Teaching Teachers to Teach Math & Science via Engineering Activities at The University of Toledo (T^{4}MS/E)

Mark Pickett, Douglas Oliver and Sammie Giles
College of Engineering, University of Toledo

1. INTRODUCTION

One of the common threads of many of the reform efforts in secondary mathematics and science education involves an emphasis on 'hands on' team projects for science and the use of 'real-world,' yet simple applications for mathematics. In the opinion of the authors, engineers have a great deal to offer to realize both of these educational reform goals. In this paper the four engineering activities introduced are appropriate for secondary math and/or science classes.

This paper describes four projects used in a collaborative effort at the University of Toledo between engineering professors and education professors to create a successful course entitled T^4MS/E: 'Teaching Teachers to Teach Math & Science via Engineering Activities'. The course was graduate-level, targeting secondary science and mathematics teachers in local schools. The course was offered through the education college but it was taught primarily by engineering faculty. The material presented was 'hands-on' in nature and included projects where physics and engineering design may be combined in the secondary math and science curriculum.

The effort involved two semesters of work. During the first semester, participants (the math and science teachers) worked under a team of engineering faculty on a set of engineering activities which were appropriate to secondary-level physical science curricula. In the second semester, the participants worked with secondary students in a summer day-camp setting to implement the same set of activities. These secondary students came primarily from ethnic groups which are under-represented in the engineering profession.

2. BEAM BENDING

Many physical science teachers are call upon to advise a student who is constructing a light-weight sturdy structure such as a balsawood bridge or a tower for a science competition. One of the basic components of such a structure is a beam. Beam failure can occur in several modes, one of these is failure due to a bending moment. This project investigates a bending moment created by an concentrated force located at the center of a simply supported beam (Fig. 1).
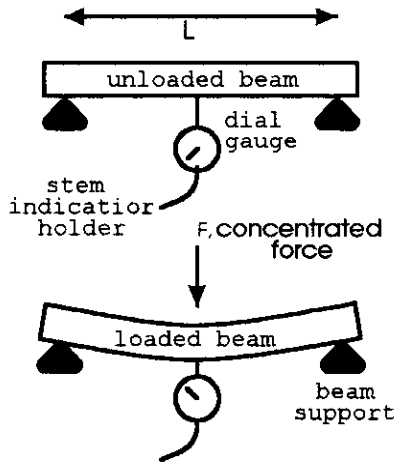
Figure 1: Beam: (top) without load, and (bottom) with load and deflection.

2.1. Theory of Beam Bending.

The position as measured by the dial gauge, y, of a beam of uniform cross section as illustrated in Fig. 1 is given by the equation[1]:

$$y = -\frac{FL^3}{48EI} + y_o \tag{1}$$

Where F is the force exerted at the center of the beam, L is the distance between beam supports, E is the modulus of elasticity of the beam, I is the second moment of area (often called by engineers 'the moment of inertia') about the neutral axis and y_o is the unloaded gauge reading. Equation 1 facilitates two simple relations depending which parameter is to be varied: a linear relation and a cubic relation.

2.2. Linear Relation.

One of the most commonly used equations in secondary mathematics courses is the slope-intercept form of a linear equation: $y = mx + b$. Beam construction and testing presents students with an opportunity to use this relation where the slope and the intercept have physical meanings which may be explored by the students.

If the supported length, L, is held constant, the maximum deflection is proportional to the applied force:

$$y = \overbrace{\left(\frac{-L^3}{48EI}\right)}^{\text{held constant}} \overbrace{F}^{\text{variable}} + y_o \tag{2}$$

---

[1]This relation may be found in most elementary texts on the mechanics of materials.

In slope-intercept format this is:

$$\underbrace{y}_{y,\,dial\,reading} = \overbrace{\left(\frac{-L^3}{48EI}\right)}^{m} \overbrace{F}^{x} + \overbrace{y_o}^{b,\,unloaded\,reading} \tag{3}$$

Students may be asked to find the value of the slope, $\frac{-L^3}{48EI}$, by plotting the gauge reading vs. the imposed force, F, and graphically obtaining the slope of the (approximate) line. Advanced students can obtain an experimental approximation for the modulus of elasticity, E, from the value of the slope with:

$$E = \left(\frac{-L^3}{48I}\right) \underbrace{\overbrace{\left(\frac{1}{m}\right)}^{inverse\,of\,the\,slope}}_{experimentally\,determined}$$

