appears to be somewhat of an outlier[20] due to the actions of many central banks in stimulating economies by holding down interest rates to atypical values as compared with the last century. Results concerning the robustness of these results[20] with alternative data set choices and of doing the analysis in real terms (adjusted for inflation) will be included with this paper's presentation.

While we do not talk about investing in detail in our classes, we do note the following points:

- There is evidence that the majority of mutual funds do not perform as well as the stock market in the long-run.
- There is substantial evidence that most individual investors do poorly when trying to time the market (buy-in and sell-out); they would have done significantly better simply buying and holding. Many individuals tend to buy when stocks are higher than average and sell when stocks are at or near their lows.[15 – 17, 26]
- There is evidence that fundamental analysis is linked to successful investing.[27 – 29] In finance texts, this is presented as a violation of the semi-strong form of the efficient market hypothesis—all publicly available information is generally reflected in market prices.
- There is evidence that on average technical analysis (relying on past trends or patterns in stock prices) doesn't support "beating the market." In finance texts, this is presented as the weak form of the efficient market hypothesis is generally true—market prices generally reflect all information about stock price movements.[28 – 30]

The implication for individuals having a defined contribution program is that the best strategy is to get a company match (where possible) and then invest in a diverse set of firms and bonds through an exchange traded fund (ETF) or a low cost mutual fund. It is difficult for individuals to compete with institutional investors (who have millions to spend on how to invest billions). The key to long-term success seems to be keeping transaction costs down and investing regularly. Fortunately, ETFs offer the individual investor a way to hold very well diversified portfolios of stocks and/or bonds that are designed to match major market indices. We also note that these ETFs include returns to investors (dividends on stocks and interest on bonds) that go beyond the simple price indices. Note that both price and total return versions of the Dow Jones and the S&P 500 indices are available. Only total return versions should be used. We note that the S&P 500 and the Russell 1000 are generally regarded as better measures than the Dow Jones, as they include more firms and they weight them by their capitalization or float, rather than simply averaging the stock market prices of 30 firms as does the Dow Jones with a divisor calculated to maintain historical continuity.

A Realistic and Valuable Bond and Stock Portfolio

We want to build a portfolio that is a combination of assets in order to manage the risk and return of the entire portfolio. In general, we want to maximize the return while minimizing the risk. Stocks, treasury bonds, and treasury bills will be combined in different proportions, using data from Table 2, to explore the results of different combinations.

Developing the Portfolio. The geometric averages and standard deviations for stocks and bonds from Table 2, along with the correlation value, is all that is needed to construct a realistic and useful portfolio model. We begin by discussing Figure 2 with detailed values in Table 3, and we will shortly present the equations that underlie it.