The value I, for a rectangular beam is given by:

$$I = \frac{(base)\,(height)^3}{12}$$

2.3. Cubic Relation. When working with more advanced students, Eq. (1) may be converted into a cubic relation by holding the force constant and varying the length of the beam :

$$y - y_o = \overbrace{\frac{-F}{48EI}}^{held\,constant} \overbrace{\left(L^3\right)}^{variable} \tag{4}$$

Typically, secondary science and math students are exposed to cubic relations primarily in the context of volume. This project allows for an additional cubic relation for the teacher's repertoire. If more than one value is used for the force then by plotting the net displacement, $y - y_o$, as a function of the length of the beam, a family of cubic curves result from Eq. (4).

2.4. Equipment Needed.

1. Small beams of wood or metal with a uniform cross-section.

2. Moveable beam supports.

3. Dial deflection gauge with flexible stem indicator holder[2].

4. Concentrated variable weight, this can be a simple as a plastic bucket into which weights are placed.

---

[2]These may be purchase through the ENCO cataloge: dial gauge (#6054080 - about $15) and flexible stem indicator holder (#6250355 - about $30).

3. WIND CHIMES

This section describes a method by which students can use a simple equation involving a square root to design an octave of wind chimes. This activity is described in more detail in 'Hollow-Tube Chimes' [1].

3.1. Length-Frequency Relations for a Wind Chime.
The length of a long rigid chime with a constant cross section is proportional to the inverse of the square root of the frequency:

$$L = \frac{k}{\sqrt{f}} \tag{5}$$

where k is a constant which depends on the geometric and material properties of the chime, L is the length of the chime, and f is the frequency of the chime.

The frequencies associated with the four octaves from C_4 (middle C) through C_8 are:

Note	Hz	Note	Hz	Note	Hz	Note	Hz
C_4	262	C_5	523	C_6	1047	C_7	2093
D_4	294	D_5	587	D_6	1175	D_7	2349
E_4	330	E_5	659	E_6	1319	E_7	2637
F_4	349	F_5	698	F_6	1397	F_7	2794
G_4	392	G_5	784	G_6	1568	G_7	3136
A_4	440	A_5	880	A_6	1760	A_7	3520
B_4	494	B_5	988	B_6	1976	B_7	3951
C_5	523	C_6	1047	C_7	2093	C_8	4186

Table 1: Frequencies for Four Octaves.

3.2. Experimental Determination of k.
If a value for k is known for a specific type of tube then the lengths of tubes required for particular note may be obtained using Eq. 5 and Table 1. A value for k, for a specific tube may be experimentally obtained by cutting a length of tube about 500 mm in length. Two small holes should be drilled at the top of the tube for a string to be passed through (Fig. 2).

The tone of the chime should be determined by holding the chime by the string, then striking the center of the chime with a small hammer (table knives work well). Then compare the pitch of the chime with various keyboard tones to identify which tone it is nearest. This step requires an 'ear' for musical pitches[3].

Once you have identified a length of tube which corresponds to a standard tone, the value of k may be determined. For example, if a chime with a length of 508 mm.

---

[3]You may do this even if you do not have a good 'ear' for pitches. If you are off pitch, the chimes will sound good by themselves, but not when accompanied by another instrument.

Strings

For best results, the strings
should not touch the chimes
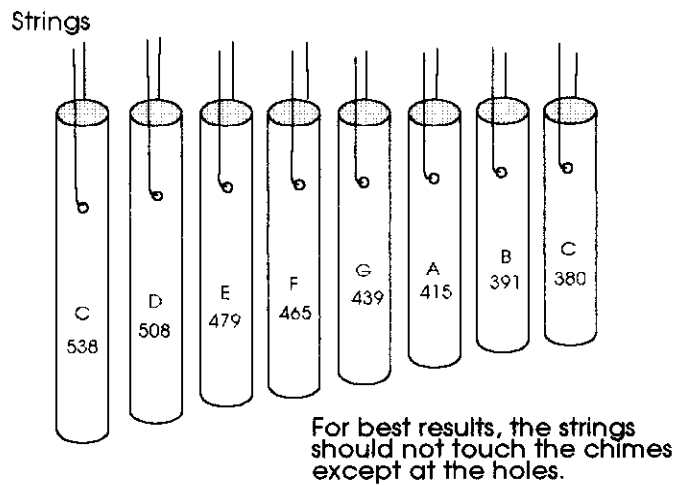except at the holes.

Figure 2: Example of a one octave of whole notes with lengths in millimeters. Your lengths may differ from those illustrated.

has a pitch which is equivalent to D_6 (1175 Hz) on the piano, the value of k for the tube may be computed from Eq. 5:

$$
\begin{aligned}
k &= L\sqrt{f} \\
&= 508\,mm\sqrt{1175Hz} \\
&= 17,413\,(mm \cdot Hz^{1/2})
\end{aligned}
$$

When k is known, the appropriate lengths for each chime may be calculated using Table 1. For example, with $k = 17,413\,(mm \cdot Hz^{1/2})$ the length for C_6 (1047 Hz) is:

$$
\begin{aligned}
L_{C_6} &= \frac{k}{\sqrt{f}} = \frac{17,413\,(mm \cdot Hz^{1/2})}{\sqrt{1047\,Hz}} \\
&= 538\,mm.
\end{aligned}
$$

Approximate chime lengths for the other whole notes in the C_6 octave are illustrated in Fig. (2).

It should be noted that the frequency which you 'hear' will not necessarily be the primary or lowest frequency. This is in part due to the fact that most people hear best frequencies near four kilohertz (4 kHz).

3.3. Equipment Needed.

1. Lengths of 1/2 inch electrical conduit tubes (available in most hardware or building supply stores),

2. Metric measuring tape,

3. Electric hand drill, hacksaw, and file or grinder,

4. tuned piano or other keyboard instrument,

5. Thin nylon string and a permanent felt-tip marker.

4. FLASHLIGHT CONSTRUCTION AND LOGIC CIRCUITS

At first glance, construction of a simple flashlight seems to be a trivial matter. However, it was found by Brown, Slater, and Adams [2, Gender Differences ...] that a major segment of college-level physics students could not combine a wire, battery, and light bulb correctly to produce light. In addition, it was found that female students tended to have significantly more difficulty in this task than did male students.

4.1. Logic Circuits.

In the T^4MS/E course, construction of a simple flashlight was used as the first step in constructing several logic circuits such as AND and OR logic circuits. The construction of flashlights is a valuable exercise for many secondary students. However, the expansion into logic circuits should be limited to more advanced students.

Even though logical circuits form a cornerstone of the digital revolution of the past few decades, few high school students have the opportunity to work with basic logic circuits. In this case, students not only work with logic circuits, but build them from simple components.

At this point, a quick review of the AND and OR logic gates is in order. Both the AND and the OR gates have two binary inputs. Each of these may be either on or off. The AND gate is off unless both of the inputs are on:

AND gate logic

$$\left.\begin{array}{l}\text{off}\\\text{off}\end{array}\right\} \Rightarrow \text{off}, \quad \left.\begin{array}{l}\text{off}\\\text{on}\end{array}\right\} \Rightarrow \text{off}, \quad \left.\begin{array}{l}\text{on}\\\text{off}\end{array}\right\} \Rightarrow \text{off}, \quad \left.\begin{array}{l}\text{on}\\\text{on}\end{array}\right\} \Rightarrow \text{on}.$$

The OR gate is on unless both of the inputs are off:

OR gate logic

$$\left.\begin{array}{l}\text{off}\\\text{off}\end{array}\right\} \Rightarrow \text{off}, \quad \left.\begin{array}{l}\text{off}\\\text{on}\end{array}\right\} \Rightarrow \text{on}, \quad \left.\begin{array}{l}\text{on}\\\text{off}\end{array}\right\} \Rightarrow \text{on}, \quad \left.\begin{array}{l}\text{on}\\\text{on}\end{array}\right\} \Rightarrow \text{on}.$$

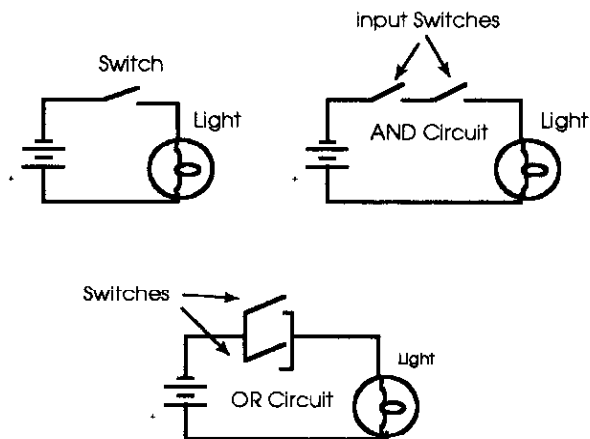Schematics for simple AND and an OR logic circuit are illustrated in Fig. (3).