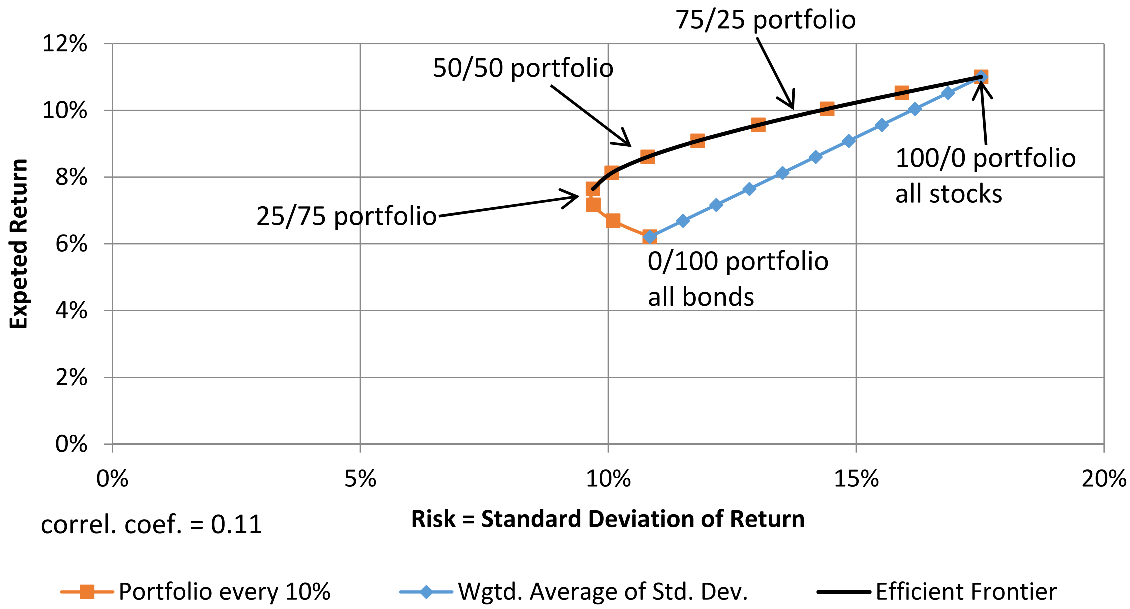

Figure 2. Risk and return for portfolios of stocks and bonds.

Table 3. Risk and return data for portfolios of stocks and bonds.

Stock	Bond	Portfolio Std Dev	Portfolio Return
100%	0%	17.5%	11.0%
75%	25%	13.7%	9.8%
50%	50%	10.8%	8.6%
25%	75%	9.6%	7.4%
0%	100%	10.8%	6.2%

First, how is the value of diversification shown in Figure 2? One way is to focus on how, for any expected return, the portfolio's standard deviations are much less (lower x-axis values) than the blue line of weighted averages of the returns and standard deviations. As detailed in Table 3, changing from all stocks (expected return = 11.0% & std. dev. = 17.5%) to 25% or 50% bonds drops the expected return by about 1.2% for each step but decreases the expected standard deviation by about 3.5% for each step.

The second way that diversification's value is shown is to compare the 50/50 portfolio with the 0/100 (pure bond) portfolio. Both have a standard deviation of 10.8%, but the 50/50 portfolio has an expected return that is 2.4% higher. We explain that the curves are smooth, as shown by the

10% steps, but the 25% steps make for a simpler, clearer link between the discussion and the figure.

The *efficient frontier* of portfolios can be defined using the principles of higher return and lower risk being better. This is the black line portion of the portfolios shown in Figure 2. These are portfolios which represent the best possible return for a given risk, or equivalently the minimum risk for a given return. In Figure 2, these portfolios have a maximum weight on bonds of about 75% and a minimum weight on stocks of about 25%. Selection of the “best” portfolio along the efficient frontier of 25% to 100% stocks depends on individual trade-offs between risk and return. Utility curves can also be used to identify the optimum portfolio, but these are often conceptual in nature and are difficult to actually estimate.

After discussing Figure 2, we present equations 2 and 3.[31] The portfolio’s expected return is a simple weighted average. The standard deviation of the portfolio is less than the weighted average of the standard deviations, except for the special case of perfectly positively correlated standard deviations.

Expected return of a two-security portfolio

$$E(R_P) = w_1 E(R_1) + w_2 E(R_2) \quad (2)$$

Standard deviation of a two-security portfolio with dependent securities

$$\sigma_P = \sqrt{(w_1^2\sigma_1^2 + w_1^2\sigma_1^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2)} \quad (3)$$

In constructing Figure 2, there is an assumption that the weights are non-negative values. We do not believe that “selling short” (or using negative weights) is a strategy that should be used for long-term investments. We believe that such strategies are best presented in advanced finance courses (they are typically only mentioned and not a significant part of introductory finance texts).

Analyzing the Portfolio. If the time budget for a particular course permits, it is useful to analyze the basic relationships. Figure 3 adds straight-lines for perfect positive and negative correlations ($\rho = +1$ or -1) between the two securities and the curve assuming independence ($\rho = 0$).

The first important lesson from Figure 3 is that perfectly correlated securities have no diversification value. Instead, both the expected return and the standard deviation are simple weighted averages of the values for the securities. This is the straight-line for $\rho = 1$.

The second lesson from Figure 3 is that independent securities with no correlation only produce small improvements over securities with a small positive correlation, such as the $\rho = 0.11$ value used here and in Figure 2.

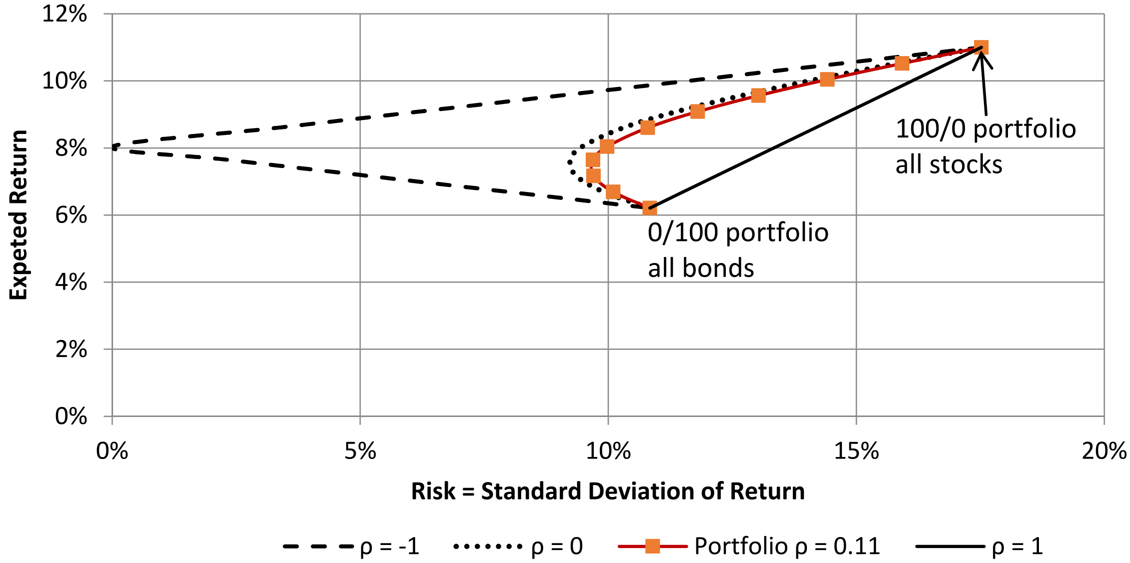

Figure 3. Optional coverage of the correlation coefficient and portfolio risk and return.

The third lesson from Figure 3 is that in theory a perfect negative correlation allows the construction of a portfolio with no risk. However, since large long-term negative correlations of major market measures do not appear to exist, this is only of theoretical interest. Even if such a historical correlation could be found, the likelihood of it continuing seems unlikely.

The large risk reductions possible with negative correlations are why a business might, for example, choose to make products for a market that sells well when oil prices are high and products for another market that sells well when oil prices are low. The negative correlations between the markets are why this may be a low risk strategy.

When time permits our next step is to introduce students to SOLVER. This Excel tool makes it easy to find the exact portfolio weights for the minimum variance portfolio. In Figure 4, we have inserted an extra row (16) with an initial weight of 25% for stocks. Cell C16 with the value of the portfolio's standard deviation is chosen in SOLVER as the cell to minimize. Then cell A16 is chosen in SOLVER as the variable to change (B16 = 1 – A16). The default of non-negative values for the variables is kept.

	A	B	C	D
1		Stock	Bond	
2	E(Return)	11.0%	6.2%	
3	Std Dev	17.5%	10.8%	
4	Coef Correlation		0.11	
5				
6			Portfolio	Portfolio
7	Stock	Bond	Std Dev	E(Return)
8	100%	0%	17.5%	11.0%
9	90%	10%	15.9%	10.5%
10	80%	20%	14.4%	10.0%
11	70%	30%	13.0%	9.6%
12	60%	40%	11.8%	9.1%
13	50%	50%	10.8%	8.6%
14	40%	60%	10.1%	8.1%
15	30%	70%	9.69%	7.65%
16	25.25%	74.75%	9.65%	7.42%
17	20%	80%	9.70%	7.17%
18	10%	90%	10.1%	6.7%
19	0%	100%	10.8%	6.2%