Figure 3: Schematic for a flashlight as well as AND and OR circuits.

4.2. Equipment Needed.

1. Bell wire with wire cutters,

2. Batteries with holders or connectors,

3. two switches,

4. a small light appropriate for the battery voltage.

5. RUBBER-BAND AIRPLANE CONTEST

Designing, building, and testing model planes is an excellent 'hands-on' method for demonstrating how experimentation coupled with mathematics and physics can be used to produce an improved product. This section describes a model airplane design competition which may be used as a group project as a component of a physical science class or as a unit of a science club at a junior high school. An airplane competition allows a secondary science teacher to incorporate: teamwork, trial-and-error analysis, as well as mathematical skills. It allows those students who are skilled with their hands, to work successfully in teams with those who are more theoretically inclined.

The students are supplied with an off-the-shelf balsa-wood airplane with a rubber band driven propeller[4]. Their task is to alter the design of the airplane so that it

---

[4]We use a model #55 Guilliow's 'Jetstream' which costs under $2.00 each. Local dealers may be obtained from the manufacturer: Paul K. Guillow, Inc. of Wakefield MA 01880.

can carry cargo, such as pennies, over short distances. Design suggestions are given in [3, Rubber-Band] and are not repeated here in detail.

The lift force is what allows a plane to stay aloft in the air. The lift supplied by a wing is approximated by the following equation:

$$Lift = \left(\frac{\rho}{2}\right) \overbrace{\left(C_L V^2 A\right)}^{\text{design parameters}} \tag{6}$$

where

- ρ is the density of the air,

- C_L is the coefficient of lift,

- V is the velocity of the plane, and

- A is the area of the wing, as see from above the plane.

Students may treat any of the variables except ρ, as a design variable to enhance the performance of their planes.

Suggested Contest Rules

Students are to work in small groups (of two or three) to design, test, and fly a rubber band driven airplane which carries pennies. Each team will have three tries, the scores for each try will be summed. The students will be scored on the number of penny-meters the plane carried aloft for each flight. For example, if a plane carried two pennies and was aloft for 3.5 meters, the score would be seven points; (2 pennies ×3.5 meters = 7 penny-meters).

- Distances are measured as the linear distance from the point of takeoff to the point at which the plane landed.

- All of the energy imparted to the plane must come from a single rubber band, through a single propeller, in the spirit of a propeller driven plane. No sling-shot type devices should be allowed. The students should not allowed to push or throw the planes.

- All planes are to be launched from ground level.

For less advanced students, it may be wise to leave out the cargo (the pennies) and limit the competition to simply the farthest distance aloft.

6. Biographical Information

Dr. Mark Pickett is an associate professor of civil engineering at the University of Toledo. He is a former naval submarine officer where he was an assistant nuclear engineer. He has developed a keen interest in bringing engineering principles into the secondary science classroom, especially through 'hand-on' activities.

Dr. Douglas Oliver is an associate professor of mechanical engineering at the University of Toledo. He is also a certified math and physics teacher who has served as a secondary school teacher in such diverse settings as Alaska and Colombia South America. He enjoys finding classroom activities which bridge the gap between mathematics and physics.

Dr. Sammie Giles is an associate professor of electrical engineering. He has degrees in both physics and electrical engineering. Dr. Giles has a strong interest in expanding the number of African-American students who successfully complete degrees in engineering.

Acknowledgment

References

[1] Oliver, D.L.R, 'Hollow-Tube Chimes', The Physics Teacher, vol. 36, April 1998, pp. 209-210.

[2] T. Brown, T. Slater, and J. Adams, 'Gender Differences with Batteries and Bulbs,' The Physics Teacher, vol. 36, December 1998, pp. 526-527.

[3] Oliver, D and T. Ng, 'Rubber-Band Driven Airplane Contest' The Physics Teacher, vol. 37, February 1999, pp. 108-109.