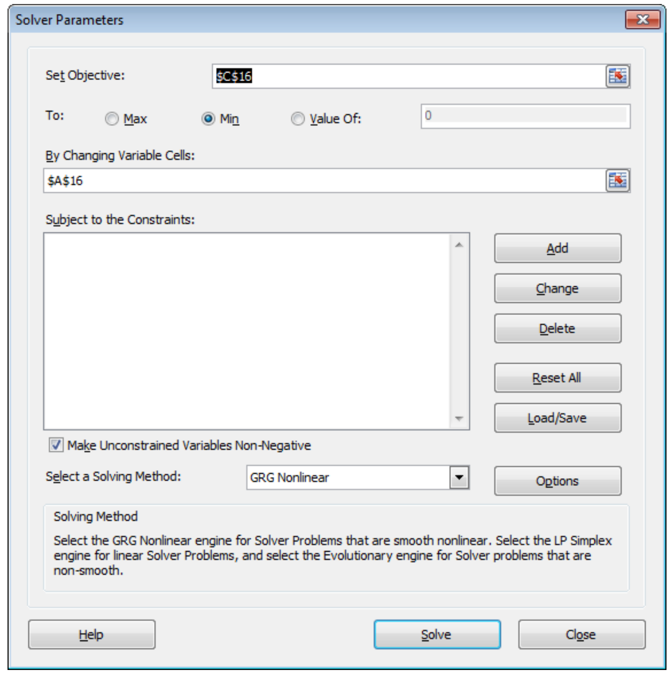

Figure 4. Finding the minimum variance portfolio.

For the data sets that we have analyzed, the weight on bonds for the minimum variance portfolio is typically in the neighborhood of 75%. Thus as shown in Figure 4, portfolios of more than 74.75% bonds are dominated by portfolios with a smaller percentage of bonds. For example, portfolios of 70% and 80% bonds both have a standard deviation of about 9.70%. However, the 80% portfolio has an expected return about 0.5% lower than the 70% portfolio.

Adding the T-bill Rate

Adding the T-bill rate supports a more complete investment model with a *better* efficient frontier. A third class of securities is made up of short-term treasury bills or T-bills. These typically have maturities of 3, 6, or 12 months. Because these are issued by the U.S. government and have short maturities, they are generally considered to be free of default risk. Their expected return is the value typically used as the “risk free” interest rate in textbook discussions of the capital asset pricing model. However, as shown by the 2.84% standard deviation reported in Table 2, these T-bills are not truly risk-free. They are still subject, for example, to inflation risk. Note that considering T-bills as real rates of return above inflation does *not* reduce the risk to zero as suggested by the term risk-free.[20]

Even though they are not truly risk free, the return and risk values for T-bills allow a “near-cash” point to be added to Figure 2, as shown in Figure 5. A second addition is a new efficient frontier line that connects the T-bill point with the point of tangency to the efficient frontier of the stock/bond portfolio. This is the solid straight-line shown in Figure 5. For the data sets we have analyzed, the point of tangency is typically in the neighborhood of a 50/50 portfolio of stocks and bonds. Thus we use a 50/50 bond/stock portfolio, since this is close to optimal for our data and it makes for a much easier explanation of portfolios. While points of tangency and line slopes could be mathematically calculated, we suggest that the accuracy of the data is better matched to simply eyeballing the line as it is added.

This new efficient frontier supports new better choices. Before, the minimum variance frontier was 74.5% bonds and 25.5% bonds with a portfolio standard deviation of 9.65% and an expected return of 7.42%. Using 30% T-bills and 35% each stocks and bonds achieves the same expected return, but lowers the standard deviation by nearly 1.2%.

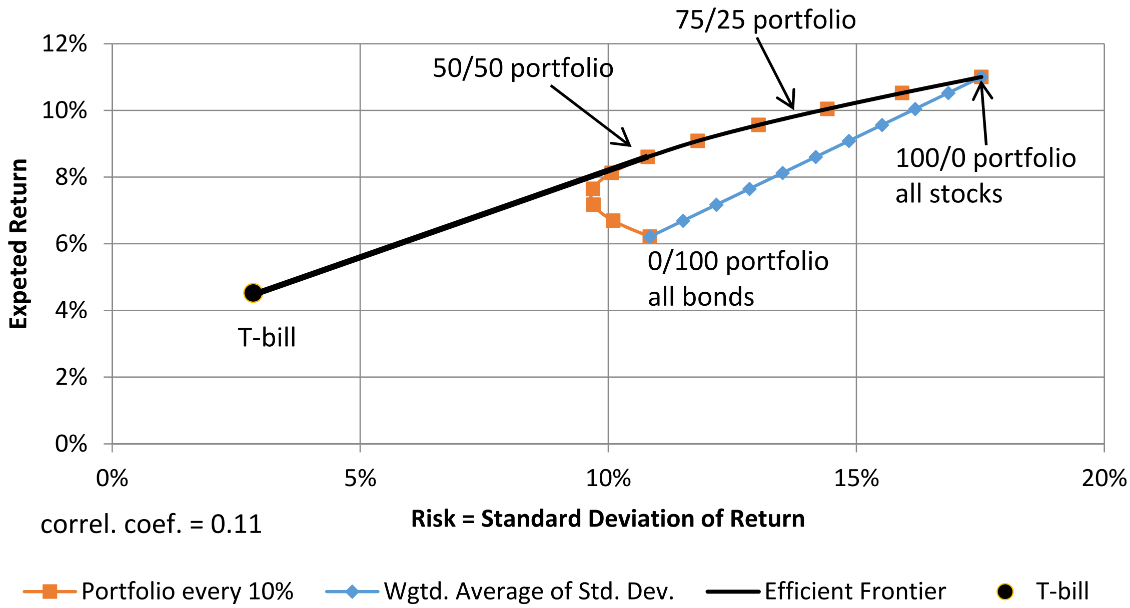

Figure 5. Adding T-bills to the stock/bond portfolio.

The addition of T-bills as a portfolio option means that lower risk portfolios can be constructed. At one extreme, holding a portfolio consisting solely of T-bills achieves the risk/return values of the T-bill point (low risk, but low return). A second important result is that the point of tangency to the bond/stock portfolio curve represents the maximum weight on bonds for the bond/stock split. It is better to include T-bills in the portfolio than to use greater weights on bonds. A new efficient frontier has been created. Moving along the line from the T-bill point to the point of tangency also represents shifting from a portfolio that is 100% T-bills to one that is 0% T-bills and 50% each for stocks and bonds. Each 10% decrease in the T-bill allocation increases stocks and bonds by 5% each.

In theory, the line can be extended to the left of the T-bill rate or to the right of the point of tangency by using negative weights for the stock/bond portfolio or for T-bills respectively. Again we do not believe that including negative weights is appropriate for this level of discussion (nor in reality if our money is involved).

One result of adding the T-bill option to the portfolio is to *debunk* one of the common guidelines for investing for older individuals. That guideline is that the minimum weight on bonds should approximately equal your age. In other words, a 20-year old should hold about 20% of a portfolio in bonds, while a 60-year old should hold about 60% in bonds. Figure 5 shows that individuals over 50 should add T-bills to their portfolio rather than further increasing their weighting on bonds.

This is consistent with newer advice on stocks and bonds after retirement. Because more people are living twenty to forty years past when they retire, it is better to keep more than historic recommendations in stocks—while continuing to shift from stocks to bonds to maintain enough available funds.

We explain to students the reasoning behind the rule of thumb. Older individuals are closer to when they need to spend their retirement savings, thus if the stock market falls they might have to sell at depressed values rather than being able to wait for the market to rise. Bonds, especially if held to maturity, change much less in value than the stock market.

Table 4 summarizes results for the new efficient frontier. Notice that portfolios with T-bills are weighted averages of the T-bill and the 50/50 portfolio values—not proportionate calculations using the original standard deviations for stocks and bonds.

Table 4. New efficient frontier values.

T-bill	Stock	Bond	Portfolio Std Dev	Portfolio Return
	100%	0%	17.5%	11.0%
	90%	10%	15.9%	10.5%
	80%	20%	14.4%	10.0%
	70%	30%	13.0%	9.6%
	60%	40%	11.8%	9.1%
0%	50%	50%	10.8%	8.6%
20%	40%	40%	9.2%	7.8%
40%	30%	30%	7.6%	7.0%
60%	20%	20%	6.0%	6.2%
80%	10%	10%	4.4%	5.3%
100%	0%	0%	2.8%	4.5%

Results from Class Use

This paper is based on attempts to improve student understanding and interest in our finance and engineering economy courses. This material was first presented to engineering economy classes in Spring 2015 at St. Cloud State University (Minnesota), the University of Alaska Anchorage, and the University of Bridgeport (Connecticut). All results are from graduate students in engineering (EM) or technology management (TM) programs. Students in the first two programs are typical of EM graduate programs (experienced, working professionals with only a few going to school full-time). Most of the students in the third program are full-time international students who recently completed their B.S.

We have not subdivided the results, as they were quite consistent over the 3 universities. The only exception was that 4 of the 6 respondents saying, "This was new to me (Figure 7)," were from the third university. Our sample was more familiar with investing than could be expected of undergraduate engineering students. Most employed students have begun saving for retirement. The EM and TM graduate students may also have had or will have a required or elective finance course. Our data indicates that the more they know about investing, the less likely they are to believe that this material should be included in engineering economy courses.

Common questions were asked to ascertain the knowledge level of students regarding investments and portfolio management before the details were presented. Students were also asked whether they recommended that this material be incorporated into an engineering economics course. Results of the surveys are included in Figures 6, 7, and 8.

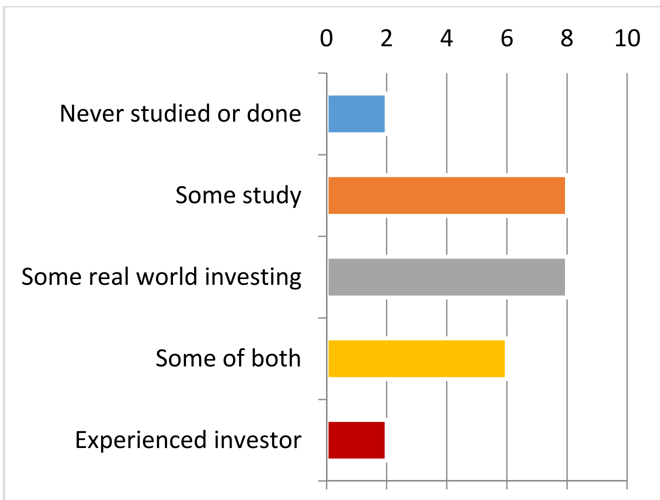

Figure 6. How familiar are you with investing? (n = 26)

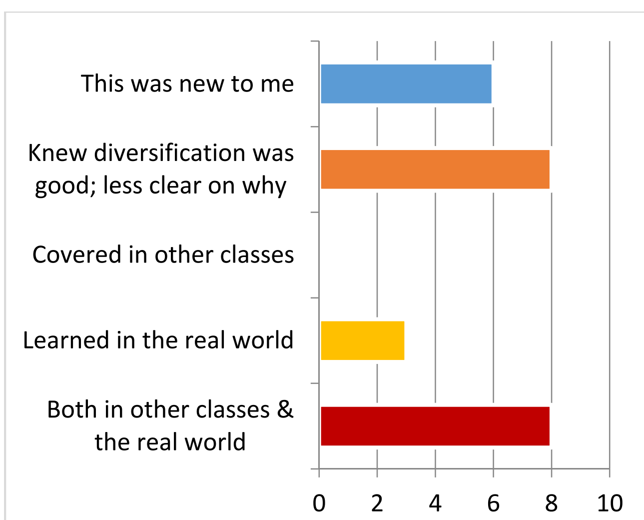

Figure 7. How familiar are you with demonstrating the value of diversifying your investments? (n = 25)

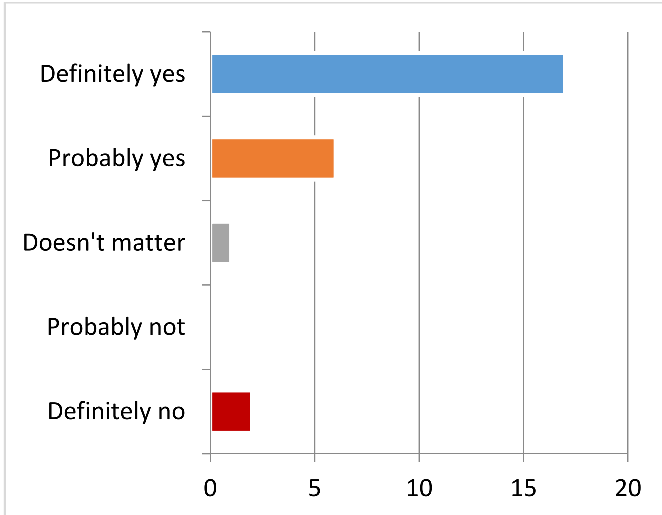

Figure 8. Should this material be part of an engineering economy course? (n = 26)

Conclusions

This paper includes more material than the time budget for many engineering economy courses will allow. It is included to help prepare instructors for student questions, and to answer questions that instructors themselves may have. We do believe that all of this material can be covered in a single 50 to 80 minute lecture.

We believe that Table 2 and Figure 2 or 4 can be covered in a brief presentation that should be part of introductory engineering economy courses. An ever shrinking number of students can expect to collect a defined benefit pension. Instead most will have to use 401(k)s and other investment vehicles to save for their retirement. Since the engineering economy course is the *only* money related course that most engineers take, we argue that some coverage of investing must be included to help students succeed in their careers and lives.

We also note that virtually every engineering economy text includes applications to student lives. This recognizes that increasing student interest and motivation increases learning and that many personal and industrial applications of engineering economy involve the same concepts.

We believe the understanding of the value of diversification in reducing risk is a core concept that should be part of engineering economy courses. It not only applies to investing where there is a mathematically rigorous framework—it also applies to engineering project portfolios. We do note, however, that strategic selection of engineering projects considering models of competition, core competencies, and diversification is not part of any introductory engineering economy text.

References

1. Rubinstein, Mark, "Markowitz's 'Portfolio Selection': A Fifty-Year Retrospective," *The Journal of Finance*, 57(3) June 2002, 1041-1045.
2. Canada, John R., William G. Sullivan, Dennis J. Kulonda, and John A. White, *Capital Investment Analysis 3rd*, Prentice Hall, 2005. Chapter 11: CAPM; Markowitz's efficient portfolio, capital allocation using lines integer programming.
3. Eschenbach, Ted, Neal Lewis, Joseph Hartman, and Lynn Bussey, *The Economic Analysis of Industrial Projects 3rd*, Oxford University Press, 2015.
4. Park, Chan S., and Gunter P. Sharp-Bette, *Advanced Engineering Economics 3rd*, Wiley, 1990.
5. Blank, Leland T., and Anthony J. Tarquin, *Engineering Economy 7th*, McGraw Hill, 2012.
6. Blank, Leland T., and Anthony J. Tarquin, *Basics of Engineering Economy*, McGraw Hill, 2008
7. Eschenbach, Ted G., *Engineering Economy: Applying Theory to Practice 3rd*, Oxford University Press, 2011.
8. Hartman, Joseph C., *Engineering Economy and the Decision-Making Process*, Prentice Hall, 2007.
9. Newnan, Donald G., Jerome P. Lavelle and, Ted G. Eschenbach, *Engineering Economic Analysis 12th*, Oxford University Press, 2014.
10. Park, Chan S., *Contemporary Engineering Economic Analysis 5th*, Prentice Hall, 2011.
11. Park, Chan S., *Fundamentals of Engineering Economics 3rd*, Prentice Hall, 2013.
12. Sullivan, William G., Elin M. Wicks, and C. Patrick Koelling, *Engineering Economy 16th*, Pearson Prentice Hall, 2015.
13. White, John A., Kenneth E. Case, and David B. Pratt, *Principles of Engineering Economic Analysis 6th*, John Wiley, 2012.
14. White, John A., Kellie S. Grasman, Kenneth E. Case, Kim LaScola Needy, and David B. Pratt, *Fundamentals of Engineering Economic Analysis*, Wiley, 2014.
15. Bodie, Zvi, Alex Kane, and Alan J. Marcus, *Investments* 10th, McGraw-Hill Irwin, 2014.
16. Benartzi, Shlomo, *Save More Tomorrow: Practical Behavioral Finance Solutions to Improve 401(k)s*, Portfolio/Penguin, 2012.
17. Nofsinger, John, *Psychology of Investing 5th*, Prentice Hall, 2014.
18. Zweig, Jason, "How the big brains invest at TIAA-CREF," *Money*, January 1998, 118.
19. Park, Chan S., and Gunter P. Sharp-Bette, *Advanced Engineering Economics 3rd*, Wiley, 1990.
20. Eschenbach, Ted, and Neal Lewis, "Exploring A Retirement Porfolio's Efficient Frontier," ISERC National Conference, Nashville, TN, June 2015.
21. Eschenbach, Ted, and Neal Lewis, "When to Start Collecting Social Security: Designing a Case Study," ASEE National Conference, San Antonio, TX, June 2012.
22. Butrica, Barbara A., Howard M. Iams, Karen E. Smith, and Eric J. Toder, "The Disappearing Defined Benefit Pension and Its Potential Impact on the Retirement Incomes of Baby Boomers," *Social Security Bulletin*, 69(3), 2009.
23. Towers Watson, "Retirement Plan Landscape Stabilizing as Fewer Fortune 500 Companies Shifting Defined Benefit Plans to 401(k)s, Towers Watson Analysis Finds," September 4, 2014. http://www.towerswatson.com/~/media/Images/Insights/Newsletters/Americas/Insider/2014/09/Towers-Watson-Insider-Retirement-in-Transition-for-the-Fortune-500-1998-to-2013-Figure-01-Large.jpg. Downloaded January 8, 2015.

24. Miller, Mark, "The vanishing defined-benefit pension and its discontents," *Reuters*, May 6, 2014. http://www.reuters.com/article/2014/05/06/us-column-miller-pensions-idUSBREA450PP20140506, Downloaded January 8, 2015.

25. Gaviria, Marcela, and Martin Smith, "The Retirement Gamble," *Frontline*, April 23, 2013. http://www.pbs.org/wgbh/pages/frontline/business-economy-financial-crisis/retirement-gamble/transcript-43/. Downloaded January 18, 2015.

26. Hirschey, Mark, and John Nofsinger, *Investments: Analysis and Behavior 2nd*, McGraw-Hill/Irwin, 2009.

27. Buffett, Warren E., and Lawrence A. Cunningham, *The Essays of Warren Buffett: Lessons for Corporate America*, 1998.

28. Brigham, Eugene F., and Phillip R. Daves, *Intermediate Financial Management 11th*, South-Western/Cengage, 2013.

29. Reilly, Frank K., and Keith C. Brown, *Investment Analysis and Portfolio Management 10th*, South-Western/Cengage, 2012.

30. Ross, Stephen A., Randolph W. Westerfield, and Bradford D. Jordan, *Essentials of Corporate Finance 8th*, McGraw-Hill, 2014.

31. Markowitz, Harry, "Portfolio Selection," *The Journal of Finance*, 7(1) (March, 1952), 77-90.